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DURELL'S ARITHMETIC

BOOK THREE

BY

FLETCHER DURELL, PH.D.
HEAD OF THE MATHEMATICAL DEPARTMENT
THE LAWRENCEVILLE SCHOOL

NEW YORK
CHARLES E. MERRILL COMPANY
1912

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PREFACE

THE main purpose in writing this series of Arithmetics has been to give the subject a broader and more vital practical meaning. By special treatment of the concrete applications of arithmetic, the series develops bread-winning power; and by cultivating a grasp of the functions of number processes, it develops a broad, cultural power.

Among the features which contribute to these ends are the *Industry Reviews*, which treat of the arithmetical element in representative industries, as agriculture, mining, and forestry. The *permanent number facts* in geography, history, physics, and other departments have been collected in tables and used repeatedly in examples. These features give a *correlation* of arithmetic with other studies.

The systematic development of principles is a primary aim of the series. All informational matter has been introduced in such a way as not to break up the regular formal development of principles. For instance, the Industry Reviews are each placed at the end of a chapter.

The all-important subject of *review* work has received special attention. In many places the examples call for both written and oral review of important parts of the preceding year's work. This repeated review of preceding work not only gives a more thorough mastery of processes, but in large measure relieves the teacher of the labor of dictating supplementary examples. If the prescribed drills and reviews are followed, practically all

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the pupils using these books should become *rapid and accurate computers.*

Special mention should be made also of those exercises which develop a grasp of principles of *efficiency and economy*, and thus prepare pupils, not only for efficient business management, but for efficient life in general.

Book Three is designed for use in Grades VII and VIII under ordinary circumstances. To suit the greater mental grasp of pupils in these grades, the treatment is mainly topical.

To avoid the repetition of material with which the pupil is already familiar, and thus the better to sustain the pupil's interest, the treatment in the early chapters is condensed largely into review exercises. The teacher, in certain cases, will find it advisable still further to condense this review work.

Special attention has been given to embodying in the book the latest business practice. See, for example, the treatment of Trade, Transportation, and Classification of Freights (pp. 235–238).

A large number of Industry Review exercises have been given. Six of these, for instance, relate to different aspects of agriculture. It is expected that each teacher will emphasize such of these Industry Reviews as are important in the community where the book is being used, and will abbreviate or omit others.

The author wishes to express his obligation to Miss Elizabeth Hall, Supervisor of the Primary Grades in the public schools of Minneapolis, and to James J. Sheppard, Principal of the High School of Commerce, New York City, for their important suggestions concerning the manuscript of the book.

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11. Find the sum of the areas of all the states in the United States which border on the Atlantic Ocean. (See p. 383.) On the Great Lakes.

12. On the Gulf of Mexico. On the Pacific Ocean.

13. The receipts of a dealer in lumber, coal, lime, and fertilizers for the days of a week were as follows:

	M.	T.	W.	Th.	F.	S.	Totals
Lumber	$120.50	$87.42	$101.37	$92.95	$16.72	$327.18	
Coal	72.40	27.60	127.32	18.72	39.37	116.42	
Lime	17.40	32.80	16.24	100.00	36.40	12.80	
Fertilizers	162.43	39.27	8.76	48.72	19.24	30.16	
Totals							

Find the total amount received on each day of the week; also the sum of these amounts. Then the total receipts during the week for each article, and hence the total for all articles.

14. Make up and work a similar example concerning the number of pounds of milk received at a creamery from six different farms during a week.

EXERCISE 4

SUBTRACTION

	1.	2.	3.	4.	5.
From	4705	13,756	21,504	34,576	28,765
take	3846	8,607	8,476	16,268	9,175

	6.	7.	8.
From	7,265,432	301,705,401	2,706,510,547,308
take	3,514,765	170,643,053	1,607,432,813,954

9. Find the difference between 58,000 and 31,762.

10. How many dollars must be added to $7489 to make $8516?

11. The distance from London to Bombay by the Cape of Good Hope is 11,220 miles. By the Suez Canal the distance is 6332 miles. How many miles does the Suez Canal save for a person traveling between London and Bombay?

12. The moon's distance from the earth varies at different times, the greatest distance being 252,830 miles, and the least 221,520 miles. Find the difference.

Fill in the missing number in each of the following:

13.	**14.**	**15.**	**16.**	**17.**
6278	$37.62	$38.92	$300.00	$3462.50
5923	276.82	56.72	592.76	598.76
8716	39.72	384.76	87.43	1543.24
127	104.27	91.82	592.74	376.84
3106	59.23	472.50	86.92	501.50
2017	117.23	600.00	519.46	384.76
150	32.18	72.67	1723.72	123.46
47629	1000.00	2174.76	4567.84	20000.00

EXERCISE 5

ORAL ADDITION AND SUBTRACTION

Add at sight:

1. 64 31	**4.** 90 57	**7.** 75 29	**10.** $1.50 1.20	**13.** 300 209
2. 87 52	**5.** 62¢ 27¢	**8.** 75 49	**11.** 532 311	**14.** 300 219
3. 60 22	**6.** 50 in. 28 in.	**9.** 100 72	**12.** $246 115	**15.** $300 195

16. At sight, subtract in Exs. 1–15.

17. Let the class practice making change, after the teacher or a pupil has written on the blackboard the amounts of various possible purchases and sums paid. Such data may also be given orally.

EXERCISE 6

MULTIPLICATION

Multiply and check:

1. $760	**2.** $809	**3.** 828	**4.** 891	**5.** 3508
5	26	567	635	983

Find the product in each of the following:

6. 32,071 × 50

7. 420 × 76,203

8. 93,761 × 305

9. 56,714 × 270

10. 508 × 56,702

11. 67,094 × 1320

12. 76,543 × 2045

13. 3029 × 67,038

14. 1432 × 67,563

15. 76,805 × 7503

16. 80,984 × 3370

17. 3502 × 66,894

18. If it costs $23,562 to build a single mile of railroad, what will it cost to build 273 miles at the same rate? 301 miles? 999 miles? (Can you think of a short way of making the last computation?)

19. By how much does the area of Alaska exceed twice that of Texas? By how much does the area of Texas exceed 5 times that of New York State? (See p. 383.)

EXERCISE 7

DIVISION

Divide and check:

1. 3)426 in.

2. 2)590 ft.

3. 4)30248

4. 5)16785

5. 8)301376

6. 8)579752

7. 11)345631

8. 11)499037

9. 12)494820

10. 2530 tons by 55

11. 6300 hours by 75

12. 7990 by 94

13. 3166 by 42

14. 17,152 by 67

15. 29,602 by 82

16. 51,428 by 92

17. 400,189 by 187

18. 106,953 by 231

19. 377,289 by 927

20. 802,392 by 998

21. 690,720 by 708

22. 8,010,555 and 17,266,376 each by 2461

23. 27,300 by 350

24. 37,800 by 1400

25. 1,663,200 by 5400

26. 3,056,000 by 3000

27. If in going from the surface of the earth to the bottom of a mine 2666 feet deep, the temperature increases 43 degrees, how many feet of depth make an increase of 1 degree, on the average?

28. In the year 1800 the number of bales of cotton produced in the United States was 155,556. In the year 1910 the number of bales was 12,132,000. The crop in 1910 was how many times as great as the crop in 1800?

EXERCISE 8

In each of the following, first state the order of operations; then make the computation:

1. $7 \times 8 + 4$

2. $12 \times 9 - 5 \times 4$

3. $18 + 17 \times 19 + 16 \times 3$

4. $18 \times 9 + 64 \div 8$

5. $7 \times 6 + 63 \div 9$

6. $9 + 40 \times 2 - 72 \div 3$

7. $80 + 121 \div 11 - 90$

8. $196 \div 14 + 348 \div 58$

9. How many examples in this Exercise can you work at sight?

EXERCISE 9

ORAL REVIEW

1. At sight, state which of the following products is greater:

(*1*) 101×7, or 999×3

(*2*) 1001×9, or 9999×5

(*3*) 998×10, or 1002×8

(*4*) 3276×5, or 992×4

2. How many figures will there be in each of the following products?

(*1*) 203 × 102
(*2*) 315 × 201
(*3*) 1005 × 35
(*4*) 2100 × 51
(*5*) 3076 × 107
(*6*) 99 × 2000

Also state the highest order of units in each of these products.

3. A farmer has 981 bushels of wheat which he expects to sell for 98¢ a bushel, and he wishes to know whether the receipts will enable him to pay a note of $1150. Can you tell him without doing any written work?

4. Will two tons of feed at 5¢ a pound cost $20 or $200?

5. If New Jersey has an area of 7815 miles, is its area greater or less than that of a rectangle 50 × 150 miles? One 60 × 140 miles?

6. Make up and work a similar example concerning the area of your own state.

7. If the breadth of the United States from east to west on the average is 3000 miles, and from north to south is 1000 miles, how many square miles are there in its area?

8. A church has 120 pews each of which will seat 5 persons, and 50 pews each of which will seat 7 persons. What is the seating capacity of the whole church?

1. **Tests of Divisibility.** We have the following aids or tests in trying to factor a given number:

A number is divisible by 2, *if the last figure is an even number or* 0;
By 3, *if the sum of the figures is divisible by* 3;
By 4, *if the number expressed by its last two figures is divisible by* 4;
By 5, *if the last figure is* 0 *or* 5;
By 8, *if the number expressed by its last three figures is divisible by* 8;
By 9, *if the sum of the figures is divisible by* 9.

Thus, 7554 is divisible by 2 since 4 is.

It is also divisible by 3 since the sum of its figures, 21, is divisible by 3. Write a number containing seven figures which is divisible by 4. Another number of seven figures divisible by 5. Another by 8. Another by 9.

EXERCISE 10

Factoring

1. Make a list of the prime numbers between 1 and 100.

Find the prime factors of

2. 50 56 60 72 64
3. 80 84 88 92 98 105
4. 108 112 124 128
5. 136 148 150 165
6. 224 396 480 600
7. 842 873 919 960
8. 1315 1599 3003 2145
9. 3696 4081 12 121

Tell by inspection whether or not each of the following numbers is divisible by 2, 3, 4, 5, 6, 8, 9, 10, or 20:

10 120 130 140 156

11. 171 217 240 498

12. 3428 7653 9345

13. 76,532 97,605 123,456

14. Write a number containing five figures and divisible by 3. By 9. By 5. By 4.

15. In the following numbers, supply a figure in place of the * so as to make each number divisible by 3:

(*1*) 32*5 (*2*) 573* (*3*) *2893 (*4*) 60*276

16. Also in Ex. 15 supply a figure to make each number divisible by 9.

EXERCISE 11

Cancellation

In each of the following, first put the work in a form ready for the use of cancellation and then compute:

1. $32 \times 15 \div 20$
2. $128 \times 30 \div 60$
3. $560 \div (64 \times 35)$
4. $18 \times 15 \div (30 \times 3)$
5. $16 \times 40 \div (20 \times 16)$
6. $24 \times 30 \times 10 \div (40 \times 60)$
7. $(32 + 16) \div (27 \times 12)$
8. $64 \div 20 \times 30 \div 24$

Reduce:

9. $\frac{6 \times 10}{20}$

10. $\frac{24}{3 \times 2}$

11. $\frac{8 \times 15}{20 \times 3}$

12. $\frac{24 \times 25}{75}$

13. $\frac{56 \times 27}{9 \times 42}$

14. $\frac{65 \times 84}{26 \times 25}$

15. $\frac{144 \times 125}{100 \times 12}$

16. $\frac{88 \times 105}{20 \times 77}$

17. $\frac{216 \times 343}{98 \times 42}$

18. $\frac{5 \times 6 \times 20}{4 \times 15 \times 2}$

19. $\frac{7 \times 9 \times 16}{8 \times 14 \times 3}$

20. $\frac{17 \times 20 \times 27}{9 \times 34 \times 5}$

21. $\frac{48 \times 68 \times 77}{84 \times 187}$

22. $\frac{60 \times 75 \times 81}{45 \times 50 \times 54}$

23. $\frac{90 \times 96 \times 98}{168 \times 630}$

24. $\frac{205 \times 666 \times 18}{81 \times 185 \times 84}$

25. $\frac{384 \times 162 \times 275}{216 \times 200 \times 99}$

26. Divide 16 × 18 × 24 × 30 by 45 × 32 × 72.

27. Divide 60 × 70 × 85 × 96 by 42 × 125 × 64.

28. Divide 128 × 132 × 150 by 275 × 48 × 84.

29. Divide 345 × 396 × 425 by 187 × 276 × 375.

30. Reduce:

(*1*) $\frac{12 + 4}{36 \times 10}$ (*2*) $\frac{16 \times 150}{20 \times 3}$ (*3*) $\frac{5 \times 18 + 8}{28}$ (*4*) $\frac{7 \times 18}{3 \times 14}$

31. If 48 apple trees yield 168 bu. of apples, how many bushels should 600 trees yield?

32. Make up and work a similar example concerning the yield of peach trees.

33. If a farmer exchanges 14 tons of hay worth $15 a ton for coal worth $6 a ton, how many tons of coal will he receive?

34. If 12 horses cost $2700, what will 14 horses cost?

35. 8 recitation periods of 45 minutes each are equal to how many periods of 40 minutes? To how many periods of one hour each?

36. If 1 bu. of wheat weighs 60 lb., find the cost of 8600 lb. of wheat at 90 cents a bushel.

37. Find the cost of 6884 lb. of oats at 40 cents a bushel (32 lb.).

38. Find the cost of 738 eggs at 32 cents a dozen.

39. If one toad can keep a piece of ground 50 ft. long and 12 ft. wide free from insects, how many toads will protect from insects a piece of ground 450 ft. long and 300 ft. wide?

40. How many watermelons can be raised on an acre, if the hills are 8 ft. apart one way and 6 ft. apart the other way, and each hill yields 10 melons?

41. A girl working 5 hours a day for 18 days, picked 110 qt. of blackberries, which she sold at 9 cents a quart. How much did she make per hour?

EXERCISE 12

ORAL

1. If 3 lb. of grapes cost 20¢, what will 9 lb. cost? 6 lb.? 12 lb.? 15 lb.?

2. If 4 lead pencils cost 25¢, what will 8 lead pencils cost? 12 pencils? 24 pencils?

3. If 6 car tickets cost 25¢, what will 12 car tickets cost? 24 tickets? 18 tickets?

4. If 2 cans of peas cost 25¢, what will 6 cans cost? 4 cans? 12 cans? 18 cans?

5. If 12 lemons cost 25¢, what will 36 lemons cost? 60 lemons? 4 doz. lemons? 6 doz.?

6. We have 42 apple trees in our orchard. I have gathered 11 bu. of apples from 3 trees. At the same rate, how many bushels will I gather from all the trees? From one half of them?

7. If 100 oysters cost 80¢, what will 50 oysters cost? 75 oysters?

8. If 6 oranges cost 15¢, what will 4 oranges cost?

9. If 4 pads of paper cost 25¢, how many pads can be bought for 50¢? For $1? For $2? For $2.50?

10. If 3 cans of corn cost 25¢, how many cans can be bought for 50¢? For $1? For 75¢?

11. If 3 ft. of wire cost 10¢, how many feet can be bought for 20¢? For 50¢? For $1? For $1.50?

12. If 3 spools of thread cost 20¢, how many spools can be bought for 40¢? For $1? For $2?

EXERCISE 13

G. C. D. AND L. C. M.

1. On the foot rule show what is meant by the G. C. D. of 4 in. and 6 in. By their L. C. M.

2. Find the G. C. D. of 84, 126, 210.

$$\begin{array}{l} 2\underline{)84,\ 126,\ 210} \\ 3\underline{)42,\ \ 63,\ 105} \\ 7\underline{)14,\ \ 21,\ \ 35} \\ \quad\ 2,\ \ \ 3,\ \ \ 5 \end{array} \qquad 2 \times 3 \times 7 = 42, \text{ G. C. D.}$$

Find the G. C. D. of the following groups of numbers:

3. 18, 42

4. 24, 60

5. 90, 198

6. 30, 42, 72

7. 32, 56, 88

8. 180, 144, 198

9. 24, 40, 56, 104

10. 40, 70, 90, 150

11. Find the L. C. M. of 14, 20, 21, 35.

$$\begin{array}{l} 2\underline{)14,\ 20,\ 21,\ 35} \\ \ 7\underline{)7,\ 10,\ 21,\ 35} \\ \ 5\underline{)1,\ 10,\ \ 3,\ \ 5} \\ \quad\ \ 1,\ \ 2,\ \ 3,\ \ 1 \end{array} \qquad 2 \times 7 \times 5 \times 2 \times 3 = 420, \text{ L. C. M.}$$

12. Find the L. C. M. in Exs. 2–9.

EXERCISE 14

REDUCTION OF FRACTIONS

Reduce the following mixed numbers to equivalent improper fractions:

1. $4\frac{1}{4}$ in.
2. $7\frac{2}{3}$ ft.
3. $7\frac{5}{9}$ rd.
4. $16\frac{4}{5}$ yr.
5. $3\frac{1}{7}$
6. $45\frac{1}{8}$
7. $17\frac{14}{15}$
8. $310\frac{4}{5}$
9. $\$65\frac{7}{8}$
10. $\$73\frac{1}{10}$
11. $270\frac{3}{5}$
12. $111\frac{19}{20}$

Reduce the following improper fractions to equivalent mixed numbers:

13. $\frac{48}{7}$ qt.
14. $\frac{59}{8}$ mi.
15. $\frac{75}{4}$ in.
16. $\frac{124}{5}$ day
17. $\frac{163}{7}$ wk.
18. $\frac{176}{9}$ gal.
19. $\frac{249}{12}$ ft.
20. $\$\frac{293}{10}$
21. $\$\frac{459}{20}$
22. $\$\frac{877}{15}$
23. $\$\frac{889}{25}$
24. $\$\frac{756}{11}$
25. $\frac{694}{13}$
26. $\frac{545}{17}$
27. $\frac{728}{27}$
28. $\frac{598}{33}$

Reduce each of the following fractions to its lowest terms:

29. $\frac{30}{54}$
30. $\frac{35}{63}$
31. $\frac{72}{88}$
32. $\frac{54}{99}$
33. $\frac{60}{70}$
34. $\frac{70}{98}$
35. $\frac{90}{105}$
36. $\frac{96}{176}$
37. $\frac{228}{950}$
38. $\frac{2310}{3570}$

Reduce:

39. $\frac{1}{3}$ to 45ths.

40. $\frac{7}{12}$ and $\frac{5}{18}$ each to 36ths.

41. I owned a farm of 240 acres and sold 200 acres of it. What fraction of the farm did I sell? What fraction of it did I have left?

42. In a book containing 324 pages a girl has read 243 pages. What fraction of the book has she read? Which is easier to picture to yourself: $\frac{243}{324}$ or $\frac{3}{4}$?

43. If the distance from San Francisco to Hawaii is 2040 miles and that from San Francisco to Manila is 4850 miles, the first distance is what fraction of the second?

44. How many examples in this Exercise can you work at sight?

EXERCISE 15

ADDITION OF FRACTIONS

1. Add $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{8}$.

$$\frac{2}{3} + \frac{5}{6} + \frac{3}{8} = \frac{16}{24} + \frac{20}{24} + \frac{9}{24} = \frac{45}{24} = \frac{15}{8} = 1\frac{7}{8} \textit{ Sum}$$

Add:

2. $\frac{3}{4}$ and $\frac{1}{6}$

3. $\frac{4}{9}$ and $\frac{5}{6}$

4. $\frac{7}{12}$ and $\frac{8}{15}$

5. $\frac{3}{5}, \frac{4}{15}, \frac{1}{3}$

6. $\frac{7}{8}, \frac{5}{6}, \frac{1}{12}$

7. $\frac{1}{4}, \frac{1}{6}, \frac{17}{18}$

8. $\frac{9}{10}, \frac{7}{15}, \frac{1}{3}$

9. $\frac{5}{12}, \frac{11}{15}, \frac{7}{20}$

10. $\frac{8}{15}, \frac{7}{18}, \frac{13}{30}$

11. $\frac{2}{3}, \frac{4}{5}, \frac{7}{30}, \frac{9}{10}$

12. $\frac{4}{9}, \frac{5}{12}, \frac{11}{27}, \frac{23}{36}$

13. $\frac{7}{8}, \frac{9}{10}, \frac{2}{3}, \frac{5}{6}, \frac{11}{15}$

Find the sum of each of the following in the shortest way:

14. $\frac{1}{3} + \frac{1}{7}$

15. $\frac{1}{5} + \frac{1}{8}$

16. $\frac{1}{3} + \frac{1}{13}$

17. $\frac{2}{5} + \frac{3}{8}$

18. $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

19. $\frac{4}{5} + \frac{1}{2} + \frac{1}{3}$

20. $\frac{3}{4} + \frac{1}{5} + \frac{1}{4} + \frac{3}{5} + \frac{1}{5}$

21. $\frac{1}{2} + \frac{1}{6} + \frac{1}{4} + \frac{3}{4} + \frac{1}{2}$

22. $\frac{1}{6} + \frac{1}{4} + \frac{5}{6} + \frac{3}{4}$

23. $\frac{1}{2} + \frac{1}{8} + \frac{3}{8} + \frac{5}{8} + \frac{1}{4}$

24. Add $11\frac{5}{8}$, $20\frac{7}{12}$, and $6\frac{1}{2}$.

$$\begin{array}{r|l} & 24 \text{ L. C. D.}\\ 11 & 15\\ 20 & 14\\ 6 & 12\\ \hline 37 & \frac{41}{24} = 38\frac{17}{24} \textit{ Ans.}\end{array}$$

The sum of the fractions is $\frac{41}{24}$, or $1+\frac{17}{24}$. The sum of the whole numbers is 37. Hence, the final sum is $37 + 1 + \frac{17}{24}$, or $38\frac{17}{24}$.

Find the value of

25. $3\frac{2}{3} + 2\frac{1}{2}$

26. $4\frac{5}{6} + 2\frac{3}{4}$

27. $5\frac{1}{5} + 7\frac{3}{10}$

28. $6\frac{3}{8} + 2\frac{5}{12}$

29. $4\frac{5}{9} + 4\frac{2}{15}$

30. $7\frac{5}{6} + 1\frac{2}{3}$

31. $5\frac{11}{12} + 6\frac{17}{18}$

32. $8\frac{5}{24} + 1\frac{7}{36}$

33. $1\frac{2}{7} + 3\frac{5}{14} + 7\frac{1}{2}$

34. $3\frac{4}{5} + 4\frac{5}{6} + 6\frac{7}{10}$

35. $8\frac{7}{12} + 5\frac{4}{15} + 10\frac{9}{20}$

36. $3\frac{1}{5} + 2\frac{1}{3} + 5\frac{1}{6}$

37. How many examples in this Exercise can you work at sight?

EXERCISE 16

SUBTRACTION OF FRACTIONS

1. Subtract $\frac{3}{8}$ from $\frac{7}{12}$.

$$\tfrac{7}{12} - \tfrac{3}{8} = \tfrac{14}{24} - \tfrac{9}{24} = \tfrac{5}{24}\ \textit{Ans.}$$

Subtract:

2. $\frac{2}{3}$ from $\frac{5}{6}$
3. $\frac{5}{9}$ from $\frac{7}{12}$
4. $\frac{8}{21}$ from $\frac{9}{14}$
5. $\frac{7}{25}$ from $\frac{29}{30}$
6. $\frac{3}{35}$ from $\frac{11}{60}$
7. $\frac{6}{25}$ from $\frac{14}{15}$

Find the value of

8. $\frac{6}{7} - \frac{1}{3}$
9. $\frac{4}{9} - \frac{5}{24}$
10. $\frac{18}{35} - \frac{8}{25}$
11. $\frac{5}{18} - \frac{4}{81}$
12. $\frac{61}{64} - \frac{55}{72}$
13. $\frac{47}{120} - \frac{23}{75}$

14. A board is $\frac{7}{8}$ in. thick. How much of it must be planed off to make it $\frac{3}{4}$ in. thick?

15. If a man owns $\frac{3}{64}$ of a sailing vessel, how much would he need to buy in order to own $\frac{1}{2}$ of the vessel?

16. Subtract $12\frac{2}{3}$ from $17\frac{1}{4}$.

$$\begin{array}{rcrcr} 17\frac{1}{4} & = & 17\frac{3}{12} & = & 16\frac{15}{12} \\ 12\frac{2}{3} & = & 12\frac{8}{12} & = & \underline{12\frac{8}{12}} \\ & & & & 4\frac{7}{12}\ \textit{Difference} \end{array}$$

Find the value of

17. $3\frac{1}{5} - 2\frac{1}{2}$
18. $6\frac{1}{5} - 2\frac{1}{3}$
19. $8\frac{1}{4} - 5\frac{4}{5}$
20. $2\frac{5}{12} - 1\frac{11}{18}$
21. $4\frac{2}{3} - 3$
22. $6\frac{1}{16} - 2\frac{1}{2}$
23. $3 - \frac{1}{2}$
24. $3 - 1\frac{1}{3}$
25. $14 - 5\frac{5}{9}$
26. $13\frac{6}{7} - 9\frac{3}{5}$
27. $25\frac{11}{24} - 8\frac{13}{16}$
28. $15\frac{19}{1000} - 7\frac{17}{100}$

29. $\frac{3}{4} + \frac{1}{2} - \frac{1}{3}$
30. $\frac{1}{5} + \frac{3}{8} - \frac{3}{10}$
31. $\frac{2}{3} - \frac{1}{6} + \frac{4}{9}$
32. $1\frac{1}{2} - \frac{2}{3} + \frac{5}{6}$
33. $2\frac{1}{6} + 1\frac{1}{2} + 2\frac{1}{3}$
34. $\frac{3}{5} + 7\frac{1}{4} - 5\frac{3}{10}$
35. $2\frac{1}{2} + 3\frac{2}{3} - 3\frac{3}{4}$
36. $7\frac{1}{3} - 2\frac{1}{2} - 4\frac{1}{6}$
37. $8\frac{2}{9} - 3\frac{1}{3} + 2\frac{5}{6}$

38. How many examples in this Exercise can you work at sight?

EXERCISE 17

MULTIPLICATION OF FRACTIONS

1. Multiply $\frac{5}{6}$ by $\frac{4}{7}$.

$\frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{7} = \frac{10}{21}$ *Product*

For $\frac{1}{6} \times \frac{1}{7} = \frac{1}{42}$.
Hence, $\frac{1}{6}$ of $\frac{4}{7} = 4$ times $\frac{1}{6} \times \frac{1}{7}$, or $\frac{4}{42}$.
And $\frac{5}{6}$ of $\frac{4}{7} = 5$ times $\frac{4}{42}$ or $\frac{20}{42}$, or $\frac{10}{21}$.

Find the value of

2. $\frac{3}{5} \times 4$
3. $\frac{4}{7} \times 14$
4. $10 \times \frac{3}{16}$
5. $12 \times \frac{5}{21}$
6. $\frac{15}{44} \times 33$
7. $\frac{3}{7} \times \frac{2}{3}$
8. $\frac{4}{9} \times \frac{1}{2}$
9. $\frac{7}{11} \times \frac{3}{5}$
10. $\frac{5}{6} \times \frac{12}{25}$
11. $\frac{6}{7} \times \frac{21}{22}$
12. $\frac{48}{49} \times \frac{63}{64}$
13. $\frac{3}{5} \times \frac{4}{9} \times \frac{15}{16}$
14. $\frac{14}{15} \times \frac{20}{21} \times \frac{9}{16}$
15. $\frac{20}{49} \times \frac{63}{70} \times \frac{7}{18}$
16. $\frac{72}{121} \times \frac{55}{56} \times \frac{77}{90}$

17. If a man owned $\frac{3}{16}$ share in an oil well and sold $\frac{3}{4}$ of what he owned, what fraction of the well did he sell? What fraction did he have left?

18. If each turn of a screw advances the screw $\frac{3}{16}$ inch into a piece of wood, how far will $\frac{1}{2}$ of a turn advance the screw? $\frac{3}{4}$ of a turn? $\frac{2}{3}$?

19. Multiply $2\frac{3}{4}$ by $5\frac{1}{3}$.

$$2\frac{3}{4} \times 5\frac{1}{3} = \frac{11}{\cancel{4}} \times \frac{\overset{4}{\cancel{16}}}{3} = \frac{44}{3} = 14\frac{2}{3} \text{ \textit{Product}}$$

Find the value of

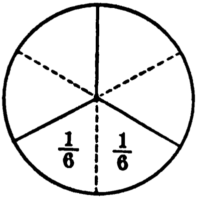
20. $10 \times 4\frac{3}{11}$
21. $28 \times 7\frac{5}{6}$
22. $2\frac{1}{2} \times 4\frac{2}{5}$
23. $6\frac{1}{4} \times 1\frac{1}{15}$
24. $8\frac{5}{9} \times 5\frac{8}{11}$
25. $20\frac{4}{13} \times \frac{39}{44}$

26. Work at sight as many of the examples in this Exercise as you can.

EXERCISE 18

Division of Fractions

1. Draw a circle and divide it into 3 equal parts by lines drawn from the center. Draw other lines from the center to show that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.



Hence, show that $\frac{1}{3} \div 2 = \frac{1}{3 \times 2} = \frac{1}{6}$.

Draw other dotted lines to show that

$$\frac{1}{3} \div 4 = \frac{1}{3 \times 4} = \frac{1}{12}$$

2. Divide $\frac{10}{17}$ by 15.

$$\frac{10}{17} \div 15 = \frac{\overset{2}{\cancel{10}}}{17 \times \underset{3}{\cancel{15}}} = \frac{2}{51} \text{ } \textit{Ans.}$$

Divide:

3. $\frac{12}{17}$ by 6 5. $\frac{24}{29}$ by 40 7. $\frac{48}{71}$ by 120
4. $\frac{9}{16}$ by 6 6. $\frac{8}{27}$ by 20 8. $\frac{3}{4}$ by 5

9. Divide $\frac{6}{5}$ by $\frac{3}{10}$.

$$\frac{6}{5} \div \frac{3}{10} = \frac{\overset{2}{\cancel{6}}}{\cancel{5}} \times \frac{\overset{2}{\cancel{10}}}{\cancel{3}} = 4$$

$\frac{1}{10}$ is contained in one 10 times. Hence, $\frac{3}{10}$ is contained in one, $\frac{1}{3}$ of 10 times, or $\frac{10}{3}$ times. If $\frac{3}{10}$ is contained in one $\frac{10}{3}$ times, it is contained in $\frac{6}{5}$, $\frac{6}{5} \times \frac{10}{3}$ times, or 4 times.

In general, to divide a whole number or a fraction by a fraction, *invert the divisor and multiply.*

Divide:

10. $\frac{3}{4}$ by $\frac{1}{12}$ 11. $\frac{7}{10}$ by $\frac{7}{12}$ 12. $\frac{15}{56}$ by $\frac{60}{49}$

Find the value of

13. $\frac{3}{8} \div \frac{3}{4}$ 15. $10 \div \frac{5}{2}$ 17. $12 \div \frac{6}{7}$
14. $\frac{5}{16} \div \frac{7}{24}$ 16. $20 \div \frac{5}{4}$ 18. $\frac{91}{140} \div \frac{169}{196}$

19. For a room 18 ft. wide, how many strips of carpet each $\frac{3}{4}$ yd. wide will be needed?

20. A man owns $\frac{3}{4}$ of a coal mine and his son owns $\frac{1}{8}$ of the same mine. The father's share is how many times as large as that of the son?

Divide:

21. $1\frac{3}{5}$ by $1\frac{1}{3}$	**23.** $13\frac{3}{4}$ by $15\frac{1}{8}$	**25.** $92\frac{4}{7}$ by 6
22. $5\frac{1}{4}$ by $4\frac{1}{5}$	**24.** $20\frac{5}{6}$ by $52\frac{1}{12}$	**26.** $247\frac{2}{5}$ by 7

Put each of the following in a form ready for the use of cancellation and then compute:

27. $5\frac{2}{5} \times 3\frac{1}{3} \div 7\frac{7}{8}$

29. $3\frac{1}{5} + 2\frac{1}{2} \div (\frac{9}{10} \times 6\frac{1}{3})$

28. $\frac{2}{3} \times 1\frac{1}{5} \div (\frac{3}{4} \times \frac{16}{45})$

30. $18\frac{3}{4} \times 4\frac{3}{5} \div (62\frac{1}{2} \times 7\frac{3}{10})$

Simplify:

31. $$\frac{\dfrac{2\frac{5}{6} - 1\frac{5}{8}}{2\frac{5}{12}}}{8\frac{1}{3} - 7\frac{5}{12}}$$

32. $$\frac{\dfrac{\frac{4}{7} \text{ of } 9\frac{4}{5}}{9\frac{3}{10} - 5\frac{1}{15}}}{1 \div 6\frac{1}{12}}$$

33. How many examples in this Exercise can you work at sight?

EXERCISE 19

ANALYSIS WITH FRACTIONS

1. If $\frac{5}{6}$ of a rectangular piece of land is worth $450, what is the whole piece of land worth? (Draw diagram.)

2. Make up and work a similar example.

3. If $\frac{5}{6}$ of a piece of pie is worth 15 cents, what is the whole pie worth? (Draw diagram.)

4. If I pay $24.50 for the use of a certain sum of borrowed money for $\frac{5}{12}$ of a year, what ought I to pay for its use for a year? (Draw diagram.)

5. Make up and work a similar example.

6. Tin melts at 440°, which is $\frac{11}{70}$ of the temperature at which iron melts. At what temperature does iron melt?

7. After selling $\frac{3}{5}$ of his farm, a man had 120 acres left. How many acres did he have at first?

$\frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

$\frac{2}{5}$ of the farm = 120 acres
$\frac{1}{5}$ of the farm = 60 acres
$\frac{5}{5}$ of the farm,
or the whole farm = 300 acres *Ans.*

Draw a diagram to represent this solution.

8. A woman sold $\frac{2}{7}$ of her chickens and had 135 chickens left. How many chickens did she have at first?

9. Make up and work a similar example.

10. A steer, when killed and dressed, weighed 728 lb. If in this process $\frac{3}{10}$ of the weight of the steer was lost, what did the live steer weigh?

11. A basketball team won 16 games and lost the remainder, which was $\frac{3}{7}$ of the whole. How many games did the team play?

12. A man first sold $\frac{1}{2}$ of a lot of ground and then $\frac{1}{4}$ and had 600 sq. ft. left. How many square feet were in the lot originally? (Diagram this.)

13. If $\frac{2}{3}$ of a piece of land is worth $1500, what is $\frac{3}{4}$ of it worth?

14. If $\frac{3}{4}$ yd. of silk costs 90 cents, what will $\frac{5}{6}$ yd. cost?

15. If $\frac{5}{12}$ of a turn advances a screw $\frac{1}{8}$ of an inch in a board, how far will $\frac{2}{3}$ of a turn advance it? Make up and work a similar example.

16. If $2\frac{1}{2}$ lb. of sugar cost 15 cents, what will $7\frac{1}{2}$ lb. cost? (What is the shortest way of working this example?)

17. If $12\frac{1}{2}$ yd. of dress goods cost $10, what will $37\frac{1}{2}$ yd. cost? 25 yd.? $62\frac{1}{2}$ yd.?

18. Make up and work a similar example.

19. What is $\frac{3}{5}$ of a lot worth, if the rest of the lot sells for $1250? (Diagram this.)

20. A capitalist spent $12,000 in putting an invention on the market. In return, the inventor gave him $\frac{3}{5}$ of the value of the invention. What was the value in money of the share of the inventor, who owns the remaining interest?

21. 54 days of vacation are gone, or $\frac{3}{4}$ of the whole. How many days in $\frac{1}{2}$ of the vacation? (Diagram this.)

22. How many examples in this Exercise can you work at sight?

EXERCISE 20

ORAL REVIEW

Write the following on the blackboard:

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{7}$ $\frac{1}{8}$ $\frac{1}{9}$ $\frac{1}{10}$ $\frac{1}{11}$ $\frac{1}{12}$ $\frac{2}{3}$ $\frac{2}{5}$ $\frac{2}{7}$ $\frac{2}{9}$

$\frac{2}{11}$ $\frac{3}{4}$ $\frac{3}{5}$ $\frac{3}{7}$ $\frac{3}{8}$ $\frac{3}{10}$ $\frac{3}{11}$ $\frac{4}{5}$ $\frac{4}{7}$ $\frac{4}{9}$ $\frac{4}{11}$ $\frac{5}{6}$ $\frac{5}{7}$ $\frac{5}{8}$ $\frac{5}{9}$

$\frac{5}{11}$ $\frac{5}{12}$ $\frac{6}{7}$ $\frac{6}{11}$ $\frac{7}{8}$ $\frac{7}{9}$ $\frac{7}{10}$ $\frac{7}{11}$ $\frac{7}{12}$ $\frac{8}{9}$ $\frac{8}{11}$ $\frac{9}{10}$ $\frac{9}{11}$ $\frac{10}{11}$ $\frac{11}{12}$

As the teacher points

1. Add each pair of fractions. Subtract each pair.
2. Multiply each pair. Divide each pair.
3. Multiply each fraction by 2. By 3, 4, 5, 6, 7 in turn.
4. Divide each fraction by 2. By 3, 4, 5, 8, 9 in turn.
5. Give the value of each of the following:

(1) $\frac{1}{2} + \frac{1}{3}$
(2) $\frac{1}{4} + \frac{2}{3}$
(3) $\frac{3}{7} + \frac{2}{3}$
(4) $4\frac{1}{2} + \frac{3}{5}$
(5) $3\frac{2}{3} + 2\frac{1}{2}$
(6) $\frac{5}{2} - 2$
(7) $\frac{11}{2} - \frac{2}{3}$
(8) $\frac{3}{2} - \frac{1}{4}$
(9) $2\frac{4}{5} - 1\frac{3}{5}$
(10) $2\frac{1}{2} - \frac{1}{3}$
(11) $\frac{2}{3} \times 12$
(12) $\frac{1}{2} \times \frac{3}{5}$
(13) $14 \times 2\frac{1}{2}$
(14) $12 \times 3\frac{3}{4}$
(15) $1\frac{1}{2} \times 2\frac{2}{3}$
(16) $2 \div \frac{1}{3}$
(17) $\frac{3}{2} \div 6$
(18) $1\frac{1}{2} \div 3$
(19) $2\frac{2}{3} \div \frac{1}{2}$
(20) $\frac{3}{4} \div 1\frac{1}{2}$

6. If the heart of a frog beats 10 times in a minute, the heart of a man 72 times, and the heart of a bird 120 times, what fraction is the first rate of the second? Each of the first two is what fraction of the last?

7. Is it easier to visualize, or picture, $\frac{31}{4}$ or $7\frac{3}{4}$?

8. If a boarding house uses $\frac{3}{8}$ of a barrel of flour in a week, how many weeks will 6 barrels of flour last?

9. If a horse eats $1\frac{1}{2}$ pecks of oats per day, how long will 10 bushels of oats last him?

10. Find the value of $12\frac{1}{2} \times 50 + 37\frac{1}{2} \times 50$.

11. The rainfall in Colorado is approximately twice as great as that in Arizona; also that in New York is 5 times as great as that in Arizona. Compare the rainfall in New York with that in Colorado.

12. Which would you rather own: $\frac{3}{20}$ or $\frac{4}{25}$ of a sailing vessel?

13. If $\frac{3}{4}$ lb. of candy cost 30¢, what will a pound cost? 2 lb.? $2\frac{1}{2}$ lb.?

14. What is the value of $212 \times 5\frac{3}{4}$¢ $- 212 \times 5\frac{1}{4}$¢?

15. A man bought a $\frac{1}{3}$ share in an oil well, and later $\frac{1}{6}$. How much more would he need to buy in order to own $\frac{2}{3}$ of the well?

EXERCISE 21

INDUSTRY REVIEWS: TELEGRAPH, TELEPHONE, POST OFFICE

1. Find the cost of a telegram containing 14 words @ 40–3.

@ 40–3 means a cost of 40¢ for the first 10 words in the telegram and 3¢ for each additional word. Hence, at the given rate 14 words would cost 40¢ + 4 × 3¢, or 52¢ *Ans.*

Find the cost of the following telegrams:

2. 15 words @ 50–3

3. 45 words @ 60–4

4. 40 words @ 25–2

5. 19 words @ 50–3

6. 25 words @ 25–2

7. 16 words @ 25–1

8. 32 words @ 100–7

9. 18 words @ 25–2

10. 29 words @ 50–3

11. 41 words @ 75–5

12. Find the day and night rates from your home telegraph office to New York City and other important cities; make up and work five examples using these rates. Write at least one message on a telegraph blank.

A night message (called a night letter) which does not exceed 50 words may be sent for the cost of a day message of 10 words. For every 10 words over 50, one fifth of the first cost of the message is added.

Find the cost of the following night letters:

13. 50 words @ 25¢ **14.** 75 words @ 25¢ **15.** 38 words @ 50¢

16. Find the cost of a cablegram of 9 words at 25¢ a word. At 37¢. At $1.12. At $1.33.

17. Find the cost per word of cablegrams from New York to important foreign cities; make up and work three examples using these rates.

18. In long-distance telephoning, what does a charge of 20–5 mean?

19. Find the cost of using the long-distance telephone 12 minutes at 60–15, 6 minutes at 100–25, and 8 minutes at 150–40. If three trips by railroad cost $28.50, $37.75, and $40.60 respectively, find their total cost. How much is saved by using the telephone three times as indicated, instead of taking the three railroad trips?

20. A family pay $24 a year for unlimited service in the town in which they live, and 10¢ for each message to towns near by. Find the telephone bill of the family for a year, if during the year they telephone 43 times to towns near by, and use the long-distance telephone as follows: 3 minutes at 40–10, 5 minutes at 20–5, and 7 minutes at 100–25.

In working the following examples use these rates:

$\frac{1}{16}$¢ per ounce (*i. e.* 1¢ per lb.) for newspapers and periodicals mailed by the publishers. (*Second* class.)

¼¢ per ounce for newspapers and periodicals mailed by any one but the publishers. (*Second* class.)

½¢ per ounce for books, proof sheets, etc. (*Third* class.)

1¢ per ounce for general merchandise. (*Fourth* class.)

2¢ per ounce for letters to addresses in the United States. (*First* class.)

2¢ per ounce for letters sent to Mexico, Great Britain, and Germany or their dependencies.

5¢ per ounce for letters mailed to other foreign countries.

A fraction of an ounce is counted as one ounce additional.
Thus, the postage due on a book weighing 8½ oz. is 5 cents.
A package sent by mail must not weigh over 4 lb.

The cost of sending a letter by special delivery is 10¢ extra; of registering a letter, 10¢.

21. Assuming that the wrappings are included in the weights given, find the cost of the postage on the following articles:

(*1*) A book weighing 12½ oz.

(*2*) A magazine weighing 3½ oz., and a letter weighing 1½ oz.

(*3*) ½ lb. candy, and a package of newspapers weighing 11½ oz.

(*4*) A pair of slippers weighing 9¾ oz., a registered letter weighing ¼ oz., and magazines weighing 1 lb. 2 oz.

(*5*) A letter weighing ½ oz. sent by special delivery, and a pair of skates weighing 3 lb. 2 oz.

22. Make up and work five examples similar to those in Ex. 21.

23. If the school has a set of postal scales, or if a set can be secured by a member of the class, practice estimating the weight of various articles, and hence the postage due on each article. Then test your estimates by use of the scales.

24. If a copy of a newspaper weighs $3\frac{1}{2}$ oz., what will it cost the publisher to mail 20,000 copies? What will it cost to mail 1 copy? To mail 10 copies in one package? Would the publisher of the newspaper put stamps on the copies mailed by him?

25. A certain scientific society published an annual volume of Proceedings which weighed 11 oz. If the society had 450 members, what was the entire cost of mailing a copy of the Proceedings to each member? If the volume should be divided into four parts and mailed as a quarterly periodical, what would the cost of mailing be?

26. Is it cheaper for me to send a MS. weighing 3 lb. 4 oz. by express at a cost of 40¢, or by mail? One weighing 2 lb. 3 oz., if the expressage on it is 40¢ also?

What would be the greatest possible weight of a manuscript which could be sent by mail for 40¢? For 25¢?

27. A child ten years old bought a postal savings card for 10 cents and three postal savings stamps for 10 cents each. How many more postal savings stamps must he buy before his card and the stamps on it are together worth one dollar?

When the card has thus become worth one dollar, the child can open a savings account at the post office.

28. In the year 1789 there were 75 post offices in the United States. In 1910 there were 59,580 post offices. If the population in 1789 was 3,930,000 and in 1910 was 91,972,267, on the average how many people were served by one post office in each of these years?

29. In 1910 in the United States, 14,004,577,271 pieces of postal matter were handled. How many was this, on the average, for each person?

30. In England a package mailed by parcels post costs 6¢ for the first pound and 2¢ for every pound over that up to the limit of 11 lb. What would it cost to send a 4 lb. package of merchandise in England by mail (*i. e.* by parcels post)? In the United States by mail?

31. In a recent year the United States life-saving service cost $1,885,110 and saved $11,056,805 worth of property. Find the difference. On the average, each of the 280 life-saving stations saved how much property? 3,733 persons were also saved. How many were saved per station?

32. Ascertain the numerical facts concerning some other branch of the United States government, as the army, navy, department of agriculture, coast or geological survey, land reclamation. Make up and work examples involving these facts.

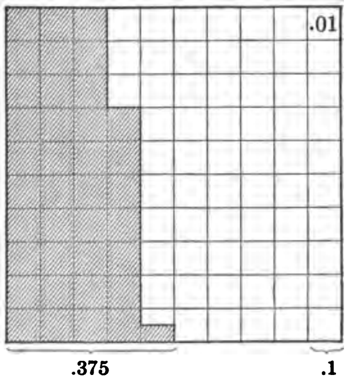
CHAPTER II

REVIEW EXERCISES IN DECIMAL FRACTIONS AND BILLS AND ACCOUNTS

Decimal Fractions

EXERCISE 22

Numeration and Notation of Decimal Fractions



1. On squared paper mark off a square having ten spaces on each side. On this square shade the part which stands for .375, as in the diagram.

2. In like manner on squared paper show the meaning of .8; .27; .673; .07; .025; .183.

3. In .78543, if we take away the 7 and place the decimal point next to the 8, what change is made in the value of 8? Of 5? Of 4? Of 3? Hence, how many items of work does the 7 do?

4. Compare the work of writing .6785 with that of writing $\frac{6785}{10000}$.

Ten thousands	Thousands	Hundreds	Tens	Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
7	3	9	2	8	.	4	6	7	8	0	5
Integers						Decimals					

Using the above numeration and notation table, read the following:

5. .78; 1.4142; 1.73205; 108.0301; .003001.

6. Cork is .24 as heavy as water.

7. A cubic foot of water weighs 62.43 pounds.

8. The circumference of a circle is 3.1416 times as long as the diameter. More exactly, the circumference of a circle is 3.141592 times the diameter.

9. A kilogram is 2.2046 of a pound.

Write the following in figures as decimal fractions:

10. Forty-six hundredths.

11. Ten and sixteen hundredths.

12. Seven and fifty-one thousandths.

13. Thirty-six millionths.

14. Five and five millionths.

15. Seventy-five and forty-two ten-thousandths.

16. Five hundred-thousandths.

17. Five hundred thousandths.

18. One thousand five hundred and one tenth.

19. Four hundred and four hundred one thousandths.

20. The area of New Jersey is one hundred fifty eight thousandths that of New York. Show the meaning of this statement by the use of squared paper.

EXERCISE 23

Addition and Subtraction

Add:

1.	$8.92	2.	$27.05	3.	5.571	4.	1.0071
	7.63		123.74		93.428		.0382
	5.49		6.73½		.96		5.917
	27.38		2.04½		.407		41.0328
	6.25		38.7		8.14		17.51

5. $125.74 + $307.06 + $51.075 + $6.305.

6. 270.01 + 31.0031 + .0073 + 25 + 43.0106 + 4.008.

7. The rainfall at a certain place during the months of a year beginning with January was 5.15 in., 4.07 in., 4.5 in., 3.045 in., 1.4 in., 1.1 in., .95 in., .82 in., 2.76 in., 2.8 in., 3.005 in., and 4.03 in. Find the rainfall for the entire year.

8. In building a railroad, a track-laying gang lays in successive days 1.245 mi., 1.87 mi., 2.105 mi., 1.925 mi., 2.2 mi., and 2.005 mi. How much track does it lay in a week?

9.	From $7.35	10.	$18.196	11.	71.44 in.	12.	65.03
	take 3.21		4.75		38.67 in.		47.903

Compute:

13. .54 − .37

14. 9.53 − 7.99

15. 70.07 − 6.408

16. 2 − .010203

17. On three days a man deposited in a bank $27.54, $35.97, and $71.16, and on the fourth day withdrew $49.73. How much of his money then remained in the bank?

18. Find the difference between six hundred twenty-eight thousandths and four hundred and sixty-nine thousandths.

19. The English bushel contains 2218.192 cu. in., and the American bushel 2150.42 cu. in. Find the difference.

20. The equatorial diameter of the earth is 7926.592 miles, and the polar diameter is 7901.476. Find the difference.

EXERCISE 24

MULTIPLICATION AND DIVISION

1. Multiply 53.17 by 1.8.

$$\begin{array}{r} 53.17 \\ \underline{1.8} \\ 42536 \\ \underline{5317} \\ 95.706 \end{array}$$

The position of the decimal point in the product is determined by the fact that the number of decimal places in the product equals the sum of the decimal places in the multiplier and the multiplicand taken together.

Multiply:	**2.** 8.3	**3.** 3.5	**4.** 7.6	**5.** 10.4	**6.** .88
	5	1.2	.05	.15	.7

7. Multiply each of the following by 2.05:

.32 .036 10.08 200.04 35.1 71.09

8. 5 × .4 × 30 × 1.8 = ?

9. 1.1 × 4 × 5.5 × .02 = ?

10. 5.3 × 2.01 × .46 = ?

11. 5.8 × .025 × 1.003 = ?

12. What is the simplest method of multiplying a decimal fraction by 10? By 100? By 1000?

13. What is the simplest method of multiplying a decimal fraction by .1? By .01? By .001?

14. If one meter is 39.37 in. long, how long are 3.23 meters?

15. If 1 cu. ft. of water weighs 62.5 lb., and ice weighs .92 as much as water, what will be the weight of 3 cu. ft. of ice? Of 2.3 cu. ft.?

16. On squared paper show the meaning of 3 × .15. Also of .2 × .3.

17. If one perch of stone contains 24.75 cu. ft. and stone is 2.7 times as heavy as water, how much will a perch of stone weigh? (See Ex. 14.)

18. What is the cost of 3075 cu. ft. of gas at $.80 per thousand cubic feet?

19. Divide .0221 by .013.

If we multiply each of the given numbers by the same number (that is, move both decimal points the same number of places to the right), the value of the quotient will be unchanged.

```
              1.7  Ans.
      .013.).022.1
             13
             --
              91
              91
              --
```

Divide:

20. .084 by .12

21. .096 by .008

22. .007 by .025

23. 16.8 by .021

24. 46.5 by .015

25. 10.11 by .01011

26. 700 by 6.25

27. 4.64 by .145

28. 87.098 by 4.07

29. 30030.3 by .0375

30. .0456 by .0076

31. 100 by .001

Divide to four decimal places:

32. 7.101 by 19

33. 31.76 by 23

34. 101.5 by 30.7

35. .0077 by .058

36. If 1 cu. ft. of iron weighs 468 lb., how many cubic feet are in a ton of iron? (Find to three decimal places.)

37. The area of Great Britain and Ireland is 121,371 sq. mi. What decimal part of the area of Texas is this? (Find to three decimal places.)

38. If 1 mark of German money is worth \$.238, \$250.50 equals how many marks?

39. In a recent year in the United States 45,211,000 A. were planted with wheat and produced 637,087,000 bu. Find the yield per acre to two decimal places.

EXERCISE 25

Analysis; Common and Decimal Fractions

1. If .7 of a rectangular piece of ground is worth \$1400, what is the whole plot worth? (Diagram this.)

2. Make up and work a similar example.

3. If .07 of a piece of ground is worth \$210, what is the whole piece worth? What is .15 worth? .65?

4. A farmer sold 210 bu. of wheat, which was .03 of what he had. How many bushels did he have?

5. What is .35 of a piece of land worth, if the rest sells for \$1365? Make up and work a similar example.

6. If a county pays .3 of the cost of a macadam road and the state the rest, which is \$14,700, what does the county pay?

7. The population of a certain city was 31,800 at the end of a given year. This was .06 more than it was at the beginning of the year. What was the population at the beginning of the year?

8. Change .56¼ to the form of a common fraction.

$$.56\tfrac{1}{4} = \frac{56\frac{1}{4}}{100} = \frac{\frac{225}{4}}{100} = \frac{225}{400} = \frac{9}{16}\ \textit{Ans.}$$

$$\text{Or,}\qquad .56\tfrac{1}{4} = .5625 = \frac{5625}{10{,}000} = \frac{9}{16}$$

Note that .56¼ means 5 tenths 6 hundredths and ¼ of a hundredth (not ¼ of a thousandth).

Reduce each of the following to its equivalent common fraction in its lowest terms:

9. .32 **11.** .006 **13.** $.14\frac{2}{7}$

10. .275 **12.** $.62\frac{1}{2}$ **14.** 6.4124

15. Express $.33\frac{1}{3}$ of a yard as inches; also .25 yd.; $.16\frac{2}{3}$ yd.; .75 yd.

16. Make up and work a similar example about a decimal part of a bushel and quarts.

17. On squared paper show that $.4 = \frac{2}{5}$; $.75 = \frac{3}{4}$; $.35 = \frac{7}{20}$; $.16 = \frac{4}{25}$.

18. Change $\frac{7}{8}$ to the form of a decimal fraction.

$$8\overline{)7.000}$$
$$.875 \textit{ Ans.}$$

19. Reduce each of the following to an equivalent decimal fraction:

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{1}{8}$ $\frac{5}{8}$ $\frac{7}{16}$ $\frac{11}{16}$ $\frac{8}{25}$ $\frac{21}{25}$ $\frac{19}{32}$ $\frac{31}{32}$

20. Find correctly to five decimal places the value of

$\frac{1}{6}$ $\frac{2}{7}$ $\frac{5}{9}$ $\frac{15}{17}$ $1\frac{4}{15}$ $\frac{9}{400}$ $\frac{87}{1000}$

21. What decimal fraction must be added to 3.141592 to make $3\frac{1}{4}$?

22. What decimal fraction of 80 is 20? Of 200 is 16?

23. Express 3 mo. as the decimal part of a year. Also 6 mo. 9 mo. 8 mo. 10 mo.

24. Express 6 days as the decimal part of a month (of 30 days). Also 15 da. 12 da. 21 da. 2 da. 18 da.

25. What decimal part of 360 days is 72 days? Is 3 months? 12 days? (Take 1 month as equal to 30 days.)

26. Make up a similar example concerning ounces and a pound and solve it.

27. A boy increased $1\frac{5}{8}$ in. in height in one year and his friend increased 1.64 in. Which of the boys increased the more? How much more?

28. Write as a decimal fraction: $\frac{5}{16}$; $\frac{3}{20}$; $\frac{1}{2}$ of $\frac{3}{4}$.

29. Write as a common fraction: .08; .28; $.27\frac{1}{2}$.

30. On squared paper show the meaning of .03. Of $.03\frac{1}{2}$. Of $.0\frac{1}{2}$.

31. Find the sum of $.27 + \frac{8}{10} + \frac{9}{100}$.

Aliquot Parts of a Dollar

2. An **Aliquot Part** of a number is a number which is exactly contained in a given number. Thus, 6 is an aliquot part of 18.

An aliquot part may be a mixed number. Thus, $33\frac{1}{3}$ is an aliquot part of 100, since $33\frac{1}{3}$ is contained in 100 exactly 3 times.

By the method of Ex. 8, Exercise 25, it is readily shown that

$.06\frac{1}{4} = \frac{1}{16}$
$.12\frac{1}{2} = \frac{1}{8}$
$.16\frac{2}{3} = \frac{1}{6}$
$.25 = \frac{1}{4}$
$.33\frac{1}{3} = \frac{1}{3}$
$.37\frac{1}{2} = \frac{3}{8}$
$.50 = \frac{1}{2}$
$.62\frac{1}{2} = \frac{5}{8}$
$.66\frac{2}{3} = \frac{2}{3}$
$.75 = \frac{3}{4}$
$.87\frac{1}{2} = \frac{7}{8}$

Ex. 1. Find the cost of 48 lb. of butter at $\$.33\frac{1}{3}$ per pound.

$$\$.33\tfrac{1}{3} \times 48 = \$\tfrac{1}{3} \times 48 = \$16 \textit{ Ans.}$$

Ex. 2. At $37\frac{1}{2}$¢ per pound, show how many pounds of butter can be bought for \$60.

$$\$60 \div \$.37\tfrac{1}{2} = \$60 \div \$\tfrac{3}{8} = \$60 \times \$\tfrac{8}{3} = 160 \textit{ Ans.}$$

EXERCISE 26

Compute the following by the method of aliquot parts (to the nearest cent when the result is not exact):

1. 48 times $\$.12\frac{1}{2}$ Times $\$.06\frac{1}{4}$ $\$.62\frac{1}{2}$ $\$.33\frac{1}{3}$ \$.25

2. 43 times $.50   Times $.33⅓ $.66⅔ $.25 $.37½

3. 24.8 times $.25   Times $.87½ $.66⅔ $.50 $.06¼

4. Make up and work two similar examples.

Divide:

5. $3 by $.12½   By 37½¢ $.25   50¢   62½¢ $.87½

6. $24 by 33⅓¢   By 37½¢   25¢ $.12½ $.62½   .87½

7. $42.50 by $.66⅔   By $.62½   25¢   33⅓¢ $.12½   37½¢

8. Make up and work a similar example.

By the method of aliquot parts of $1, find the cost of

9. 44 lb. @ $.25

10. 54 lb. @ $.37½

11. 50 oz. @ $.33⅓

12. 94 ft. @ $.66⅔

13. 68 bu. @ $.87½

14. 95 qt. @ $.12½

15. 83 gal. @ $.75

16. 54 lb. @ $.16⅔

17. 77 pt. @ $.37½

18. 61 lb. @ $.12½

19. 38 qt. @ $.16⅔

20. 91 ft. @ $.75

Find the cost of

21. 48 lb. of butter at 37½¢. At 50¢. At 33⅓¢.

22. 23 gal. of molasses at 62½¢. At 50¢. At 87½¢.

23. 60 bushels of potatoes at 75¢. At 66⅔¢. At 87½¢.

State how many of the following can be bought:

24. Yards of muslin at 12½¢ for $2.50. $5.

25. Pounds of butter at 33⅓¢ for $5. $7.

26. Collars at 16⅔¢ for $3. $5.50.

27. Bushels of corn at 87½¢ for $140.

28. Penknives at $1.37½ for $33.

29. On the six days of a week a boy works 8, 6, 7, 7, 8, 5 hours respectively. Find his earnings for the week at 12½¢ an hour.

30. Find the cost of 24 collars at the rate of 6 collars for $1.

31. If a commission merchant buys 200 baskets of peaches at $1.12½ and sells them at $1.25, how much does he make?

32. Multiply 224 by .37½ in the long way. Also in the short way. Compare the amount of work in the two processes. In which of the two processes are you less likely to make a mistake?

33. How many examples in this Exercise can you work at sight?

BILLS AND ACCOUNTS

3. Price and Cost. In a sale of a given quantity of material, the **price** is the amount paid for one unit, as for one pound, or one quart, of the given quantity.

The **cost** is the amount paid for the entire quantity.

Thus, the *price* of one pound of coffee might be 38¢; then the *cost* of 17 pounds would be $6.46.

In Exercise 26, Exs. 20–25, point out the price and also the cost.

4. A **Bill** is a written or printed statement of the items of a sale or of services rendered.

The *essential parts* of a complete bill are the *place* of the sale, the time or *date* at which the bill is made out, the name of the *buyer*, the name of the *seller*, the *price*, *quantity*, and *cost* of each item, and the *total cost* of all the items.

What is a debtor? A creditor? What is the meaning of the following abbreviations used in connection with bills and accounts?

@ Cr. Dr. Pay't No. or #
a/c Bal. Per Rec'd

Ex. On Sept. 2, 1912, Walter Howard bought of M. P. Estill & Co., of Baltimore, Md., 24 boys' suits at $8.50; 30 men's suits at $16.75; 18 overcoats at $18.50; 35 white vests at $3.50. What was the entire cost of his purchases?

The purchases may be arranged as a bill and summed up as follows:

Baltimore, Md., Sept. 2, 1912.

Mr. Walter Howard,
Bought of M. P. Estill & Co.

	24 boys' suits	$ 8.50	$204	00
	30 men's suits	16.75	502	50
	18 overcoats	18.50	333	00
	35 white vests	3.50	122	50
			$1162	00

Non-essential parts of a bill are often added as a matter of convenience. Thus, the person or persons who make out the bill for a firm may put on a bill a statement of the business they carry on, give their telephone number, or state the terms of the sale as cash, C. O. D., or 30 days, etc.

Let the pupils obtain and bring to class bill-heads containing some of the non-essential parts just named.

Bills are written in two principal forms:

(1) **Bill of sales form.** See the above example.

(2) **The debit or Dr. form.** Bills for labor or professional services are usually written in the debit form.

Ex. Cleveland, Ohio.

J. L. McLean
To James Mills, D.D.S. Dr.

May 1, 1913.	Dental work during April	$17	00

5. **Receipting a Bill** is writing on a bill a statement that the bill has been paid.

Thus, the bill at the top of this page might have been receipted by writing (or stamping in part) the following across it, or near its foot:

Received payment,
M. P. Estill & Co.,
Per R. S.

Do you know what the "per R. S." means?

The date when the receipting is done may or may not be stated. Sometimes a receipt is written, not on the bill, but on another piece of paper. In this case the sum paid and the name of the person paying it must appear in the receipt.

In computing bills, it is well to learn to perform mentally as many of the multiplications as possible.

EXERCISE 27

Find the amount of each of the following bills:

1. Mobile, Ala., Apr. 10, 1913.

Horace Perry,
Bought of Wallace, Brown and Co.

6 lb. nails	5½¢		
12 ft. zinc	7½¢		
15 doz. screws	4⅔¢		
40 squares tin	17¢		

2. Philadelphia, Sept. 18, 1913.

Mrs. Walter Edwards,
Bought of Finley Acker & Co.

26 jugs syrup	30¢		
7 lb. coffee	28¢		
9 lb. rice	9½¢		
45 lb. oatmeal	4½¢		
30 jars of fruit	48¢		
50 lb. sugar	5½¢		
2½ lb. tea	80¢		

3. In Exs. 1 and 2, point out the essential parts of each bill. Add some non-essential item to the bill in Ex. 1. Add a different non-essential to the bill in Ex. 2.

Make out a bill for the following items and find its amount:

4. Mrs. James Richards bought of Hunt and Co., of Philadelphia, on Jan. 3, 1914:

37 yd. Brussels carpet @ \$1.65
14 yd. Axminster carpet @ 2.85
$41\frac{1}{2}$ yd. Ingrain filling @ .75
9 small rugs @ \$2.75
45 step pads @ $.27\frac{1}{2}$

5. On April 11, 1913, Mrs. Marion Rutledge bought of Hatfield and Co., of Chicago:

61 yd. muslin @ $11\frac{1}{2}$¢
55 yd. table linen @ \$1.25
23 yd. silk @ \$1.15
10 yd. lining @ 23¢
31 yd. carpet @ \$1.50

6. On Dec. 12, 1913, a restaurant keeper, Samuel Morris, bought of Atwood Sons, of Kansas City, Mo., 30 lb. beefsteak at 18¢, 25 lb. chicken at 17¢, 15 lb. veal at $16\frac{1}{2}$¢, $21\frac{1}{2}$ lb. ham at 22¢.

Suppose yourself the proprietor of a grocery store, and make out bills for the following sets of items, filling in prices with probable numbers where they are lacking. Find the amount of each bill. Also receipt the bills in various possible ways.

7. 6 lb. crackers @ $12\frac{1}{2}$¢
3 loaves bread @ 8¢
3 cakes @ 45¢
4 lb. wafers @ 15¢

8. $\frac{1}{2}$ lb. tea @
2 lb. cheese @
$\frac{3}{4}$ lb. pepper @
3 lb. coffee @

9. 3 lb. butter @ 45¢
3 bunches celery @ 15¢
$3\frac{1}{2}$ doz. eggs @ 30¢
$\frac{1}{2}$ bu. potatoes @ 45¢

10. 7 lb. oatmeal @
$1\frac{1}{2}$ bu. potatoes @
$\frac{1}{2}$ doz. cans tomatoes @
1 gal. olive oil @

11. Suppose yourself the proprietor of some business of your own selection, as dry goods, or coal and lumber, and make out complete bills such as might be required in that business.

12. Make out two bills in each of which two essentials are missing. After the teacher has inspected these bills, supply the parts that are missing.

13. If you should buy several articles at a store and pay for them at the time of purchase, what would be the advantage of having a bill made out and receipted and of taking it away with you?

14. Find out, if you can, what is meant by the loose-leaf method of keeping accounts and the advantages of it.

15. State as many different ways of receipting a bill as you can.

16. Do you know what a check is, as used in connection with banks in paying bills? Get a blank check on some bank and write a copy of it.

17. What are the necessary parts of a check?

18. What are the advantages of paying by check?

19. Collect as many different kinds of bills, statements, etc., as you can, and point out the differences in them.

20. What are the uses of bills?

6. **Accounts.** Instead of paying for articles when they are bought, it is often a convenience to have a record kept of such purchases and to make a payment or payments for them at other times.

An **account** is a record of various articles bought and payments made.

A **statement** is a record of an account up to a specified date, as the first day of a month. The sending of a statement to a customer usually means that the payment of the balance of the account is desired.

The following is an example of an account:

Harristown, Pa., March 2, 1914.

Mr. Walter Morris,
Watertown, Pa.
To McCullough, Tompkins & Co.,
Dealers in Fruits and Vegetables,
47 Linden St.

Telephone 8R

			Dr.		Cr.	
Feb. 1	Balance due		$317	52		
4	20 bbl. cranberries	$6.25	125	00		
7	30 bbl. apples	$2.20	66	00		
7	Cash				300	00
15	40 bu. sweet potatoes	70¢	28	00		
18	Cash				125	00
23	20 crates oranges	$2.10	42	00		
28	Cash to balance				153	52
			578	52	578	52

EXERCISE 28

1. During the month of October last, at the department store of Harrison, Hunt, and Co., of New Orleans, La., the following items were charged to the account of William P. Baker: Oct. 1, balance due $78.52; Oct. 3, $7\frac{1}{2}$ yd. dress goods @ $1.12, a pair of blankets $8.75, 8 sheets @ 90¢; Oct. 8, books $12.72; Oct. 17, bedstead $12.40. Also the following sums were paid: Oct. 5, $78.52; Oct. 15, $25.

Make out an account for the above items similar to that above.

2. Make up a similar account for the purchaser at the same store for the month of November.

3. Make up an account for the purchases of a family at a fruit and vegetable store during the month of April. Also of September.

4. What are some of the advantages in keeping an account at a store? What are some of the disadvantages?

5. What is meant by consulting Dun's or Bradstreet's? Find out the advantages of doing so.

6. Can you suggest a convenient form in which a boy or a girl could keep an account of personal receipts and expenditures? Make up such an account containing at least five receipt items and five expenditures.

7. Make up an account of a boy's or a girl's probable receipts and expenditures during the month of July. Of December.

8. Make an account for a boy who, as treasurer of a baseball association, received and paid out the following sums: Received, subscriptions of members, $42; gate receipts, $327.75; from advertisements on tickets and programs, $27.50. Paid out, for rent of grounds, $62; traveling expenses, $72.46; cost of printing, $17.50; suits, $147.30.

9. Make a similar account for a boy who cultivated a garden during one summer. Also for a girl who acted as treasurer of a dramatic association during the winter.

Or let each pupil make up and work a set of accounts for any enterprise with which he is familiar.

10. Keep a cash account for a week and show it to your father, your teacher, or some friend.

EXERCISE 29

ORAL REVIEW

1. Write the following on the blackboard:

.6   1.2   .3   2.4   .36   .9   .48   3.36   1.5   7.2   12.63

As the teacher points, add 1 to each. Also add in turn 2 .3 1.2 .003

2. Subtract .1 from each number in Ex. 1. Subtract .2 .02 .003

3. Multiply each number in Ex. 1 by .2. By .1. .02 .01 .05

4. Divide by 2. By .2 .002 .3 .03 .01

5. Give the value of each of the following:

(*1*) $4 \div .2$ (*4*) $4 \div .02$ (*7*) $.6 \div 2$ (*10*) $.06 \div 2$
(*2*) $4 \times .2$ (*5*) $4 \times .02$ (*8*) $6 \div .2$ (*11*) $.09 \div .3$
(*3*) $4 - .2$ (*6*) $4 - .02$ (*9*) $.6 \div .2$ (*12*) $2.37 + 1$

6. If a sick person has a temperature of 103.5°, how much is this above normal (98.7°)?

7. If .4 of a building lot costs $80, what is the cost of the whole lot, at the same rate?

8. Give the value of

(*1*) $\$84 \div \$.12\frac{1}{2}$ (*2*) $84 \times \$.12\frac{1}{2}$ (*3*) $66 \times .33\frac{1}{3}$ (*4*) $24 \times .37\frac{1}{2}$

9. Express each of the following as a decimal fraction:

$\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{10}$ $\frac{1}{5}$ $\frac{4}{5}$ $\frac{3}{5}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{2}{5}$ $\frac{5}{8}$ $\frac{7}{10}$

10. Express each of the following as a common fraction:

.2 .4 .5 .25 .75 .6 .8 .125 .12 .05 .375

11. How many feet of wire at $.005 per foot can be bought for $2?

12. How many figures are used in writing 17 thousandths as a decimal fraction? In writing 3 millionths? 7 hundred-millionths?

13. Add the following, first as common fractions, then as decimal fractions:

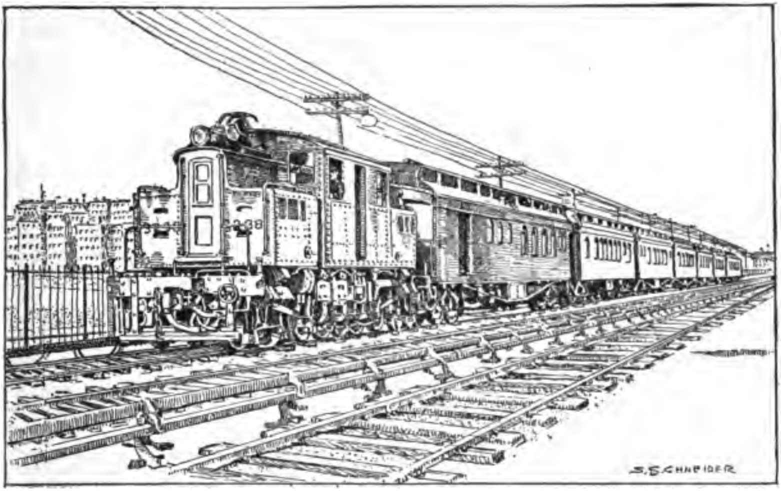
(*1*) $\frac{1}{2} + .25$ (*3*) $\frac{1}{3} + .33\frac{1}{3}$ (*5*) $2\frac{1}{2} + .125$
(*2*) $\frac{3}{5} + \frac{1}{10}$ (*4*) $.75 + \frac{1}{2}$ (*6*) $.66\frac{2}{3} + \frac{1}{6}$

14. Give an example of a common fraction which is easier to work with than the equivalent decimal fraction. Give two other such examples.

15. Give an example of a decimal fraction which is easier to work with than the equivalent common fraction. Give two other such examples.

EXERCISE 30

INDUSTRY REVIEW: RAILROADS



1. If the fare from New York to San Francisco by railroad is $77.50 and the distance is 3250 miles, what is the fare per mile (find to three decimal places)? If the round trip fare is $120, how much is saved by using a round trip ticket?

2. If it takes a letter 4 days and 9 hours to go between the two places, what is the average rate per hour at which the letter travels?

3. If there are 250,000 miles of railroad in the United States, how many times would this reach from New York to San Francisco?

4. If there are 250,000 miles of railroad in the United States, how many miles is this per 100 square miles of territory? (See p. 384.)

5. If New Jersey has 30.22 miles of railroad per 100 square miles of territory, how much is this above the average in the United States?

6. 5 men whose wages together are $18 per day run a freight train 90 miles. How much is saved per ton in wages of the train crew, if 6000 tons of freight are taken on the train instead of 350 tons?

7. The distance from New York to Philadelphia is 90 miles. How much more will rails weighing 100 lb. per yard cost for one track between these places, than rails weighing 80 lb. per yard, at $27 a ton?

8. The first locomotive built weighed $4\frac{1}{2}$ tons. A large locomotive now weighs as much as 100 tons. The second of these locomotives is how many times as heavy as the first? What is the advantage in using large locomotives?

9. A division of a certain railroad was equipped with electric power at an expense of $3,000,000. By this means, the operating expenses of the railroad were reduced $600,000 a year. The second number is what fraction of the first? What decimal fraction?

10. Before electrification, 90 lb. of coal were required to send a train 1 mile by steam; but on the electrified road only 40 lb. were needed. If the division had been using 45,000 tons of coal per year, how many tons of coal were saved per year?

11. On the electrified road, the time of a train carrying commuters to a city was reduced from 54 minutes to 37 minutes. How many hours per year were saved to a man making 300 round trips to the city in that time?

What was this saving worth to 2000 people in a year, counting time as worth 50¢ an hour to each passenger?

Can you think of any other advantage in the electrified road? Any disadvantages?

12. In a certain year there were 225,000 miles of railroad in the United States, and 400,000 miles in the whole world. The first number is what fraction of the second number?

13. If the freight rate by rail from Jacksonville, Florida, to a northern seaport is $1.25 per 100 pounds, and the distance is 1077 miles, what is the rate per ton for a mile? If the rate by steamer is $3 per ton, the rate of the railroad is how many times as great as that by steamer?

14. 44,000 tons of coal have been floated down the Ohio and Mississippi rivers at a cost of $\frac{1}{30}$¢ per ton per mile. If the distance is 2000 miles, what was the entire cost? The cost per ton of coal?

15. If the cost of carrying freight by water is, on the average, $\frac{1}{7}$ of the cost by rail, what would be saved by sending by water freight which costs $28,000,000 when sent by rail?

16. In a recent year there were 2,500,000 miles of public road in the United States, of which 108,000 were gravel, and 30,000 macadam. What decimal fraction of the whole was gravel and what macadam?

17. In England and France, where the wagon roads are largely macadam, the cost of hauling 1 ton 1 mile by wagon averages 7¢. In the United States the cost is 25¢. In the United States, how much would be saved in 1 year in hauling 300,000,000 tons 1 mile on the public roads, if the cost were reduced to that in England?

18. If the average cost of hauling a ton of freight in the United States by railroad is $.0078 per mile, how much is saved by hauling 1,000,000,000 tons of freight in this way instead of by horse and wagon? (See Ex. 17.)

19. A motor truck delivered 91 tons of coal in one day at a cost of $18.24. What was the cost per ton? 40 tons were delivered by horse and wagon at a cost of $15.84. Find the cost per ton.

20. How much would be saved by delivering 1000 tons of coal by motor truck instead of horse and wagon at the rates of Ex. 19?

21. Make up and work a set of examples relating to a railroad trip between two important cities, as Chicago and New Orleans, or New York and Los Angeles.

CHAPTER III

COMPOUND NUMBERS

EXERCISE 31

1. Compare the labor of using a 100-foot tape three times with that of measuring with a yard stick the distance covered by use of the tape.

2. In the following list, point out the distances which are best measured by the inch. By the foot. By the yard. By the mile.

(*1*) The width of a windowpane.
(*2*) The width of a building lot.
(*3*) The distance between New York and Chicago.
(*4*) The distance to the moon.
(*5*) The height of a man.
(*6*) The diameter of a baseball.
(*7*) The length of a tennis court.
(*8*) The height of an office building.

3. What unit of length is first used in measuring the length of a board? What other unit is used in completing the measurement?

4. Which is easier to visualize (or picture to yourself): 294 sq. in., or 2 sq. ft. 6 sq. in.?

5. Which is easier to multiply by 23: 294 sq. in., or 2 sq. ft. 6 sq. in.? Perform both of these operations.

7. A **compound number** is a number expressed in terms of two or more units of the same class but of different sizes and having special names; as 9 feet 7 inches.

Since units expressed in this way have special names, numbers expressed in terms of such units are sometimes called *denominate numbers*.

A **simple number** is a number expressed in terms of a single unit; as 115 inches.

WEIGHT

8. **Avoirdupois Weight** is used in weighing all ordinary objects, the exceptions being the precious metals, jewels, and drugs when retailed.

The fundamental unit in avoirdupois weight is *pound avoirdupois*.

The word *avoirdupois* is often shortened to *av.*

TABLE

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight = 1 ton (T.)

The pound avoirdupois is also regarded as made up of 7000 grains.

The *long ton*, or 2240 pounds, is used in weighing objects in the United States custom houses, and in wholesale transactions in coal and iron.

9. **Troy Weight** is used in weighing gold, silver, and jewels.

The fundamental unit is a *pound Troy*, which is the weight of a certain piece of brass kept in the custody of the United States mint.

Troy weight is often shortened to *T. weight.*

TABLE

24 grains (gr.) = 1 pennyweight (pwt. or dwt.)
20 pennyweight = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

A Troy pound = 5760 grains.

Diamonds and other jewels are also weighed in terms of another unit, the *carat*. The carat = $3\frac{1}{6}$ grains.

The term *carat* is also used in another sense, viz.: to express the parts of gold contained by a given metal alloy. In this sense, one carat means one twenty-fourth part. Gold 18 carats fine contains 18 parts of pure gold and 6 parts of some other metal.

10. Apothecaries' Weight is used in mixing and selling drugs and medicines at retail (these being purchased by druggists at wholesale by avoirdupois weight).

The fundamental unit is the same as in Troy weight, but it is divided somewhat differently.

TABLE

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (lb.)

Length, Area, and Volume

11. Long Measure. The fundamental unit of length in the United States and Great Britain is the *yard*. This is the distance between two marks on a bar of metal kept in the Exchequer Office of Great Britain.

From the yard are derived the following units of length:

TABLE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)

$$\left.\begin{array}{r}5\frac{1}{2}\text{ yards}\\ \text{or }16\frac{1}{2}\text{ feet}\end{array}\right\} = 1\text{ rod (rd.)}$$

320 rods = 1 mile (mi.)
Hence, 1 mile = 1760 yd. = 5280 ft.

In measuring land, the unit of length employed is often 1 *chain* (ch.).

1 chain = 4 rods = 66 feet = 792 inches.

In gunnery, instead of 1 mile, 1000 yards is often taken as the unit of length or distance.

The foot is frequently divided decimally instead of into inches. Thus, instead of 2 ft. 6 in., we write 2.5 ft.

Instead of the contractions ft. and in., the signs ′ and ″ are frequently used. Thus, instead of 2 ft. 6 in., we write 2′ 6″. The latter method is especially useful on diagrams where the figures and signs should occupy as little space as possible.

Linear Units is a general name for units of length (as one inch, one foot, etc.) as distinguished from units of area or of volume.

12. Surface or Square Measure.

TABLE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Hence, 1 acre = 43,560 sq. ft. = 4840 sq. yd.

In measuring the area of land, 1 square chain = 16 sq. rd. Hence, 10 square chains = 160 sq. rd. = 1 acre.

5 sq. yd. is sometimes expressed as 5 □ yd. or 5 yd^2. Other units in square measure may be expressed similarly.

13. Cubic Measure.

TABLE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

CAPACITY

14. In **Dry Measure** the fundamental unit is the *bushel,* which contains 2150.42 cu. in.

TABLE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

15. In **Liquid Measure** the fundamental unit is the *gallon*. The gallon contains 231 cu. in.

TABLE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
63 gallons = 1 hogshead (hhd.)

Barrels and hogsheads in common use often vary greatly from the standard size.

The *gill* is little used. The pint is usually divided into 16 fluid ounces, instead of into gills.

A common glass or teacup contains 8 fluid ounces.

What is called a *tablespoonful* contains about ½ of a fluid ounce.

What is called a *teaspoonful* contains about ⅛ of a fluid ounce.

VALUE

16. A **unit of money** is a certain amount of gold used to measure the values of things.

Instead of gold, other metals, as silver, nickel, and copper, are sometimes used in a limited way to measure value.

Coin is metal that has been weighed and stamped by the government for use as money; as a silver dollar, or a gold eagle, or an English sovereign.

Paper money consists of printed promises to pay a certain amount of coin to the bearer.

17. In **United States Money** the fundamental unit is the *gold dollar*. The gold dollar weighs 25.8 grains, $\frac{9}{10}$ of this being gold.

TABLE

10 mills = 1 cent (ct. or ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle

The coins in use in the United States are as follows:
Bronze: the cent.
Nickel: the five-cent piece.
Silver: the dime, quarter dollar, half dollar, and dollar.
Gold: the quarter eagle, half eagle, eagle, and double eagle.

18. In **English Money** the fundamental unit is the *pound,* whose value in United States money is $4.8665.

TABLE

4 farthings (far.) = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound (£)
21 shillings = 1 guinea (G.)

The coins used in Great Britain are as follows:
Copper: penny, half-penny.
The copper farthing was once used, but is no longer coined.
Silver: threepence, sixpence, shilling, florin (2 s.), double florin (4 s.), half crown ($2\frac{1}{2}$ s.), crown (5 s.).
Gold: half sovereign (10 s.), sovereign (20 s.).

19. In **French Money** the unit is the *franc,* the value of which in United States money is $.193.

100 centimes (c.) = 1 franc (f.)

In Belgium and Switzerland the unit of money is also the *franc.* In Italy it is the *lira,* and in Spain the *peseta,* each of which has the same value as the franc.

20. In **German Money** the unit is the *mark,* the value of which is $.238.

100 pfennige (pf.) = 1 mark (m.)

In Austria the unit of money is the *crown,* the value of which is $.203.

Time and Other Measures

21. Measures of Time.

TABLE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (h.)
24 hours = 1 day (da.)
365 days = 1 common year (yr.)
366 days = 1 leap year
100 years = 1 century (cen.)

Repeat the rhyme which tells the number of days in each month.

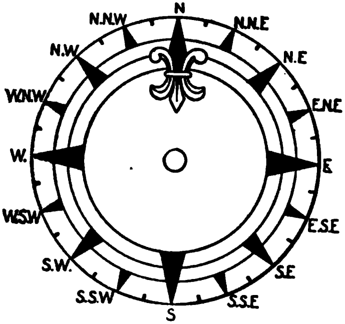
Every year divisible by 4 is a *leap year*, except for century years. Only those century years divisible by 400 are leap years.

22. Circular Measure.

TABLE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circumference (c.)

The use of circular measure is illustrated by the points of the compass. These are used to indicate the direction of one object from another on the earth's surface. On the diagram, the space between N. and E., for example, is divided into 8 equal parts. Hence, each of these parts equals $\frac{1}{8}$ of 90, or $11\frac{1}{4}^\circ$, or 11° 15°.



Points of the Compass

23. Longitude and Latitude. Each great circle on the surface of the earth (as, for instance, the equator) is divided into 360 degrees. A degree of the earth's equator is called a degree of longitude.

$\frac{1}{60}$ of a degree of longitude is called a geographical mile or *knot.*

Longitude is measured east or west from a fixed point on the equator. This is usually the point where the meridian of Greenwich crosses the equator.

The **meridian** of a place on the earth's surface is the circle which passes through any given place and the north and south poles. Each meridian is divided into 360 degrees. A degree of a meridian is called a degree of latitude.

Latitude is measured north or south of the equator on a meridian.

24. **Numbers**

12 units = 1 dozen (doz.)
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

Sheets of Paper

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

PROCESSES WITH COMPOUND NUMBERS

25. Reduction Descending.

Ex. Reduce 2 lb. 7 oz. 14 pwt. to pwt.

2 lb. = 2 × 12 oz. = 24 oz.
24 oz. + 7 oz. = 31 oz.
31 oz. = 31 × 20 pwt. = 620 pwt.
620 pwt. + 14 pwt. = 634 pwt. *Ans.*

or

$$\begin{array}{l}
\text{2 lb. 7 oz. 14 pwt.}\\
\underline{12}\\
24\text{ (oz.)}\\
\underline{7}\\
31\text{ (oz.)}\\
\underline{20}\\
620\text{ (pwt.)}\\
\underline{14}\text{ (pwt.)}\\
634\text{ pwt. } \textit{Ans.}
\end{array}$$

26. Reduction Ascending.

Ex. Change 634 pwt. to higher units.

634 pwt. = $\frac{634}{20}$ oz. = 31 lb. + 14 pwt.
31 lb. = $\frac{31}{12}$ lb. = 2 lb. + 7 oz.
2 lb. 7 oz. 14 pwt. *Ans.*

or

20)634 pwt.
12)31 (oz.) + 14 pwt.
2 (lb.) + 7 oz.
2 lb. 7 oz. 14 pwt. *Ans.*

EXERCISE 32

Reduce to the lowest unit given in each example:

1. 8 lb. 7 oz. (av.)
2. 3 oz. 4 pwt. 20 gr. (T.)
3. 5 bu. 3 pk. 6 qt.
4. 2 gal. 3 qt. 1 pt.
5. 44 rd. 3 yd. 2 ft.
6. 4 da. 17 hr. 50 min.
7. 6 sq. rd. 4 sq. yd. 5 sq. ft.
8. 35° 17′ 25″
9. 2 cu. yd. 5 cu. ft. 200 cu. in.
10. 5 lb. 8 oz. 10 pwt. 9 gr.
11. 10 bu. 2 pk. 6 qt. 1 pt.
12. £10 7s. 3d.

Change to higher units:

13. 233 pt. (dry m.)
14. 712 qt. (liq. m.)
15. 120,650 oz. (av.)
16. 18,136 gr. (T.)
17. 1710 in.
18. 17,188 sq. yd.
19. 21,026″
20. 57,330 sec.
21. 14,198 ft.
22. 166,413 cu. in.
23. .608 d.
24. 380,341 in.

25. What is the cost of 2 lb. 9 oz. of beefsteak at 24¢ a pound? At 20¢? At 22¢?

26. How many grains are there in an ounce avoirdupois? Ounce Troy? Ounce apothecaries'?

27. Which is heavier and how much: a pound avoirdupois or a pound Troy? An ounce avoirdupois or an ounce Troy?

28. How many bricks 8″ × 4″ are needed for a pavement 24′ × 8′?

29. If a man in a balloon has ascended 28,000 ft. above the surface of the earth, how many miles is this and how many feet over?

30. Which have the least number of days: the spring months, the summer months, or the fall months?

31. On squared paper, or by folding paper, construct an angle of 45°.

32. How many units in 1 great gross? How many dozen in 9 score?

33. How many acres and what decimal of an acre in 156 sq. ch.? In 218 sq. in.?

34. One bin is 25′ × 6′ × 4.5′ and another is 20′ × 8.5′ × 5′. What is the easiest way of finding out which bin will hold the more? Determine this.

35. A pair of gloves cost 4.5 francs in Paris. How much was that in United States money?

36. How many days are there in 18 years, 5 of which are leap years?

37. If the sun's diameter as seen from the earth is a little over 30′, what fraction is this of a semicircle of the sky? About how many minutes will it take the sun to pass over a space equal to its own diameter?

38. If the earth's equator contains 24,902.302 miles, how many miles are there in 1 degree? In a geographical mile?

39. If the earth turns on its axis once in 24 hours, through how many degrees does a point on the equator turn in one hour? In two hours? In $3\frac{1}{2}$ hours?

40. If four carloads of coal contain 62,500 lb., 74,560 lb., 84,240 lb., 66,720 lb., respectively, how many tons and what decimal of a ton do they contain together? What is the coal worth at $4.72 per ton?

41. If corn is planted in hills 42 inches apart each way, how many hills will there be in an acre? If two grains of corn are planted in a hill and there are 2300 grains in a quart, how many bushels will be required to plant 20 acres?

42. If I pay 75 cents for a ream of typewriter paper, how many sheets do I get for one cent?

43. Sixteen sheets is what decimal part of a quire?

44. A space of at least 600 cubic feet should be allowed for each pupil in a schoolroom. What is the largest number of pupils which should be placed in a room 48′ × 33′ × 12′? (Use cancellation.)

45. Treat your schoolroom by the rule for space given in Ex. 44.

46. A book sold for 32 marks in Berlin. What is the equivalent of this price in United States money?

47. If a railroad train travels 60 miles an hour, how many feet is this per second? (Use cancellation.) How many yards?

48. How many cubic feet in $\frac{1}{3}$ cu. yd.? In 3 cu. yd.? In $2\frac{2}{3}$ cu. yd.?

49. What is the volume of a box whose dimensions are 3, 5, and 6 inches? 4, 6, and 7 feet?

50. What is the volume of a 3-inch cube? Of a 4-foot cube?

51. By cutting and folding paper, make an angle of 22° 30′. Also one of 11° 15′.

52. Can you find out what a protractor is, and what it is used

for? Let some pupil (or the teacher) bring a protractor to school and show the rest of the class what it is.

53. If a dozen pairs of gloves bought in Paris cost 37.62 fr., find the cost of 1 pair in United States money. In English money. In German money.

54. "Boxing the compass" is naming the points of the compass in order. Box the compass from N. to E. From W. to S.

55. On a certain date the record for the mile run was 4 min. $15\frac{3}{5}$ sec. How many feet per second is this, on the average?

56. By how many cubic inches does $1\frac{1}{4}$ cu. ft. differ from one bushel? What decimal part of a bushel is this difference?

57. If you breathe 25 cu. in. of air 20 times per minute, how many cubic feet of air will you breathe in two hours?

58. How heavy is a piece of ice $12'' \times 12'' \times 7\frac{1}{2}''$ if 1 cu. ft. of ice weighs 60 lb.?

59. How many degrees and minutes between two consecutive points of the compass?

60. If the wind shifts from N. E. to N. W., through how many degrees does it shift?

61. Make up and work two examples similar to Ex. 60.

62. If a horse eats 3 qt. of oats per day, how many weeks will 12 bu. 2 pk. last him?

63. A man traveling in Europe has $20 with which to buy an overcoat. What would this be in English money? In French money? In German money?

64. Make up and work a similar example concerning a woman who has $42.50 with which to buy a dress.

65. If a clock loses 1 minute 22 seconds from May 1 to June 1, how many seconds does it lose per day?

66. Three places, A, B, C are given. B is 100 yd. north of A, and C is 100 yd. east of B. What direction is C from A?

67. If 4 ears make 1 qt. of shelled corn, how many ears will a farmer have to select in order to get $12\frac{1}{2}$ bu. of shelled corn?

68. State as many advantages as you can in having the watches and clocks in a city indicate the same hour at the same time.

69. If 1 gal. of ice cream costs $1.20, what should 2 qt. 1 pt. cost?

70. Find the volume and surface of a cubic tin can whose edge is 6 in. Also of one whose edge is 12 in. The volume of the second can is how many times the volume of the first? The surface of the second can is how many times the surface of the first? Is it cheaper, therefore, to ship goods in large or small packages?

71. Cut out and fold pasteboard so as to form a cubic inch.

EXERCISE 33

ORAL

In avoirdupois weight, tell how many

1. Ounces in 2 lb. In $2\frac{1}{2}$ lb. $\frac{3}{4}$ lb. $\frac{5}{8}$ lb.

2. Pounds in 32 oz. In 80 oz. $1\frac{1}{2}$ tons. $2\frac{1}{4}$ tons.

State how many

3. Inches in 6 ft. In $7\frac{1}{2}$ ft. 1 yd. 2.5 ft.

4. Feet in 5 yd. In 2 rd. 1 mile. 60 in.

5. Square feet in 3 sq. yd. In $5\frac{1}{3}$ sq. yd. 2 sq. rd.

6. Square inches in 4 sq. ft. In 25 sq. ft. 1 sq. yd.

7. Pints in a gallon. In a peck. In a bushel.

8. Quarts in a gallon. In a barrel. In 4 gal. 3 qt. $2\frac{1}{2}$ pk. 1.75 bu.

9. Gallons in 36 qt. In 48 qt. 3 bbl.

10. Days in 6 weeks. In 96 hours. 3 years.

11. State how many ounces in 1 lb. 3 oz. In 2 lb. 8 oz. 3 lb. 3 oz.

12. Bushels in 12 pk. In 64 qt. 64 pk.

13. How long is the day when the sun rises at 7 A. M. and sets at 4 P. M.? How long, when it rises at 4 A. M. and sets at 7 P. M.?

14. What part of a foot is 3 in.? 4 in.? 6 in.? 8 in.? 9 in.? 10 in.?

15. If a pint of water weighs a pound, and a glass of water is half a pint, how much does a glass of water weigh? A quart fruit jar full of water? A gallon fruit jar?

16. Which is the larger measure: a dry pint or a liquid pint?

17. If a pint of wheat weighs a pound, how many pounds would a bushel of wheat weigh?

18. What part of a yard is 9 in.? 12 in.? 18 in.? 24 in.? 27 in.?

19. What instant is the exact middle of the week? Of July? Of November?

20. If potatoes are mainly water, what ought a bushel of potatoes to weigh?

21. What part of a mile is 40 rd.? 80 rd.? 200 rd.? 280 rd.?

22. Compare 3 square feet with 3 feet square. 6 square yards with 6 yards square.

23. How many ounces of water in a glassful? How many 2 oz. bottles can be filled from a glassful of water?

24. How many chains are there in 400 rd.? In 110 rd.?

25. How many tons in 8000 lb.? In 7000 lb.? 6500 lb.?

26. In what year and on what day did the twentieth century begin?

EXERCISE 34

ADDITION AND SUBTRACTION OF COMPOUND NUMBERS

1. Add 4 bu. 3 pk. 5 qt.; 3 bu. 3 pk. 3 qt.; 5 bu. 2 pk. 7 qt.

bu.	pk.	qt.
4	3	5
3	3	3
5	2	7
14	1	7 *Sum*

The sum of the quarts is 15 qt., which, reduced to pecks, is 1 pk. 7 qt. We set down the 7 qt. and add the 1 pk. with the pecks column.

Let the pupil complete the explanation.

2. Subtract 2 bu. 3 pk. 4 qt. from 5 bu. 2 pk. 7 qt.

bu.	pk.	qt.	
5	2	7	
2	3	4	
2	3	3	*Difference*

4 qt. from 7 qt. leaves 3 qt.

3 pk. cannot be subtracted from 2 pk. Hence, we take 1 bu. (or 4 pk.) from the 5 bu., and add it to the 2 pk., giving 6 pk.

3 pk from 6 pk. leaves 3 pk.
2 bu. from 4 bu. leaves 2 bu.
Hence, the difference is 2 bu. 3 pk. 3 qt.

3. Find the difference in time between Nov. 19, 1912, and June 16, 1916.

In finding the difference in time, a month is usually taken as 30 days.

yr.	mo.	da.	
1916	6	16	
1912	11	19	
3	6	27	*Difference*

Let the pupil supply an explanation.

Add:

4.

bu.	pk.	qt.
7	3	5
4	2	6
10	3	7

5.

£	s.	d.
5	10	6
3	15	11
7	14	8

6.

cu. yd.	cu. ft.	cu. in.
7	20	1115
3	19	1076
10	13	263

Subtract:

7.

bbl.	gal.	qt.
9	24	2
1	19	3

8.

gal.	qt.	pt.
27	2	0
18	3	1

9.

48°	19′	7″
39°	41′	32″

10. The latitude of New York is 40° 45′ 23″ N. and that of London is 51° 25′ 38″ N. Find the difference. Also find the difference in latitude in miles, if 1° = 69.15 miles.

11. If Rio Janeiro is 22° 54′ 24″ S. latitude, find the difference in latitude between it and New York. Also find the difference in latitude in miles, if 1° = 68.85 miles.

12. At New York City on June 21, the sun rises at 4.27 A. M. and sets at 7.36 P. M. How long is the day?

At Charleston, S. C., on June 21, the sun rises at 4.52 A. M. and sets at 7.10 P. M. How long is the day at Charleston? Find the difference between this and the day at New York.

13. At New York City, on Dec. 21, the sun rises at 7.21 A. M. and sets at 4.35 P. M. How long is the day?

At Charleston, S. C., on Dec. 21, the sun rises at 7 A. M. and sets at 4.59 P. M. How long is the day? Find the difference between this and the day at New York.

14. Find the interval between Oct. 14, 1066, and each of the following dates: Oct. 12, 1492; Dec. 21, 1620; Apr. 19, 1775; Oct. 19, 1781; Apr. 30, 1789; May 1, 1898. What important event happened on each of these dates?

15. Add 7 ft. 9 in. to 8 ft. 6 in. Now add 7.75 ft. and 8.5 ft. Which process is easier? Why?

Make up a similar example showing that it is easier to add United States money than English money.

16. The standard width of a railroad track is 4 ft. $8\frac{1}{2}$ in. How much less is this than 6 ft., which is the width preferred by some railroad engineers?

17. If the record for the pole vault is 12 ft. $10\frac{7}{8}$ in. and a boy can vault 9 ft. $11\frac{3}{4}$ in., how far below the record is he?

18. A storekeeper had 20 bushels of potatoes and has sold at different times 1 bu. 1 pk., $\frac{1}{2}$ pk., $\frac{3}{4}$ pk., 2 bu. 1 pk. How can he tell how many potatoes he has left, without taking the trouble to measure them? Find this for him.

19. Make up and work a similar example concerning a barrel of vinegar which at first contained $31\frac{1}{2}$ gallons.

20. A boy is now 5 ft. $3\frac{1}{2}$ in. tall, and his father is 6 ft. $2\frac{1}{4}$ in. tall. Find the difference.

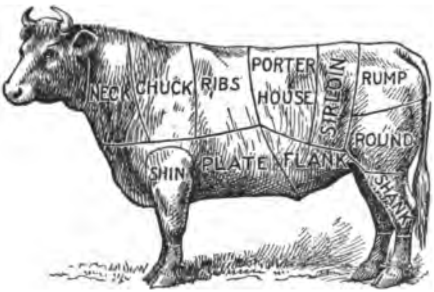
21. George Washington died Dec. 14, 1799, at the age of 67 yr. 9 mo. 22 da. What was the date of his birth?

22. Abraham Lincoln was born Feb. 12, 1809, and lived 56 yr. 2 mo. 3 da. What was the date of his death?

23. Find out the latitude of the place in which you live. By how much does this differ from the latitude of Washington (38° 55′ 15″ N.)?

24. If on a certain day it is high water at New York at 11.02 A. M., and high water at New Haven comes 3 hr. 5 min. later, at what hour is it high water at New Haven?

25. This diagram shows the parts of a beef. Draw a diagram of your own, showing the shank, round, sirloin, chuck, etc.



26. From this diagram make up and work 3 examples in the addition or subtraction of compound numbers.

27. Work again Exercise 5 (p. 13), or similar examples dictated by the teacher or some pupil.

EXERCISE 35

MULTIPLICATION AND DIVISION OF COMPOUND NUMBERS

1. Multiply 4 gal. 3 qt. 1 pt. by 5.

gal.	qt.	pt.	
4	3	1	
		5	
24	1	1	*Product*

Let the pupil supply an explanation of the process.

2. Divide 20 bu. 1 pk. 3 qt. by 7.

	bu.	pk.	qt.	
7)	20	1	3	
	2	3	5	*Quotient*

Let the pupil supply an explanation.

3. Divide 56 lb. 9 oz. 12 pwt. by 9 lb. 5 oz. 12 pwt.

Reducing each of the two compound numbers to the same lowest unit:

56 lb. 9 oz. 12 pwt. = 13,632 pwt.
9 lb. 5 oz. 12 pwt. = 2272 pwt.

$$\begin{array}{r} 6 \text{ Quotient} \\ 2272\overline{)13632} \\ \underline{13632} \end{array}$$

Multiply:

4. £8 15 s. 9 d. by 6.

5. 25 gal. 3 qt. 1 pt. by 3.

6. 3 yd. 2 ft. 8 in. by 10.

7. 7° 17′ 45″ by 15.

8. Divide 256 mi. 265 rd. by 9.

9. Divide 102 bu. 1 pk. 4 qt. by 7.

10. Divide 33° 45′ by 15.

11. Divide 42° 15′ 40″ by 15.

12. If a lighthouse uses 2 gal. 1.5 qt. of oil each day, how much oil will it use in 90 days?

13. Multiply 8 bu. 2 pk. by 17. Now multiply 8.5 bu. by 17. Which process is easier? What does this example teach us concerning the utility of decimal fractions?

14. Divide 6 gal. 1 qt. 1.4 pt. by 5. Now divide 6.425 gal. by 5. Which process is easier?

15. If a horse eats 1 pk. 4 qt. of oats per day, how long will 11 bu. 2 pk. last him?

16. In a four-mile relay race the time of the different runners was 4 min. 46 sec., 4 min. 42 sec., 4 min. 56 sec., and 4 min. 29 sec., respectively. What was the average time of the runners?

17. If a boy lives 1 mi. 200 yd. from the schoolhouse and he walks the distance twice a day 5 days in a week for 38 weeks, how far does he walk in going to school for a year?

18. How many curbstones, each 2 ft. 4 in. long, will be needed in putting a curb along both sides of a street 678 yd. 8 in. long?

19. If I save 1 hr. 15 min. each day, how much time do I save in 5 years of 300 work days each? What is this time worth at 40¢ an hour? Is the money value the only value of the time saved?

20. What is the total length in yards and feet of 45 steam pipes, each 18 ft. 3 in. long?

21. If it takes a boy 2 hr. 12 min. to walk from a farm to the city, while he can ride the distance on his bicycle in 47 min., how much time will he save by using his bicycle on 20 trips to the city and back?

22. Make up and work an example similar to Ex. 12. To Ex. 15. Ex. 19.

23. Work again Exercise 9 (p. 15).

LONGITUDE AND TIME

27. Relation of Longitude and Time. The earth revolves on its axis from west to east once in every 24 hours. As a result, the sun appears to go round the earth from east to west in the same time. Hence, if we are at a given place on the earth, at all places east of that place any particular time, as noon, is earlier, since the sun arrives there earlier; at all places to the west, noon is later, since the sun arrives there later.

Since the sun passes over 360° of longitude in 24 hours, in 1 hour it passes over $\frac{1}{24}$ of 360°, or 15° of longitude. In 1 minute of time it passes over $\frac{1}{60}$ of 15°, or 15′ of longitude; in 1 second of time it passes over $\frac{1}{60}$ of 15′, or 15″ of longitude.

TABLE

15° of longitude = 1 hour of time.
15′ of longitude = 1 minute of time.
15″ of longitude = 1 second of time.

By means of this table, if we know the difference of longitude between two places, we may determine their difference in time; and, *vice versa*, if we know their difference in time, we may determine their difference in longitude.

The navigator of a ship frequently uses the difference in time between two places to determine the longitude of the ship. Sometimes, also, when he knows the difference in longitude, from it he determines the difference in time and corrects his chronometer.

EXERCISE 36

ORAL

1. What difference in longitude corresponds to a difference in time of 3 hr.? Of 5 hr.? Of 2 hr. 30 min.? Of 40 min.? Of 50 min.? Of 1 hr. 10 min.? Of 30 sec.? Of 10 min. 30 sec.?

2. State what difference in time corresponds to a difference in longitude of 30°. Of 60°. Of 45′. Of 1° 30′. Of 2° 30′. Of 4° 30′. Of 75°. Of 135°.

3. In which direction is the earth revolving on its axis? In which direction does the sun appear to be moving?

4. When it is noon at Chicago, is it morning or afternoon at Boston? At Denver? At New York? At Omaha? At San Francisco? At London? At Paris? At Washington, D. C.? At Galveston? At St. Louis? At Montreal? At Havana? At your home?

28. I. Given the Difference in Time of Two Places, to Determine their Difference in Longitude.

Ex. The difference in time between Boston and Washington is 23 min. 47 sec. What is the difference in longitude?

23 min. difference in time corresponds to 15′ × 23, or 5° 45′, difference in longitude.

47 sec. difference in time corresponds to 15″ × 47, or 11′ 45″, difference in longitude.

$$\begin{array}{rr} \text{Adding,} & 5^\circ\ 45' \\ & \underline{11'\ 45''} \\ & 5^\circ\ 56'\ 45'' \end{array} \ \textit{Ans.}$$

29. II. Given the Difference in Longitude of Two Places, to Determine the Difference in Time.

Ex. The difference in longitude between New York and San Francisco is 48° 23′ 45″. Find the difference in time.

48° difference in longitude corresponds to $\frac{48}{15}$ hr., or 3 hr. 12 min., difference in time.

23′ difference in longitude corresponds to $\frac{23}{15}$ min., or 1 min. 32 sec., difference in time.

45″ difference in longitude corresponds to $\frac{45}{15}$ sec., or 3 sec., difference in time.

Adding 3 hr. 12 min., 1 min. 32 sec., and 3 sec., we obtain 3 hr. 13 min. 35 sec. *Ans.*

If two places are both in east longitude, or both in west longitude, *subtract* in order to get their difference of longitude; if one is in east, the other in west, longitude, *add* their longitudes in order to get the difference in longitude.

EXERCISE 37

Determine the difference in longitude, having given the difference in time, as follows:

1. 1 hr. 25 min. 10 sec.
2. 5 hr. 42 sec.
3. 7 hr. 55 min. 49 sec.
4. 9 hr. 31 min. 59 sec.

Find the difference in time between two places when the difference in longitude is as follows:

5. 43° 10′
6. 77° 40′ 15″
7. 12° 7′ 45″
8. 69° 33′ 30″

9. The difference in time between New York and Paris is 5 hr. 5 min. 20 sec. What is the difference in longitude? At which city is it noon first?

10. The difference in longitude between two cities is 42° 8′ 30″. What is the difference in time? When it is noon at the western city, what is the time at the eastern? When it is 10 A. M at the eastern city, what time is it at the western?

11. When it is 12 o'clock at San Francisco, it is 2 hr. 58 min. 23⅓ sec. P. M. at Rochester. What is the difference in longitude?

12. A ship's chronometer set at Greenwich reads 8 hr. 14 min. 56 sec. P. M. when the sun is on the meridian. What is the longitude of the ship?

13. The longitude of Galveston is 94° 46′ 34″ W. and of Mobile is 88° 1′ 19″ W. When it is 10 A. M. at Mobile, what time is it at Galveston?

14. If a boat race is decided in London (long. 0° 5′ 48″ W.) on the Thames at 5.30 P. M., and the result is known in Philadelphia (long. 75° 9′ 54″ W.) 5 minutes later, at what time is it known in Philadelphia?

15. The San Francisco (long. 122° 25′ 42″ W.) earthquake occurred at 5.13 A. M. April 18, 1906. What was the local time in New York (long. 74° 0′ 3″ W.) when it happened? In Chicago (long. 87° 36′ 42″ W.)?

16. What is the longitude of the place in which you live? Find the difference between this longitude and the longitude of New York. Express this difference in longitude as a difference in time.

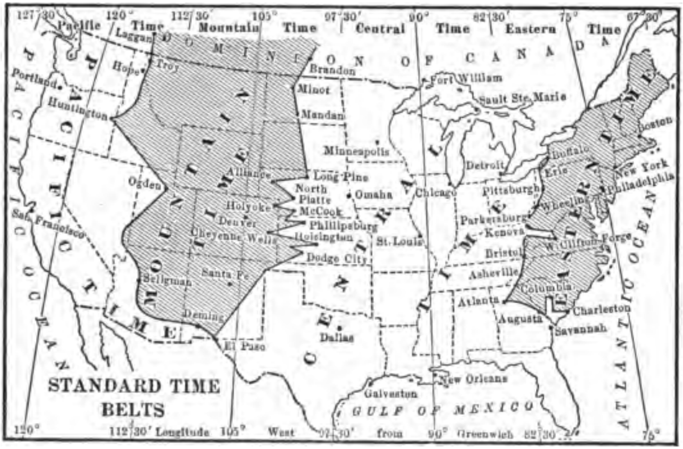
17. The longitude of Calcutta is 88° 23′ 15″ E. At 7 P. M. in Chicago, what is the time in Calcutta? At 6 P. M. in Calcutta, what is the time in Chicago? (See Ex. 15.)

18. When it is 8 hr. 12 min. 48 sec. A. M. at Jerusalem (long. 35° 32′ E.), it is 6 A. M. at Paris. Find the longitude of Paris.

19. Make up and work an example similar to Ex. 12. To Ex. 14. To Ex. 15.

20. Work again Exercise 11 (p. 17).

30. Standard Time. Since 15° of longitude correspond to 1 hour of time, it has been found convenient to divide the territory of a country into belts 15° wide, the time in each belt being determined by a meridian approximately central in the belt. Time determined in this way is called Standard or Railway time. The standard meridians in the United States and Canada are the 75th, 90th, 105th, and 120th (west from Greenwich), and the corresponding belts are said to have Eastern, Central, Mountain, and Pacific time. Standard time in the eastern part of a belt may thus be as much as half an hour ahead of true local time; and in the western part of a belt it may be half an hour behind. Travelers in passing from one belt to another must change their time by one hour. The boundaries of belts have been made somewhat irregular, owing to the configuration of the country, local conveniences, etc. The following map shows the standard time belts in the United States.



31. The International Date Line. If a person should start at London and travel around the world in a westerly direction, setting his watch back 1 hour for every 15 degrees of longitude he passed over, on reaching London again he would have set back his watch 24 times and he would be exactly one day behind time on his reckoning. Hence it is agreed that on passing the 180th meridian a person going in a westerly direction shall advance his time reckoning by one day (*i. e.* skip one day in the calendar). For example, if he crosses the 180th meridian at 2 P. M. on Aug. 18, in writing a letter at 2.05 P. M., he should date the letter Aug. 19.

In like manner, in going around the world in an easterly direction, in crossing the 180th meridian a person should deduct one day in his time reckoning (*i. e.* use one day in the calendar twice).

Hence the 180th meridian from Greenwich is called the *international date line.*

EXERCISE 38

1. By use of the map of standard time belts, state the difference in standard time between Chicago and Portland, Oregon. Chicago and Savannah. Boston and Denver. Boston and Los Angeles. Galveston and Santa Fe.

2. Which of the time belts in the United States has the greatest area?

3. How many standard time belts are needed to cover the whole of the earth's surface?

4. Why should the first standard time belt start with the meridian of Greenwich?

5. How many hours after the polls close in Philadelphia do they close in San Francisco, if the closing time in both cities is 7 P. M. (standard time)?

6. If a baseball game is finished at 6 P. M. at New York, how is it that the score can appear in a paper published at 3.30 P. M. the same day in Seattle?

7. When a traveler left New York, the clocks in a railroad station indicated 2.25 P. M., and when he arrived in Chicago the station clocks indicated 10.40 A. M. How long a time had the journey occupied?

8. If a ship leaves San Francisco on April 3 and arrives at Tokio on April 21, how many days has the voyage occupied?

9. Make up and work a similar example about a ship crossing the Pacific Ocean from west to east.

10. Work again Exercise 12 (p. 19).

EXERCISE 39

REVIEW

1. What is the difference in volume between 1 bushel and a box each inside edge of which is 13 inches? Hence, how may you visualize 1 bushel?

2. Make up and answer a similar example concerning a gallon and a box 6′ × 6′ × 6′.

3. Find the area of the top of a table $7\frac{1}{2}$ ft. long and $3\frac{1}{4}$ ft. wide.

4. If Niagara Falls is receding at the rate of 4.2 ft. per year, how long is it since the Falls were at Lake Ontario, 14 miles distant?

5. A gallon is what common fraction of a cubic foot? What decimal fraction?

6. If the United States forest reserves cover 180,000,000 acres, how many square miles is this? How many states of the size of New Jersey? Of New York?

7. If in the United States 13,000,000 bales of cotton averaging 500 pounds per bale are raised on 32,000,000 acres, how many pounds is this per acre?

8. If a storm center travels at the rate of 600 miles a day, how long does it take to travel from Denver to New York City (2100 miles)?

9. The Troy pound is what decimal fraction of the pound avoirdupois? (Find to three places.)

10. At the rate of how many miles per hour is a man walking, who passes through a street 528 yards long in 6 minutes?

11. If 1 railroad tie is used for every $2\frac{1}{4}$ ft. of track and the first tie is laid at the beginning of the mile, how many ties will be used in laying a mile of track? What will these cost at $.40 a tie?

12. A man has ascended in a ballon 33,000 ft. above the earth's surface. How many miles is this and what decimal of a mile (to 2 places)?

13. A boy can run 100 yd. in 10 sec. If he could continue to run for an hour at the same rate, how many miles would he go in the hour? (Use cancellation.)

14. State the advantages in the use of standard time as compared with local time.

15. Find the cost of 25,500 lb. of wheat at $1.05 a bushel, if 1 bu. = 60 lb. (Use cancellation.)

16. A boy hires a motor boat at $1 an hour and in proportion for part of an hour. He uses the boat from 10.30 A. M. to 1.15 P. M. How much does the boat cost him?

17. Find the area of the surface of a cube whose edge is $1\frac{1}{4}$ inches. Also find the volume of this cube.

18. How many cubic yards are there in a ton of water? How many ounces does a cubic inch of water weigh?

19. A stream is 25 yd. wide, 4 ft. deep on the average, and flows at the rate of $2\frac{1}{2}$ ft. per second. How many cubic feet of water will fall over a dam on this stream in one minute?

20. The distance from the earth to the sun is how many times as great as the distance from the earth to the moon? (See p. 385.)

21. If 10 chickens together weigh 37 lb. 12 oz., what is their average value at 22¢ a pound?

22. Find the difference in cubic inches between 8 gallons and 1 bushel.

23. If the exact length of a year is 365.2422164 days, what is the difference between 400 years and 400 times $365\frac{1}{4}$ days? Hence, how many leap years should there be in 400 years? How many are there by our present calendar?

24. Express 2 ft. 8 in. as the fraction of a yard.

25. Reduce $\frac{2}{3}$ of a mile to lower units:

$$\begin{aligned}\tfrac{2}{3}\text{ mi.} &= \tfrac{2}{3} \times 320\text{ rd.} = 213\tfrac{1}{3}\text{ rd.}\\ \tfrac{1}{3}\text{ rd.} &= \tfrac{1}{3} \times \tfrac{11}{2}\text{ yd.} = 1\tfrac{5}{6}\text{ yd.}\\ \tfrac{5}{6}\text{ yd.} &= \tfrac{5}{6} \times 3\text{ ft.} = 2\tfrac{1}{2}\text{ ft.}\\ \tfrac{1}{2}\text{ ft.} &= 6\text{ in.}\end{aligned}$$

Hence, 213 rd. 1 yd. 2 ft. 6 in. *Ans.*

26. Reduce to lower denominations:

(*1*) $\frac{7}{8}$ mi.
(*2*) $\frac{25}{72}$ degree
(*3*) .425 gal.
(*4*) .170 A.
(*5*) .45 yr.
(*6*) .725 yr.
(*7*) .06 yr.
(*8*) 23.5 ft.
(*9*) 42.67 bu.

27. See if you can write the prime numbers between 0 and 100 in less than one minute.

28. If the rainfall during a certain year was $36\frac{1}{2}$ in. and that for the three winter months $17\frac{5}{8}$ in., what was the rainfall for the rest of the year?

29. If $a = \frac{27}{16}$, $b = \frac{25}{18}$, $c = \frac{8}{35}$, find the product of a, b, c in the shortest way.

30. A piece of ribbon 1 ft. 6 in. long and another 2 ft. 4 in. long are to be cut into the largest possible pieces of equal length. Find the length of these pieces. Have you found a G. C. D. or an L. C. M.?

31. What are the advantages in knowing whether a given number, as 19, is prime or not? Give an example showing that it is useful to know that 18 can be factored.

32. Find all the exact divisors of 5280 which are less than 25. Treat 1760 and 1728 in the same way. What is the advantage in knowing these divisors?

33. How many yards of calico at $12\frac{1}{2}$¢ a yard will be received in exchange for 12 lb. of butter at 33¢ a pound?

34. If sound travels 1090 ft. per second, how many miles away was a flash of lightning, if the thunder which began with the flash reached me in $6\frac{3}{4}$ seconds?

35. Find the cost of 23,500 bricks at $7.25 per M.

36. Of 16,800 stamped envelopes at $21.40 per M.

37. Three boys gather 2 bu. 8 qt. of chestnuts. The boy who climbed the trees received twice as large a share as each of the other boys. How many quarts did each boy receive?

38. A wire fence 212 yd. 1 ft. long is fitted with uprights 2 ft. 2 in. apart. How many uprights are required?

39. If 3,500,000 lb. of sugar are bought at $4\frac{1}{4}$¢ a pound and sold at $4\frac{3}{8}$¢, find the profit in the shortest way. What would be the long way of computing this?

EXERCISE 40

ORAL REVIEW

1. A wagon with its load of hay weighs 4600 lb. If the wagon weighs 1600 lb., what is the hay worth at $16 a ton?

2. If a train travels 40 miles an hour, how far will it go in 90 minutes? In 75 minutes?

3. From a 10-pound package of tea, 3 lb. 8 oz. have been sold. How much is left?

4. Which has the greater area: a blackboard 8 ft. × 3 ft. or one 10 ft. × $2\frac{1}{2}$ ft.?

5. If a boy buys a bushel of peanuts for $1.80 and sells them at 10 cents a quart, how much does he make?

6. If I leave New York at 2.35 P. M. and arrive at Chicago at 10.45 A. M., how long did the journey occupy?

7. If a baseball game is finished at 6 P. M. in San Francisco, what is the (standard) time in New York at the close of the game?

8. A grocer buys tomatoes at 25¢ a bushel and retails them at the rate of 2 qt. for 5¢. How much does he gain on a bushel?

9. A dealer buys a half dozen saws for $7.20, and sells them so as to gain 50 cents on each saw. What is the selling price of one saw?

10. Which are the next 3 leap years? How do you tell the leap years? Will 1926 be a leap year?

11. What was the first day in the eighteenth century? The last day? In which century were you born? In which century is Dec. 25, 1900? How many leap years in the twentieth century?

12. What is the longitude of your nearest city? Its latitude?

13. Which is heavier: a pound of gold or a pound of meat? An ounce of gold or an ounce of meat?

14. Which is the greater: 6 dozen dozen or half a dozen dozen?

15. Which is greater: $\frac{1}{2}$ yd. or 1 ft. 9 in.? $\frac{1}{2}$ gal. or 10 pt.?

16. How many examples in Exercise 15 (p. 22) can you work at sight?

EXERCISE 41

INDUSTRY REVIEW: WHEAT

1. A field of wheat $\frac{1}{2}$ mile long and 140 rods wide produced 2660 bu. of wheat. How many bushels was this per acre?

2. By the use of labor-saving machinery, the time which was once required to cultivate 1 acre of wheat has been reduced from 61 hours to 3 hours. If this acre produced 20 bu., how much time was used in raising 1 bu. of wheat by each method? At 20¢ an hour, what was the labor cost of raising 1 bu. of wheat by each method?

Name as many kinds of labor-saving machines used in producing wheat as you can.

3. 1000 lb. of wheat take from the soil 24 lb. of nitrogen, 9 lb. of phosphoric acid, and 6 lb. of potash. Find the money value of these elements at 15¢, $4\frac{1}{2}$¢, and 5¢ per pound, respectively. How much is this per bushel of wheat, if 60 lb. of wheat make a bushel?

Find out some of the ways in which these elements can be restored to the soil.

4. If selected seed should add $\frac{3}{10}$ to the crop of Ex. 1, what would be the gain in value of the crop per acre, at 95¢ a bushel?

5. If a wheat harvester cuts a swath 7 ft. wide and moves at a rate of $3\frac{1}{2}$ miles an hour, how many acres will it cut and bind in 10 hours? If a wheat header cuts a swath 12 ft. wide, how many acres will it harvest in the same time at the same rate?

6. If a steam thresher threshes 3000 bu. of wheat per day of 10 hours, while the old-fashioned horse thresher turned out 6 bu. per hour, one steam thresher equals how many horse threshers?

7. Wheat bran was once thrown into the river as worthless by the flour mills at Minneapolis. It is now known that 1000 lb. of wheat bran contain 26.7 lb. of nitrogen, 28.9 lb. of phosphoric acid, and 16.1 lb. of potash. What is the value of a ton of wheat bran as a fertilizer at 15¢, $4\frac{1}{2}$¢, and 5¢ a pound, respectively, for the above elements?

8. On 20 acres of grazing land a cattleman once raised one steer in 4 years. Later this land was planted with wheat and produced 15 bu. per acre per year. If the profit on the steer was $42 and on the wheat was 37¢ a bushel, how much greater was the profit in 4 years by the second method of using the land?

9. If a certain wheat farm in North Dakota is 20 miles long and contains 90,000 acres, how wide is the farm?

10. How do the corn crop and the wheat crop of the United States compare in value, if the corn crop is 3 times as large as the wheat crop and is worth $\frac{2}{3}$ as much per bushel?

11. If the wheat crop of the United States for a certain year was 630,000,000 bu., and this was .21 of the crop of the entire world, what was the crop of the world?

12. In a given year, the yield of wheat in the United States was 14 bu. per acre, and in Great Britain was 31.8 bu. per acre.

If 45,000,000 acres were planted with wheat in the United States, by how much would the crop have been increased, if the yield per acre had equalled that in Great Britain?

13. 9 min. of labor by one man are required to raise 1 bu. of wheat, 2 min. of labor to grind it, and 5 min. to make the flour from it into bread. If one person eats in one year the bread made from 12 bu. of wheat, how many minutes of labor by one man are required to supply one person with bread for a year?

14. Make up and work five examples of your own concerning the raising of wheat.

15. If you live in the country (or visit the country), find the numerical facts concerning the production of some standard crop, as potatoes, oats, hay, flax, and make up and work a set of examples concerning this crop.

CHAPTER IV

PRACTICAL MEASUREMENTS

EXERCISE 42

1. A sea captain wishes to take his vessel over a bar at the mouth of a river. How can he find out whether it is possible for him to do this, without making an actual trial and thus running the risk of losing his vessel?

2. In like manner, how can he tell whether his vessel will go through the lock of a given canal?

3. Give an illustration of your own which shows the utility of measurement.

4. I wish to measure the length and width of a building. Name some measuring instruments which I might use. Which of these would it be most convenient to use? Why?

5. Which would be likely to give the most accurate result? Why?

6. Make up examples similar to Exs. 4 and 5 concerning the laying out of a tennis court.

7. In getting the area of a field, compare the work of marking out the field in square rods and counting the number of square rods, with the work of measuring the length and breadth of the field and taking the product of the two dimensions.

8. Make up a similar example concerning ways of finding the volume of a box.

32. Measurement is the process of finding how many times an object contains a given object or quantity of the same kind, taken as a unit.

Thus, in measuring a heap of potatoes, we find how many times a bushel measure may be filled from the heap.

The measurement of straight lines is usually much easier than any other kind of measurement. Hence, wherever possible, the measurement of areas, volumes, weights, etc., is converted into a measurement of straight lines.

Practical Measurement of Lines

33. Special Units of Length. Beside the units of length given on p. 57, certain special units of length are used in special departments of work. Thus,

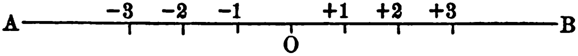
A *pace* (or long step) = 3 ft.

A *military pace* = 2 ft. 6 in.

A *knot* (or nautical mile) = 6080.27 ft., or 1.151+ mi., or $1\frac{1}{7}$ mi. approx.

Also the *fathom* (= 6 ft.) is used by seamen; and the *hand* (= 4 in.) is sometimes used in measuring the height of a horse.

34. Positive and Negative Number. *Negative* number is number opposite in some quality (as direction) from some other number taken as *positive.*



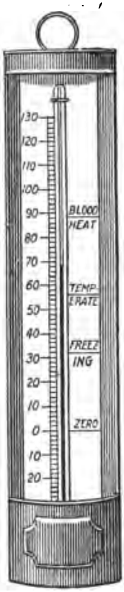
Thus, if on the line AB a distance to the right of the point O is called *plus* or *positive,* a distance to the left of O would be *minus* or *negative.*

If 35° of east longitude is plus, what will 35° of west longitude be?

If 25° of north latitude is plus, what will 25° of south latitude be?

A positive number is often indicated by placing the sign + at the left of the number, and a negative number by placing the sign − at the left. Thus, 25° south latitude may be expressed as latitude −25°.

35. Thermometer. In measuring temperature (that is, the degree of heat or cold) by means of a thermometer, temperatures above the zero point are called *positive*, and those below *negative*.



In writing a temperature, which is easier: to write 40 degrees, or to write 40°?

Point out the different marks and words on the face of a thermometer and explain their meaning.

EXERCISE 43

1. If a cannon has a range of 17,000 yd., how many miles and what decimal of a mile is this?

2. A cable 200 fathoms long is how many feet long? What decimal of a mile is its length? Answer the same for a cable 75 fathoms long.

3. A horse $15\frac{1}{2}$ hands high is how many feet and inches high? A horse $16\frac{1}{4}$ hands high?

4. If one horse is $14\frac{3}{4}$ hands high and another is $16\frac{1}{2}$, what is the difference in their height in inches?

5. How many fathoms are there in a mile? How many paces?

6. If a thermometer ranges in a single day from 23° to 52°, how many degrees does the temperature change? How many, if it varies from -7° to 19°?

7. The lowest temperature at which bananas should be shipped is 13° above freezing, and for cherries 20° below freezing. Find the difference between the two temperatures.

8. If the lowest temperature in a certain town in winter

during a given year was $-3°$, and the highest in the summer of the same year was 99°, what was the range of temperature for the year?

9. How many examples in this Exercise can you work at sight?

10. Work again Exercise 20 (p. 28) or similar examples concerning small fractions dictated by the teacher.

Practical Measurement of Areas

36. Estimating Areas of Land; the Running Foot. When the length of the front of a lot of land is the most important element in determining the value of the lot, land is often sold by the number of linear feet in its front and not by its actual area. This is called selling the land by the *running foot,* or *front foot.*

Ex. 1. At $172.50 per running foot, find the cost of a city lot whose front is 47 ft.

$$47 \times \$172.50 = \$8107.50 \textit{ Ans.}$$

The *pace* (= 1 yd.) is frequently used in making rough estimates of land areas.

Since one acre is a little less than 4900 sq. yd.,

One acre approximately equals a square each side of which is 70 *paces* (or yards).

Ex. 2. I wish to buy a certain rectangular field if it contains 5 acres. On pacing off the land, I find it to be 91 paces wide and 210 paces long. Will it pay me to hire a surveyor to determine the exact area of the piece of land?

$$\text{Approx. area of land} = \frac{\overset{13}{\cancel{91}} \times \overset{3}{\cancel{210}}}{\underset{10}{\cancel{70}} \times \cancel{70}}\text{ A.} = 3.9\text{ A.}$$

Hence, the field is too small for further consideration.

In making a rough estimate of an area, it is often an aid to remember that 1 *square mile* = 640 *acres.*

Ex. 3. A piece of forest land is $\frac{3}{4}$ mile long and $\frac{5}{8}$ mile wide. How many acres does it contain?

Area of forest = $\frac{3}{4} \times \frac{5}{8}$ sq. mi. = $\frac{15}{32} \times 640$ A. = 300 A. *Ans.*

It is also well at this point to recall that 10 *square chains* = 1 *acre.* (How long is a chain?)

EXERCISE 44

1. How many acres are there in a field 250 ft. × 300 ft.? 100 yd. × 180 ft.? 30 chains × 15 chains?

2. Think of a square each side of which is $1\frac{1}{2}$ miles long. How many acres does the square contain?

3. What is the value of a rectangular lot of ground 220 ft. long and 60 ft. wide at $.60 per square foot? What, at $120 per front foot?

4. Make up and work a similar example.

5. A tennis court is 78′ × 36′. If a margin of 15 ft. is annexed on each side and end, what will be the area of the whole?

6. A standard swimming pool is 75′ × 30′. Find the difference between the area of the swimming pool and a tennis court. Draw a diagram of the two which will show as plainly as possible this difference in area.

7. How many acres of floor space are there in a building 20 stories high, with 12,000 sq. ft. on each floor, on the average?

8. Suppose that a man without a machine can plant 1 acre of potatoes in one day and a man with a machine can plant $3\frac{1}{2}$ acres. On squared paper draw diagrams to show the relative amount

planted. (Let 1 A. = 70 × 70 sq. yd.; and one space on the paper = 10 yd.)

9. What is the area of a rectangle 24 ft. by 8 ft.? What is the area of a rectangle of double this length and width? On squared paper draw diagrams showing the two rectangles.

10. What is the area of a rectangle 18 ft. by 12 ft.? Increase each of these dimensions by one half of itself. By what fraction have you increased the area?

11. Compare the area of a field 120 paces long and 81 paces wide, with that of a field 108 paces long and 98 paces wide.

12. How many examples in Exercise 16 (p. 23) can you now work at sight?

EXERCISE 45

Oral

Tell how many acres there are in a field

1. $\frac{1}{4}$ mi. × $\frac{1}{4}$ mi.
2. $\frac{1}{2}$ mi. × $\frac{3}{8}$ mi.
3. $\frac{1}{4}$ mi. × $\frac{1}{8}$ mi.
4. $\frac{3}{4}$ mi. × $\frac{1}{2}$ mi.
5. $1\frac{1}{2}$ mi. × $\frac{1}{2}$ mi.
6. 1 mi. × $\frac{1}{16}$ mi.

If we regard a square 70 yd. × 70 yd. as an acre, tell how many acres there are in a field

7. 140 yd. × 70 yd.
8. 140 paces × 35 paces
9. 630 yd. × 280 yd.
10. 700 yd. × 35 yd.
11. 210 paces × 105 paces

Approximately in

12. 71 yd. × 133 yd.
13. 34 yd. × 71 yd.
14. 69 yd. × 210 yd.
15. 71 yd. × 141 yd.

16. Is a field 120 paces long and 40 paces wide greater or less than an acre? A field 200 paces × 60 paces greater or less than 2 acres? One 40 paces × 20 paces greater or less than half an acre?

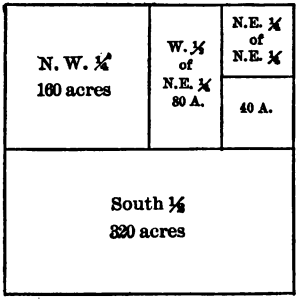
37. Division of Land into Townships and Sections. Most of the land in the northeastern part of the United States was taken by the original settlers in parts which formed no regular system. In the Western and Southern states, however, the government originally owned most of the land and divided it in a systematic way before disposing of it to the settlers.

By the plan adopted by the government, land is first divided into *townships*, each township being a square whose side is 6 miles; hence, a township contains 36 square miles. The sides of a township run east and west, and north and south.

Each township is subdivided into 36 *sections*, each section containing 1 square mile or 640 acres. The sections in a township are numbered according to a regular plan from 1 to 36.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

A Township



A Section

A section is subdivided into quarter-sections, half-quarter-sections, and quarter-quarter-sections or lots. Hence, a quarter-section contains 160 acres, a half-quarter-section 80 acres, and a lot 40 acres. A half-quarter-section is often termed "an 80," and a lot "a 40."

A *tier* of townships is a line of townships running east and west. A *range* of townships is a line of townships running north and south.

EXERCISE 46

1. Draw a plan of a township, divide it into sections, and number the sections.

By subdividing the sections on your drawing, show

2. N. E. $\frac{1}{4}$ of Sec. 20. S. E. $\frac{1}{4}$ of Sec. 33. S. W. $\frac{1}{4}$ of Sec. 36.

3. The north half of S. E. $\frac{1}{4}$ of Sec. 20.

4. The north half (N. $\frac{1}{2}$) of the S. E. $\frac{1}{4}$ of Sec. 20. The S. $\frac{1}{2}$ of S. W. $\frac{1}{4}$ of Sec. 12.

5. The S. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of Sec. 14.

6. The N. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of the school section (numbered 16 on the drawing).

7. Write and locate three other parts of the various sections in a township.

8. How many acres are there in 3 sections? In 5 sections? $\frac{1}{2}$ section? $\frac{3}{8}$ section? $2\frac{5}{8}$ sections?

9. How many rods of fence are necessary to inclose a section? A half-section? A quarter-section?

10. If all the lines on the diagram of a township on p. 94 are public highways, how far by road is it from the S. W. corner of Sec. 1 to the N. E. corner of Sec. 33?

11. How far from the N. E. corner of Sec. 2 to the nearest point of the school section (16)?

12. Rewrite Exs. 3, 4, 5, and 6, using the terms 40 and 80, wherever they are appropriate.

13. How many examples in Exercise 21 (p. 29) can you now work at sight?

38. In **paving**, the unit of measurement is the *square yard*. In **roofing, flooring**, etc., a unit called the *square* is sometimes used; this equals 100 square feet.

In roofing with shingles, the average shingle is taken to be 18 inches long and 4 inches wide, with 5 inches exposed to the weather. 1000 shingles, or 1 bundle, are allowed for 1 square.

The cost of paving is sometimes computed by the running foot.

EXERCISE 47

Find the cost of

1. Paving a walk with concrete, if the walk is 20 ft. 6 in. long and 9 ft. wide, at 90¢ per square yard.

2. Shingling a roof 18 yd. × 5 yd. at $9.25 a square.

3. Tinning a roof 32′ 9″ × 16′ 6″ at $1.30 per square yard.

4. Paving a city street $2\frac{1}{4}$ mi. long and 42 ft. wide at $1.20 per square yard.

5. If it takes 42,800 shingles, each 3 in. wide, to build a given house, how many shingles each 4 in. wide would be needed?

6. A roof is 42 ft. long and 15 ft. wide. How many 4-inch shingles exposed 5 in. to the weather will it take to cover the roof? If shingles cost $18 per thousand and it costs $4.20 to have the shingles put on the roof, what will be the cost of the roof? Would it be more economical to have the roof shingled at $20 a square?

7. How many bricks, each 8″ × 4″, are needed in making a pavement $24\frac{1}{2}'$ × 6′? (Use cancellation.)

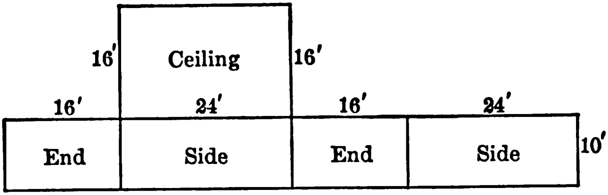
8. Find the cost of shingling in your own neighborhood, and make up three examples of different kinds concerning it.

9. If pieces of slate are 9″ wide and are laid 6″ in length to the weather, how many pieces are needed for a roof 40′ × 18′?

10. How many examples in Exercise 19 (p. 26) can you work at sight?

39. Plastering and Painting are usually estimated by the *square yard.* If the total area of openings for windows and doors in a room is small, no deduction is usually made for them. In other cases, part or all of the area of such openings is deducted. In the following examples, no deduction is to be made unless special mention is made of it.

Ex. What will it cost to plaster the walls and ceiling of a room 24 ft. long, 16 ft. wide, and 10 ft. high at 27¢ a square yard, a deduction of 19 sq. yd. being made for windows and doors?



The walls (that is, the sides and ends combined) make a rectangle 2 × 24 ft. + 2 × 16 ft., or 80 ft. long, and 10 ft. wide.

Hence, area of walls = 80 × 10 sq. ft. = 800 sq. ft.
area of ceiling = 24 × 16 sq. ft. = 384 sq. ft.
Total area = 1184 sq. ft.
Deduction for openings = 171 sq. ft.
Net area = 1013 sq. ft.

$$\text{Cost of plastering} = \frac{1013 \times \$.\overset{3}{\cancel{27}}}{\cancel{9}} = \$30.39 \textit{ Ans.}$$

EXERCISE 48

Find the cost of

1. Painting the walls and ceiling of a room 19 ft. long, 12 ft. wide, and 9 ft. 6 in. high at 12¢ a square yard.

2. Kalsomining a room 22 ft. long, 15 ft. 6 in. wide, and 8 ft. 6 in. high at 6¢ a square yard.

3. Plastering a room 24 ft. 9 in. long, 16 ft. wide, and 9 ft. 6 in. high at 20¢ a square yard.

4. Plastering a room 20 ft. long, 12 ft. wide, and 9 ft. high, if 14 sq. yd. are deducted for doors and windows, at 20¢ a square yard.

5. Painting a room 40 ft. long, 31 ft. 6 in. wide, 9 ft. 6 in. high at 5¢ a square yard for the floor and 7¢ for the rest, allowing a deduction of 10 sq. yd. for windows and doors.

6. Plastering the walls and ceiling of an audience room whose inside dimensions are 60 yd. 2 ft., 40 yd. 1 ft. 6 in., 30 ft., at 9¢ per square yard, if $\frac{1}{10}$ of the wall and ceiling surface is deducted for openings.

7. Measure the schoolroom and estimate the cost of plastering, painting, or kalsomining its walls, after ascertaining the local prices for such work.

8. Work again Exercise 23 (p. 36).

40. Carpeting. Carpeting and matting are bought by length of material and without regard to width; that is, they are bought by the *running yard* (see Art. 36, p. 91).

In ascertaining the number of yards of carpet needed to cover a given floor,

Determine the number of strips required, and multiply the length of each strip by the number of strips.

A fraction of a strip (as to width) counts the same as a whole strip. When the carpet is figured, in order to match the figures, strips of carpet must usually be made a little longer than the length of the room, the unused ends being cut off or folded under.

Ex. How many yards of carpet, $\frac{3}{4}$ yd. wide, will be required to cover a floor 26 ft. long and 17 ft. wide, if the carpet runs lengthwise and $\frac{1}{6}$ yd. per strip is wasted in matching patterns?

$$\text{No. of strips} = \frac{17}{3} \div \frac{3}{4} = 7\frac{5}{9}, \text{ or 8 strips.}$$

$$\text{Length of a strip} = \frac{26}{3}\text{ yd.} + \frac{1}{6}\text{ yd.} = \frac{53}{6}\text{ yd.}$$

$$\text{No. of yards} = \overset{4}{\cancel{8}} \times \frac{53}{\underset{3}{\cancel{6}}}\text{ yd.} = \frac{212}{3} = 70\frac{2}{3}\text{ yd. } \textit{Ans.}$$

EXERCISE 49

1. In a room 19 ft. wide, how many strips of carpet 1 yd. wide are required? How many strips $\frac{3}{4}$ yd. wide?

2. How many of each of these are required in a room 17 ft. 6 in. wide?

3. In a room 22 ft. long and 17 ft. wide, how many yards of matting 1 yd. wide and running lengthwise are needed?

4. In a room 25 ft. 9 in. long and 16 ft. 8 in. wide, if the strips of carpet run lengthwise, how many yards of carpet $\frac{3}{4}$ yd. wide are needed?

5. If carpet is $\frac{3}{4}$ yd. wide, strips run lengthwise, and $\frac{1}{6}$ yd. per strip is wasted in matching patterns, how many yards must be bought for a room 9 yd. long and 5 yd. wide?

6. How many yards of carpet $\frac{7}{8}$ yd. wide will be required for a room 17 yd. × 17 ft., if strips run lengthwise? If strips run crosswise?

7. Find the cost of carpeting a room 19 ft. × 14 ft., with carpet $\frac{3}{4}$ yd. wide and costing $1.50 a yard, when the strips run crosswise and there is a waste of $\frac{1}{4}$ yd. per strip in matching.

8. A room 13 ft. × $10\frac{1}{2}$ ft. is to be carpeted with carpet $\frac{3}{4}$ yd. wide and worth $2.25 a yard. There will be waste of $\frac{3}{16}$ yd. per strip in matching. Will it be cheaper to run the strips lengthwise or across the room? What is the difference in cost?

9. Measure the floor of the schoolroom and then estimate the cost of carpeting it with carpet of various widths, as 1 yd., $\frac{3}{4}$ yd., $\frac{5}{6}$ yd., at probable prices.

10. Work again Exercise 24 (p. 37).

41. Papering. In estimating wall paper, the unit used is the *roll.* A roll is usually 18 inches (or $\frac{1}{2}$ yard) wide and 8 yards long. At times, use is made of the double roll, which has the same width and twice the length of the single roll.

One roll is usually allowed for from 30 to 34 sq. ft. of wall surface, differences being due to the various losses in turning corners, matching patterns, and fitting window and door frames.

Since only entire rolls are sold, a part of a roll counts as a whole roll.

Ex. A room is 32 ft. long, 18 ft. 6 in. wide, and 10 ft. high. Allowing 1 roll for 32 sq. ft., find the cost of the paper for the walls of this room at $16\frac{2}{3}$¢ a roll.

$$\text{Area of walls} = 10\,(2 \times 32 + 2 \times 18\tfrac{1}{2})\text{ sq. ft.} = 1010\text{ sq. ft.}$$

$$\text{No. of rolls} = \frac{1010}{32} = 31+,\text{ or }32.$$

$$\text{Cost of paper} = \$.16\tfrac{2}{3} \times 32 = \frac{\$1 \times 32}{6} = \$5.33\ \textit{Ans.}$$

EXERCISE 50

Find the cost of paper for the walls and ceiling of

1. A room 22 ft. long, 16 ft. wide, and $8\frac{1}{2}$ ft. high at 20¢ a roll, allowing one roll for 34 sq. ft. of surface.

2. A room 20 ft. 6 in. long, 15 ft. 3 in. wide, and 9 ft. high at 25¢ a roll, allowing one roll for 32 sq. ft. of surface.

3. A room 20 ft. long, 16 ft. wide, and 10 ft. high, if a roll is allowed for 30 sq. ft. of surface, and wall paper costs 24¢ a roll and ceiling paper 18¢ a roll.

4. What will be the cost of the paper for the walls of a room 40 ft. long, 32 ft. wide, and 11 ft. high, at 45¢ a roll, allowing 3 rolls for 100 sq. ft. of surface?

5. What will be the cost of paper for the walls and ceiling of a room 21½ ft. long, 16½ ft. wide, and 15 ft. high, at 60¢ a roll, allowing 3 rolls for 100 sq. ft. of surface?

6. The walls of a room 28 ft. long, 25 ft. wide, and 15 ft. high are to be papered with wall paper selling at 75¢ a roll. There is a border of 2½ ft. width at 4¢ a yard, and a baseboard 6 in. wide. If one roll is allowed for 30 sq. ft. of surface, what is the cost of the paper?

7. Make up and work three examples concerning papering or decorating the walls of a room.

8. Measure the schoolroom and estimate the cost of papering it with various qualities of paper, using probable prices.

PRACTICAL MEASUREMENT OF VOLUMES

42. Board Measure. In measuring lumber and boards, the unit used is the *board foot.* A board foot is a rectangular piece of wood 1 inch thick and 1 foot square. A board foot is often called a foot of lumber.

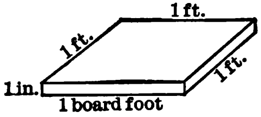
As far as possible, boards are cut 16 feet long.

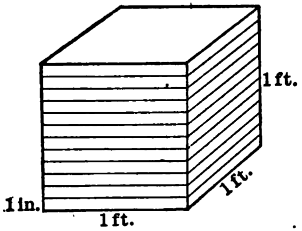
A large quantity of lumber is usually sold by the *thousand feet.* The letter M is often used to express one thousand board feet. (Why should the letter M be used for this purpose, rather than any other letter?)

The term "board feet" is often shortened to "feet."

Boards less than 1 inch in thickness are estimated as if they were 1 inch thick.

All *square* lumber, as planks, joists, beams, etc., is estimated in terms of board feet. All *round* timber, as masts, round pillars, etc., is estimated in cubic feet.





1 cubic foot or 12 board feet

To find the number of board feet in a given piece of square timber,

Reduce two of the dimensions to feet and the other dimension to inches, and multiply.

Ex. 1. Find the number of board feet in a plank 14 ft. long, 15 in. wide, and $2\frac{1}{2}$ in. thick.

$$\text{The width} = \frac{15}{12}\text{ ft.} \qquad \text{No. board feet} = \frac{\overset{7}{\cancel{14}} \times \overset{5}{\cancel{15}} \times 5}{\underset{4}{\cancel{12}} \times \cancel{2}} = 43\tfrac{3}{4}\ \textit{Ans.}$$

Ex. 2. How many feet are there in a board 14 ft. long, 10 in. wide, and $\frac{7}{8}$ in. thick?

Since $\frac{7}{8}$ inch is taken as 1 inch,

$$\text{No. board feet} = \frac{\overset{7}{\cancel{14}} \times \overset{5}{\cancel{10}} \times 1}{\underset{\cancel{6}\,3}{\cancel{12}}} = 11\tfrac{2}{3}\ \textit{Ans.}$$

EXERCISE 51

Find the number of board feet

1. In a plank 16 ft. long, 14 in. wide, and 2 in. thick.
2. In a plank 18 ft. long, 16 in. wide, and $2\frac{1}{2}$ in. thick.

3. How many board feet in a board 16 ft. long, 8 in. wide, and $\frac{5}{8}$ in. thick? $\frac{3}{4}$ in. thick? $\frac{7}{8}$ in. thick? $1\frac{3}{4}$ in. thick? $1\frac{1}{4}$ in. thick?

4. Find the number of lumber feet in a plank $14\frac{1}{2}$ ft. long, 10 in. wide, and $2\frac{5}{8}$ in. thick.

Find the number of board feet in a piece of lumber

5. $16' \times 8'' \times 3''$

6. $16' \times 8'' \times 1''$

7. $16' \times 5'' \times \frac{3}{4}''$

8. $10' \times 15'' \times 1\frac{1}{2}''$

9. $9\frac{1}{2}' \times 1' \times 1''$

10. $10\frac{3}{4}' \times 8'' \times 2''$

11. $10.5' \times 1.5' \times 3''$

12. $8\frac{1}{2}' \times 1' \times 2\frac{1}{4}''$

13. $12\frac{3}{4}' \times 1\frac{1}{4}' \times \frac{1}{4}'$

A piece of lumber 2 in. thick and 8 in. wide is often described in a short way as a 2 × 8 (read "a two by eight").

14. How many feet of lumber in a 2 × 8, 12 ft. long? In a 3 × 9, 8 ft. long? In a 4 × 10, 6 ft. long?

15. How many feet of lumber in a 9 × 12, 40 ft. long?

16. What is the cost of a board 16 ft. long, 10 in. wide, and 1 in. thick at 10¢ a foot? What is the cost, if the board is $\frac{3}{4}$ in. thick? $1\frac{1}{4}$ in. thick?

17. What is the cost of 10,400 ft. of hemlock at $26 per M?

Compute the cost of the following, using cancellation wherever possible:

	Number of Pieces	Dimensions	Cost per M
18.	10	$8' \times 4'' \times 2''$	$24
19.	40	$16' \times 10'' \times \frac{3}{4}''$	$60
20.	18	$12' \times 6'' \times 4''$	$32.50
21.	342	$16' \times 8'' \times \frac{3}{4}''$	$21.75
22.	22	$12' \times 6\frac{1}{2}'' \times 4\frac{1}{2}''$	$32.25

23. Make up and work three similar examples.

Write a receipted bill for each of the following:

24. On Dec. 12, 1914, Horace Hillman bought of Applegate and Co., of Dayton, Ohio, 1750 ft. of hemlock at $40 per M; 2865 ft. of white pine at $72 per M; 1650 ft. of cypress at $42 per M.

25. On March 3, 1914, Smith Williams bought of Applegate and Co., of Nashville, Tenn., 28,600 ft. of cedar at $48; 20,450 ft. of yellow pine at $22.50; 15,400 ft. of oak plank at $48; 28,600 shingles at $18 per M.

26. Obtain a similar bill from a dealer and compute its amount.

27. If the lumber in one of the big trees in California is equivalent to a stick 18 ft. square and 300 ft. long, what is this lumber worth at $30 per M? Is such a tree usually more valuable as lumber, or as an object to be visited and admired?

28. A single stick of lumber used in a dredge at Panama Canal was 3 ft. 4 in. square and 92 ft. long. How many board feet did it contain? What was it worth at $36 per M?

29. A log has been hewn into a timber 5 ft. square and 98 ft. long. How many board feet were there in this stick of timber?

30. If one acre of cedar timber contains 300 trees worth $3.75 on the average, how much is a square mile of such timber worth?

31. Give the meaning of sill, beam, post, stud, joist, rafter, ridgepole, gable, and siding. Make a drawing of the framework of a house, sufficiently accurate to enable you to point out these parts of the frame.

32. Work again Exercise 29 (p. 49) or similar examples concerning decimal fractions.

43. Wood Measure. The unit in measuring firewood, bark, etc., is the *cord.* A cord is a pile of wood 4 *feet wide,* 4 *feet high, and* 8 *feet long.*

Wood piled as in the picture is sometimes sold by the running foot.

EXERCISE 52

1. How many cubic feet in a cord? In $2\frac{1}{4}$ cords?

2. I paid \$15 for $2\frac{1}{2}$ cords of wood, and \$4 for having it sawed and split. What did it cost me per cord?

Tell how many cords of 4 ft. wood there are in a pile with length and height as follows:

3. 12 ft. × 4 ft.
4. 48 ft. × 6 ft.
5. 120 ft. × 4 ft.
6. 100 yd. × 5 ft.

7. A certain sailing vessel can carry an amount of wood equivalent to a rectangular solid $60' \times 18' \times 12'$. How many cords of wood is this?

8. How many cords of wood are there in a pile 30 ft. long, 18.5 ft. wide, and 12.75 ft. high?

9. A woodhouse $20' \times 15' \times 8'$ is three fourths full of sawed wood. What is this wood worth at \$8.50 per cord?

10. The wood burned by a certain stove would fill a woodbox $4' \times 3' \times 2'$ four times a week. How many cords of wood will be needed by the stove in a season of 22 weeks? What will this cost at \$4.25 per cord?

A coal stove using $2\frac{1}{2}$ tons, at \$6.50 a ton, would supply the same heat. Which kind of fuel would cost more, and how much more?

11. If the big tree mentioned in Ex. 27 (p. 104) were cut into cord wood, what would it be worth at \$3.25 a cord?

44. The **Capacity of a Bin** is usually stated as a certain number of bushels. By Art. 14, a bushel contains 2150.42 cu. in. Hence, to find the number of bushels in a bin,

Find the volume of the bin in cubic inches and divide by 2150.42.

Ex. 1. How many bushels will be contained by a bin 24 ft. long, 8 ft. wide, and 4 ft. 6 in. deep?

$$\text{No. bushels} = \frac{1728 \times 24 \times 8 \times 4.5}{2150.42} = 694.27+$$

$1\frac{1}{4}$ cu. ft. = 2160 cu. in., which exceeds one bushel by a small margin.

Hence, in many cases a sufficiently accurate determination of the number of bushels in a bin is obtained by dividing the number of cubic feet in the bin by $1\frac{1}{4}$, that is, by multiplying it by $\frac{4}{5}$. Hence,

The approximate number of bushels in a bin = $\frac{4}{5}$ *the number of cubic feet in the bin.*

Ex. 2. Find the approximate number of bushels in the bin described in Ex. 1.

$$\text{Using } \tfrac{9}{2} \text{ instead of 4.5, approx. no. bu.} = \frac{24 \times 8 \times 9 \times \overset{2}{\cancel{4}}}{\cancel{2} \times 5} = 691\tfrac{1}{5}\ \textit{Ans.}$$

Grain, seeds, and small fruits are sold by *stricken measure* (that is, by a measure filled evenly with the material sold). Coarser materials, as potatoes, corn in the ear, etc., are sold by *heaped measure* (that is, by a measure on which has been piled all the material that will stay).

The number of bushels by heaped measure = $\frac{4}{5}$ *the number by stricken measure.*

Ex. 3. How many bushels of corn in the ear will a bin 15′ × 8′ × 6′ hold?

$$\text{Approx. no. bushels} = \frac{\overset{3}{\cancel{15}} \times 8 \times 6 \times 4 \times 4}{\cancel{5} \times 5} = 460\tfrac{4}{5}\ \textit{Ans.}$$

45. Capacity of Cisterns. The unit of capacity in a cistern is usually *one gallon.*

By Art. 15, a gallon contains 231 cu. in. Hence, to find the number of gallons a cistern will hold,

Divide the number of cubic inches in the cistern by 231.

It is often convenient to use the following approximate equivalent:

$$1\ \textit{cu. ft.} = 7\tfrac{1}{2}\ \textit{gal.}$$

EXERCISE 53

1. How many bushels of oats can be put into a bin 18′ × 5′ × 4½′ by accurate computation? How many, by approximate computation?

2. How many bushels of wheat can be put into a bin 24′ × 16′ × 10′? (Approximate.)

3. A bin 22′ × 20′ × 8′ is full of potatoes. About how many bushels are there in the bin?

4. A pile of corn in the ear is estimated as 35 ft. long, 10 ft. wide, and 3½ ft. deep. How many bushels of shelled corn does the pile contain, if we take 2⅕ cu. ft. of corn in the ear as equal to 1 bu. of shelled corn?

5. A building 40′ × 40′ × 90′ will hold how many bushels of wheat, if we allow 1½ cu. ft. for each bushel?

6. A bin 6′ × 7′ × 8′ was ⅔ full of wheat. It was bought by approximate measurement at 70¢ a bushel, and was sold under accurate measurement at 90¢ a bushel. What was the gain?

7. One barrel (31½ gal.) contains how many cubic feet?

8. If 2⅕ cu. ft. of corn on the cob will make 1 bu. of shelled corn, how many bushels of shelled corn will be obtained from a crib which contains 1000 cu. ft. of corn on the cob?

Tell how many gallons there are in a cistern

9. Containing 125 cu. ft.
10. Containing 13.2 cu. yd.
11. 12′ × 4′ × 2′
12. 12′ 6″ × 4′ 6″ × 2″
13. 8.5′ × 4.2′ × 2.5′
14. 8′ 6″ × 4.5′ × 2.75′

15. How many gallons are there in a swimming pool 70 ft. long and 25 ft. wide, if the water is 6 ft. deep?

16. Make up and work three examples similar to such of the above examples as the teacher may indicate.

17. How many examples in this Exercise can you work at sight?

46. Excavations and Embankments. In moving earth, as in digging a cellar or making an embankment, the unit is one *cubic yard.* Often a cubic yard of earth is called a *load.*

Ex. At 15 cents a load, find the cost of excavating a cellar 18′ × 24′ × 6′.

$$\text{Cost} = \frac{\overset{2}{\cancel{18}} \times 24 \times \overset{2}{\cancel{6}} \times \$.15}{\underset{\cancel{3}}{\cancel{27}}} = \$14.40 \ \textit{Ans.}$$

47. Masonry and Concrete. No uniform method is followed in the measurement of masonry. Sometimes masonry is determined by the *perch,* which equals 24¾ cubic feet; at other times by the cubic foot. In computing the volume of a wall, some masons make allowance for corners. Other masons make no such allowance, but get the volume by taking the entire length of the outside wall and multiplying by the product of the thickness and the height.

Loose stone is often measured by the *cord.*

A cord = 99 cu. ft. = 4 perches.

A cord is sometimes taken as 100 cu. ft. of wall.

Bricks vary considerably in size and shape, but an ordinary brick is 8″ × 4″ × 2″. From 21 to 23 ordinary bricks are used in making a cubic foot of wall (the variation is due to the amount of mortar used).

Concrete is often used in the place of masonry. Concrete is estimated by the *cubic yard.*

EXERCISE 54

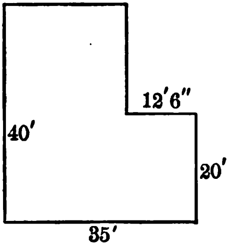
1. What is the cost of excavating a cellar 20′ × 18′ 6″ × 5′ at 10¢ per cubic yard?

2. How many cords (100 cu. ft.) of stone are in a wall 275 ft. long, 4 ft. high, and 2 ft. thick?

3. How many bricks would be required to build the wall in Ex. 2, allowing 21 bricks to the cubic foot? What would these bricks cost at $18.75 per M?

4. The adjoining diagram represents the outline of a cellar which is 5 ft. deep. How many loads of dirt would be removed in digging the cellar?

5. What is the length of the wall around the cellar? If the cellar wall is 15″ thick, and rises 2′ above ground, what would it cost to build it of concrete at $5 per cubic yard, no allowance being made for corners or openings?



6. What would it cost to build this wall (Ex. 5) of brick at $18 per M, 22 bricks per cubic foot being allowed, the cost of mortar being $12 and of labor $75?

7. How many bricks (no allowance for mortar), each 8″ × 4″ × 2″, are needed in making a wall 36′ × 6′ × 2′? (Use cancellation.)

8. How many cartloads of earth are needed to cover $1\frac{1}{2}$ ft. deep a lot 80′ × 30′?

9. Make up and work two examples concerning a stone wall and a brick wall built about the lot described in Ex. 8.

10. Work again Exercise 33 (p. 67) or a similar Exercise.

VOLUME AND WEIGHT

48. Capacity Determined by Weight. It is often convenient to *determine volume from weight.*

In making such determinations, the equivalents to be used are fixed by law in different states, and vary somewhat. The following are used in many states:

1 bu. wheat = 60 lb.
1 bu. potatoes = 60 lb.
1 bu. beans = 60 lb.
1 bu. clover-seed = 60 lb.
1 bu. shelled corn = 56 lb.
1 bu. rye = 56 lb.
1 bu. barley = 48 lb.
1 bu. oats = 32 lb.
1 bu. coarse salt = 56 lb.
1 bbl. flour = 196 lb.
1 bbl. pork or beef = 200 lb.

49. Weight from Volume. On the other hand, it is often convenient to be able to *determine weight from volume.* Thus, in shipping goods, 40 cubic feet of general merchandise are generally regarded as a ton.

A ton of coal is equivalent to from 35 to 40 cubic feet, according to the quality of the coal, soft coal being in general lighter, and therefore measuring more cubic feet per ton.

A ton of hay varies from 270 to 500 cubic feet, according to the firmness with which it is packed.

A pint of water weighs about a pound. Since a great variety of articles in common use are composed largely of water (as fruits, grains, meats), it is convenient to remember the popular rhyme,

A pint's a pound,
The world around.

EXERCISE 55

1. A box 6′ × 5′ × 4′ is full of ordinary coal. If a ton occupies 38 cu. ft., how many tons are there in the box? How many tons would there be, if it were full of Lehigh white-ash coal (1 ton = 34½ cu. ft.)?

2. If a ton of hay occupies 400 cu. ft., how many tons can be stored in a mow 40 × 28 × 20 ft.?

3. How many tons of bituminous coal are there in a train of 42 cars averaging 36 × 7 × 5 ft.? (Allow 40 cu. ft. to the ton.)

4. A farmer hauled 60 loads of hay, which averaged 18′ × 9′ × 10′ per load. What was the value of the hay at $18 a ton, if 480 cu. ft. of the hay equalled 1 ton?

5. What advantage is there in determining the number of tons of hay or coal by measuring the dimensions of the mows or bins? In what other ways might these weights be determined?

6. Which is the heaviest: 7 bu. of barley, 5½ bu. of wheat, 10 bu. of oats, or 5$\frac{6}{7}$ bu. of rye?

7. If 1 cu. ft. of water weighs 62.5 lb., how much more than a ton will 40 cu. ft. weigh?

8. If a certain kind of stone is 2.62 times as heavy as water, what will 40 cu. ft. of the stone weigh?

9. If maple wood is .67 as heavy as water, what will 40 cu. ft. of maple wood weigh?

10. Find the specific gravity of other materials (see p. 388) and make up examples concerning 40 cu. ft. of each.

The specific gravity of an object is the quotient obtained by dividing the weight of the object by the weight of an equal bulk of water.

11. If a bushel of potatoes weighs 60 lb., what is the weight of the potatoes in a bin 36′ × 4′ × 4′? (Use 1 bu. = $1\frac{1}{4}$ cu. ft.)

12. How many bushels of shelled corn are there in a ton?

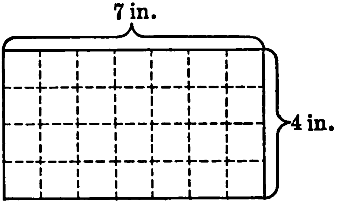
13. If shelled corn weighs 56 lb. per bushel, can 2 tons of corn be put into a bin 12′ × 4′ × 3′?

14. If 1 bu. of oats weighs 32 lb., can 3 tons of oats be shipped in a car which is 40′ × 8′ × 8′?

15. Find the weight of a gallon of canned fruit by use of the rhyme of Art. 49.

16. Work again Exercise 40 or a similar Exercise.

50. **Formulas concerning Area.** If a rectangle is 7 inches long and 4 inches wide, the area of the rectangle = 1 sq. in. × 7 × 4.



In like manner, in general,

Area of a rectangle = (1 *sq. unit*) × (*no. linear units in length*) × (*no. linear units in width*), like units being used in each case.

It is convenient to express this rule in a briefer form by the use of symbols. Let A = no. of square units in area; l = no. of linear units in length; and w = no. of linear units in width.

Then the above rule becomes the formula:

$$A = l \times w$$

In working a given example, after reducing all the numbers to like units, the 1 square unit of the rule may be omitted until the end of the process.

Ex. The area of a rectangle contains 432 sq. in. and the width is 1 ft. 6 in. Find the length.

$$\begin{aligned} A &= 432 \text{ sq. in.} \qquad w = 18 \text{ in.} \\ 432 &= l \times 18 \\ 432 \div 18 &= 24 \\ \text{length} &= 24 \text{ in. } \textit{Ans.} \end{aligned}$$

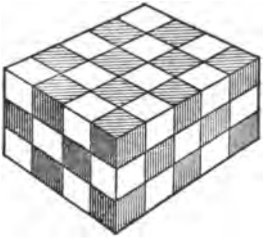
Hence, in general $l = \dfrac{A}{w}$.

In like manner, $w = \dfrac{A}{l}$.

51. Drawing to Scale. If each linear space in the diagram of Art. 50 represents 10 ft., the whole diagram will represent a building lot or a garden whose dimensions are 40 ft. and 70 ft.

On many government maps, 1 inch represents 1 mile or 63,360 inches. The scale of such a map is said to be 1 to 63,360.

Drawing to scale is making a drawing of an object so that every line of the object is represented on the diagram by a given fractional part of the line.



52. Formulas concerning Volume. If a box-shaped solid is 5 inches long, 4 inches wide, and 3 inches deep, the volume of the solid

$$= 1 \text{ cu. in.} \times 5 \times 4 \times 3.$$

Hence, if V denotes the volume of the solid, and l, w, d its length, width, and depth respectively,

$$V = l \times w \times d$$

Hence, following the method of Art. 50,

$$l = \frac{V}{w \times d}, \qquad w = \frac{V}{l \times d}, \qquad \text{and } d = \frac{V}{l \times w}.$$

Ex. A bin is 14 ft. long, 4 ft. wide, and is to contain 168 cu. ft. How deep must the bin be?

$$V = 168 \text{ cu. ft.}, \qquad l = 14 \text{ ft.}, \qquad w = 4 \text{ ft.}$$

$$168 = 14 \times 4 \times d$$

$$d = \frac{168}{14 \times 4} = 3$$

Hence, depth of bin = 3 ft. *Ans.*

EXERCISE 56

1. A rectangle is to contain 512 sq. in. and is to be 32 in. long. Find the width of the rectangle.

Make a drawing of this rectangle on the scale of 1 to 8.

2. Make up and work a similar example.

3. A garden is to be 22 yd. wide and is to contain $\frac{1}{4}$ of an acre. Find the length of the garden.

4. A certain field is to contain 48 acres and is to be $\frac{1}{8}$ mile wide. Find the length of the field in yards.

Make a drawing of this field so that 1 inch represents 100 yards.

Find the missing side, having given

	Area	One Side		Area	One Side
5.	5 A.	70 yd.	**8.**	2 A.	70 paces
6.	10 A.	145 yd.	**9.**	3.4 A.	103.2 yd.
7.	21 A.	210 yd.	**10.**	4.32 A.	317.5 ft.

11. A bin is to contain 360 cu. ft. and is to be 4 ft. deep and 15 ft. long. Find the width of the bin.

12. A bin 16 ft. long and 4 ft. wide is to contain 250 bu. (1 bu. = $1\frac{1}{4}$ cu. ft.) How deep must the bin be?

13. A cistern 6 ft. square is to contain 2000 gal. of water. How deep must the cistern be?

Find the missing dimensions in the following:

	Volume	Length	Width	Depth
14.	2000 cu. in.	25 in.	?	8 in.
15.	1 cu. ft.	20 in.	7 in.	?
16.	28 cu. yd.	?	7 ft.	2 yd.
17.	384 cu. ft.	4 yd.	4 ft. 6 in.	?

EXERCISE 57

REVIEW

1. How many board feet are there in 40 rafters, each of which is 8″ × 2″ × 18′?

2. A railroad passes through a farm, taking a strip 1½ miles long and 66 ft. wide. What is the value of this land at $80 an acre?

3. What is the cost of digging a trench 3 mi. long, 8 ft. deep, and 4 ft. wide at 15 cents per cubic yard?

4. A box is made of 2-inch material, and the outside dimensions are 12 × 10 × 9 inches. How many cubic inches are there in the material of the box, including the lid? How many cubic inches will the box contain?

5. A parlor is 50 ft. × 38 ft., and 14 ft. high. If carpet for this room is $\frac{7}{8}$ yd. wide, costs $2.50 a yard, is put down lengthwise, and every strip wastes $\frac{1}{4}$ yd. in matching, find the cost of the carpet.

6. What will be the cost of paper for the walls of this parlor at 90 cents a roll, if 60 sq. ft. are deducted from the area of the walls for openings and one roll is allowed for 30 sq. ft. of surface?

7. What will painting the ceiling (Ex. 5) cost at 18 cents a square yard?

8. At $8.40 a square, what will it cost to roof a barn whose rafters are 18 ft. 6 in. long, and whose ridgepole is 35 ft. long?

9. What will it cost to cement the floor and side walls of a cellar 25 × 24 ft., and 8 ft. deep, at 48 cents a square yard?

10. A county in one season built 17 mi. of macadamized road 12 ft. wide, at the rate of 75 cents per square yard. What was the total cost?

11. What will be the cost of the palings necessary to inclose a lawn 36 × 41 yd., if the palings are 2 in. wide, are placed 2 in. apart, and cost $2.75 a hundred?

12. A bin $15 \times 8\frac{1}{2} \times 6\frac{2}{3}$ ft. was full of apples, from which enough cider was pressed to fill three quarters of a tank $9\frac{1}{3} \times 4\frac{1}{2} \times 3$ ft. The apples were bought at 36 cents a bushel ($= 1\frac{1}{4}$ cu. ft.) and after the cider had turned to vinegar, it was sold at 33 cents a gallon. What was the gain?

13. What will be the cost of plastering the walls and ceiling of a room 16 ft. 6 in. long, 11 ft. 4 in. wide, 10 ft. high, at 36¢ a square yard, allowing 15 sq. yd. for doors and windows?

14. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what will a barrel of water weigh?

15. If a mixture of salt and ice used in freezing ice cream has a temperature of $-16°$ and the outside air has a temperature of 86°, what is the difference?

16. A manufacturer, by a change of method, saves $\frac{1}{16}$¢ per yard in making 1,000,000 yd. of muslin. How much does he save in all?

17. If one wall of a house is 24 ft. long and 22 ft. high, how many square feet of weatherboarding will it require, if we add $\frac{1}{6}$ to the area to allow for the overlapping of boards?

18. If 1000 gallons of water are put into a tank 8 ft. long and 2 ft. wide, how deep will the water be? (1 cu. ft. = 7½ gal.)

19. A tank which contains 19.85 cu. ft. holds approximately how many gallons?

20. In 1909 in New York City a plot of land containing 1825 sq. ft. sold for $1,000,000. What was the price per square foot?

21. If a large ocean steamer burns 537 tons of coal in one day, how much will the steamer burn in 5 da. 3 hr. at the same rate? How long would this last a family which burned 19 tons of coal in a year?

22. What is the advantage in weighing a large amount of milk instead of measuring it by the quart?

23. If each person in the United States on the average consumed 6½ bu. of wheat in a given year, how many bushels were consumed by the entire population of 91,500,000 persons?

How many bushels, out of a crop of 690,000,000 bu., were left for export?

24. If 1 cu. ft. of water weighs 62.5 lb., what will 1 liquid pint of water weigh?

25. The diameter of the sun is 108 times the diameter of the earth. On the blackboard draw a circle ¼ in. in radius to represent the earth. On the same scale draw a line to represent the diameter of the sun.

26. On a map, a scale of 1 to 1,000,000 would be how many miles to the inch?

27. If sound travels three times as fast in water as in air, how long does it take sound to travel a mile in water? (See p. 388.)

28. In running 100 yd. in 9⅗ seconds, how many feet were passed over in 1 second, on the average?

29. A body floating in water weighs the same as the water which it displaces. If a log containing 24.5 cu. ft. was $\frac{7}{10}$ under water, how much did the log weigh? (Weight of 1 cu. ft. of water = 62.5 lb.)

30. A bricklayer who was laying 120 bricks per hour, by following improved methods, was able to lay 350 bricks per hour. How many hours would he save in laying 100,000 bricks by the new method?

31. What is the advantage in being able to find the L. C. M. of three numbers, as of 6, 10, 15? Make up and work an example showing this advantage.

32. Make up and work an example showing how cancellation is useful.

33. In order that a door may look well, its width should be $\frac{2}{5}$ of its height. If a given door is to be 6 ft. $5\frac{1}{2}$ in. high, how wide should it be?

34. If a ton of merchandise on the average occupies 40 cu. ft. of space, how many tons can be put into a freight car 36 ft. 9 in. long, 8 ft. 4 in. wide, 8 ft. 9 in. high?

35. A man who owned $\frac{11}{32}$ of a vessel sold $\frac{1}{4}$ share in the vessel, then bought $\frac{5}{16}$, then sold $\frac{3}{8}$. What share did he then own? What is the short way of working this example? Also point out the long way.

36. A bin $12\frac{1}{2}' \times 3\frac{1}{2}' \times 2\frac{3}{4}'$ will hold approximately how many bushels?

37. If one gallon of milk weighs 8.622 lb., what will one pint of milk weigh?

38. An area of 600 sq. mi. is drained by the aqueduct which connects the Catskill Mountains with New York City. If 36 in. of rain per year fall on this area, how many gallons would this be per day? How many persons would this supply with water, at 60 gal. for each person per day?

39. A certain electric locomotive weighs 165 tons. This is equivalent to the weight of how many horses, if one horse weighs 1200 lb.?

40. Compare the rectangles 12′ × 3′, 6′ × 6′, and 9′ × 4′ as to area. Also as to perimeters.

41. What is the least fraction which, added to the sum of $\frac{17}{12}$ and $\frac{9}{5}$, will produce a whole number?

42. Simplify $\dfrac{32 + .52}{3 + \frac{1}{2}}$

43. If .325 of mortar is lime, how many pounds of lime are there in one ton of mortar?

44. Add 200 pt. and 5 gal. 2 qt. Put the result in a form easy to visualize. Also put the result in a form easy to multiply by 7.

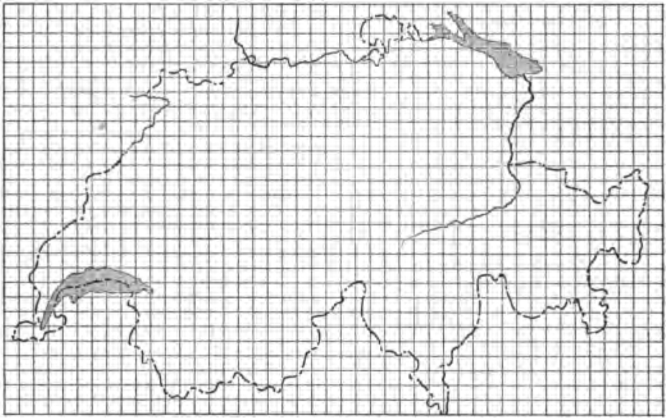
45. Find x, or the missing number, in each of the following:

	l	w	A
(1)	7 ft.	3 yd.	x
(2)	x	6 ft.	24 sq. yd.
(3)	x	$\frac{3}{4}$ ft.	48 sq. ft.
(4)	6.5 ft.	x	20 sq. ft.
(5)	x	3.2 in.	1 sq. yd.

	l	w	d	V
(6)	3′	3′ 3″	10″	x
(7)	8″	4″	x	128 cu. in.
(8)	10′	x	4′	$22\frac{2}{9}$ cu. yd.
(9)	$4\frac{2}{5}$ ft.	2 ft.	2.5 ft.	x
(10)	x	1′ 3″	2.5′	1 cu. yd.

46. By taking a map of a country and tracing the outline of the country on squared paper, the area of the given country may often be ascertained with sufficient accuracy for many purposes. In this process, it is necessary to note the scale of the map, to determine the area represented by one square space on the squared paper, and to count the number of spaces included in the traced outline. In the counting, a part of a square that is less than a half square is neglected, and a part greater than a half square is counted as a whole square.

The following is an outline of Switzerland. Find the area of Switzerland, if each small square represents 25 sq. mi.



How much does your result differ from the correct area of Switzerland?

47. By use of your geography and squared tracing paper, find the area of the state in which you live. Also of other states or countries, or of such bodies of water as the teacher may select.

EXERCISE 58

ORAL REVIEW

1. Give a rough estimate of the product of 3999 by 98.7.

SUG. The product of 4000 by 100 = 400,000. Hence, the required product must be a little less than 400,000.

2. Give a rough estimate of the quotient of 6000 by 19.

SUG. 6000 ÷ 20 = 300. Hence, required quotient is a little more than 300.

At sight give approximate answers in whole numbers:

3. $19\frac{7}{8} \times 20$

4. 100×996

5. $10,000 \div 998$

6. $23\frac{7}{8} \times 50\frac{1}{4}$

7. $\$300 \div 24\frac{7}{8}\text{¢}$

8. 999×99

9. 1001×99

10. $60 \times 80\frac{3}{4}$

11. 2000×99

12. $2000 \div 99$

13. $\$840 \div 33\frac{1}{2}$

14. $120 \times 12\frac{3}{8}$

15. $.012\frac{1}{8} \times 100$

16. $48 \times 62\frac{5}{8}$

17. 5001×999

18. $5001 \div 999$

19. $6.01 \times .602$

20. $8002 \div .49\frac{7}{8}$

21. $72.5 \times .50\frac{1}{8}$

22. 6.999×10.12

23. $\$300.65 \div .25\frac{1}{8}$

24. $500 \times .37\frac{5}{8}$

25. $1001 \times .099$

26. $1001 \div .099$

27. 60.12×60.12

28. $4500 \div .62\frac{3}{8}$

29. $64.004 \times .375$

30. 100.02×10.1

31. Change 40 thousand acres into square miles.

In order to obtain a rule for solving an example of this kind, we proceed as follows:

$$\text{No. of sq. mi. in 40,000 A.} = 40 \times \tfrac{1000}{640} = 40 \times \tfrac{25}{16} = 40\left(\tfrac{16}{16} + \tfrac{8}{16} + \tfrac{1}{16}\right)$$
$$= 40\left(1 + \tfrac{1}{2} + \tfrac{1}{16}\right) = 40 + 20 + 2\tfrac{1}{2} = 62\tfrac{1}{2}$$

In general, to change thousands of acres into square miles, *to the given number* (of thousands of acres) *add ½ and $\frac{1}{16}$ of itself.*

32. Change 80 thousand acres into square miles.

No. sq. mi. in 80 thousand acres $= 80 + 40 + 5 = 125$.

33. At sight change the following into square miles: 32 thousand acres; 48,000 A.; 120,000 A.; 16,000 A.; 92,000 A.; 50,000 A.

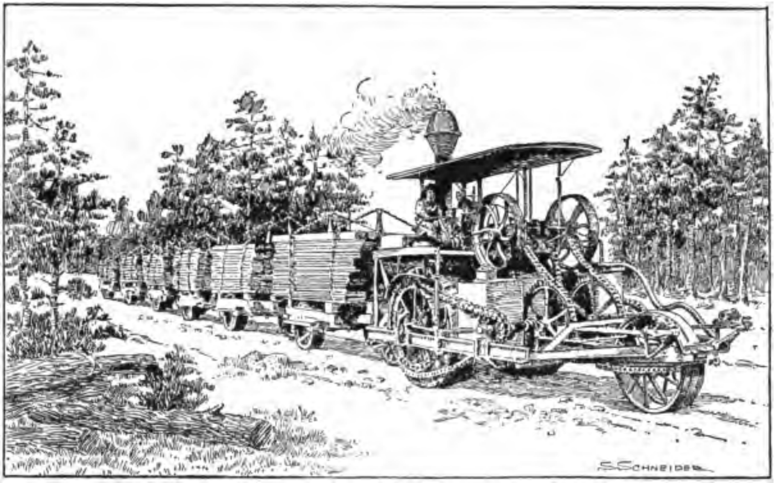
34. In one year the Pennsylvania Railroad planted 24,000 acres in trees to grow railroad ties. How many square miles was this?

35. How many acres are there in a meadow ½ mi. × ½ mi.? ⅝ mi. × ¼ mi.?

36. What is the office, or function, of the denominator of a fraction? Of the numerator?

37. What linear units are in common use? What square units?

38. If Central Park in New York City is 2½ miles long and ½ mile wide, how many acres does it contain?



EXERCISE 59

INDUSTRY REVIEW: FORESTRY

1. A man bought 27.32 acres of hardwood forest at $52.75 an acre, and 30¾ acres of softwood forest at $42.50. What did the whole cost him?

2. He cut the timber and sold 52,500 ft. of oak lumber at $68.50 per M, 27,250 ft. of hickory at $72 per M, 44,750 ft. of white pine at $67.40 per M. He also sold 247 cords of spruce for making wood pulp at $12 per cord, 144 cords of firewood at $2.75 per cord, and 14,300 lb. of bark for tanning purposes at $6 per ton. What were his total receipts for the wood cut?

3. He paid $1272.40 for the labor of cutting the trees, $437.50 for hauling the logs to the mills, and $372 for the cost of sawing. After the timber was cut, the land was sold at $22.50 per acre. What did he make by the entire transaction?

4. The Doyle rule for estimating the number of board feet in a log is as follows:

Deduct 4 inches from the diameter (of the log, inside the bark), multiply $\frac{1}{4}$ of the remainder by itself, and multiply the result by the length of the log in feet.

For example, the number of board feet in a log 12 ft. long and 10 in. in diameter is found thus:

$$10 - 4 = 6$$

$$\frac{6 \times 6}{4 \times 4} \times 12 = 27$$

27 board feet. *Ans.*

5. By the method of Ex. 4, find the number of board feet in a log 18 ft. long and 15 in. in diameter. Also in one 20 ft. long and 18 in. in diameter.

6. It is estimated that, when the United States was first settled, .62 of the land was covered with forest. How many square miles was this? How many states as large as the state in which you live would it make?

7. The forests in the United States have been cut and burned off so that now only .28 of the land is woodland. Make from this fact an example similar to Ex. 6. State some of the bad effects which will result if too large a part of our forest land is cleared.

8. In the year 1910, 17,506,000 pupils were enrolled in the public schools of the United States. If each pupil should plant 10 trees on the average on Arbor Day and the trees should be 10 ft. apart, how many acres of forest would this make? How would the ground covered compare in size with the state in which you live?

9. In Germany, in the year 1830, the amount of wood produced by each acre of forest during the year was 20 cu. ft. In

a recent year, under better management of the forests, the yield was 48 cu. ft. The gain was how many lumber feet per acre?

10. There are 180,000,000 acres of forest reserve in the United States. An increase of 15 cu. ft. per acre in the lumber cut on it would be worth how much, at $20 per M?

11. The annual loss caused by forest fires in the United States is $50,000,000. If a forest service costing $5,000,000 per year would prevent half this loss, what would be the gain in money in one year? Would there be any other advantage in this?

12. In making trees into lumber by the present methods, the loss is $\frac{1}{4}$ in logging and $\frac{1}{3}$ of the remainder in sawing. If trees containing 50 billion feet are cut in one year in the United States, what is the loss in money in these ways at $30 per M?

13. In one year the United States Government Forest Service planted 5 tons of tree seeds on 10 square miles of land. How many pounds of seed was this per acre?

14. The forest area of the United States is 550,000,000 acres, on which the annual growth is 12 cu. ft. per acre. If this growth were increased to 35 cu. ft. per acre by better management, what would the increase be worth at $20 per M of board feet?

15. If our country should have 450,000,000 acres of forest, on which the annual growth per acre was 50 cu. ft., how many board feet would this be for each person in a population of 200,000,000 people? Of 500,000,000?

16. It is estimated that the rivers of the United States annually carry 27,000,000,000 cu. ft. of soil into the sea. If .3 of the soil thus lost could be saved by proper management of our forests, streams, and agricultural lands, how many square miles of land would the soil cover 2 in. deep? How much would it add to the value of the land at $10 per acre?

17. In many cases, the United States government will send an expert to study privately owned forest lands, to study markets, freights, etc., and work out a plan for forest planting and management. How does this pay the government?

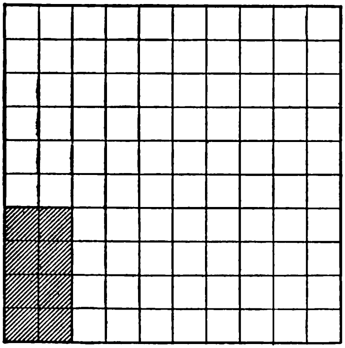
18. If you live in the country (or visit the country), find out the two most important kinds of forest trees in your neighborhood, and at what prices the useful parts of these trees are sold. Also collect or make up examples concerning the cutting and sale of these trees.

19. Find out if your state has any forest reserves and make up examples concerning them.

CHAPTER V

PERCENTAGE

EXERCISE 60



1. On squared paper mark off a square each side of which contains ten spaces. How many small squares are in the diagram? Shade 8 of these small square spaces.

2. If you should shade 8 per hundred of 300 small squares, how many would you shade? How many would 8 per hundred be in 400? In 350?

3. "Per cent" is a short way of writing "per hundred."

The Latin word for *one hundred* is *centum*, and *cent* is a short form of *centum*.

4. What is 8 per cent of 300? Of 400? Of 700? Of 450?

5. How many hundredths of the above diagram are shaded? What per cent? Is 8 per cent of the whole figure the same as 8 hundredths of it?

6. Draw a similar diagram to show 6 per cent. 10 per cent. 80 per cent.

7. What per cent of the diagram on p. 34 is shaded?

8. A stock raiser had 650 cattle and sold 8 out of every hundred; how many did he sell in all?

9. In working Ex. 8, the natural method is to take the number of hundreds, 6.5, and multiply it by 8.

Show that 650 × .08 gives the same result. The reason for this is as follows:

$$\frac{650}{100} \times 8 = 650 \times \frac{8}{100} = 650 \times .08$$

Which of the two forms, 6.5 × 8 and 650 × .08, contains the number 650 unchanged? Which of the two numbers, 8 and .08, suggests that 100 is the basis of the computation?

10. For what two reasons, then, is it better to multiply 650 by .08, than 6.5 by 8?

11. Compute 4 per cent of 350 in the most advantageous way. Also 6 per cent of 350. 2 per cent. 12 per cent. 40 per cent.

12. $1200 invested in a mortgage brings in $60. What does each $100 of the $1200 bring in?

13. $800 invested in railroad stock brings in $48. What does each $100 of the $800 bring in?

14. Which of the investments described in Exs. 12 and 13 is the more profitable?

15. Why do we now make definitions, symbols, and rules?

53. **Percentage** is the process of computing with 100 as a basis.

The **rate** per cent is the number of hundredths of a given number which is taken.

The **base** is the number of which a certain per cent is taken.

The **percentage** is the number obtained by taking a certain per cent of the base.

The symbol used for per cent is %.

Hence, $8\% = \frac{8}{100} = .08.$

The relation of the terms *rate, base,* and *percentage* may be illustrated as follows:

Ex. A poultry raiser had 400 ducks and sold 20% of them. How many did he sell?

$$\begin{array}{r r l} \text{Number of ducks owned,} & 400 & = \text{the base} \\ & .20 & = \text{the rate} \\ \cline{2-2} \text{Ducks sold,} & 80.00 & = \text{the percentage} \end{array}$$

It is often convenient to represent the base, rate, and percentage by the symbols, b, r, and p respectively.

The **amount** is the sum of the base and the percentage.

The **difference** is the difference between the base and the percentage.

Proceeds is a general term for either amount or difference.

54. Uses of Percentage. The use of 100 as a basis of comparison makes it easy to determine which of two given rates of gain, or loss, is larger. (See Exs. 12–14, p. 127.) Also the repeated use of certain per cents of gain, or loss, gives these rates a sharp and definite meaning, which aids in their rapid and accurate use.

Other advantages in percentage will become evident later.

55. I. Given the Base and Rate, to Find the Percentage or Proceeds.

Ex. 1. A school contains 240 pupils, of whom 45% are boys. How many boys are there in the school?

$$\begin{array}{r l} 240 & \textit{Base} \\ .45 & \textit{Rate} \\ \hline 1200 & \\ 960 & \\ \hline 108.00 & \textit{Percentage.}\quad 108 \text{ boys } \textit{Ans.} \end{array}$$

Hence, in general, to find the percentage,

Multiply the base by the rate expressed decimally.

In symbols, $p = b \times r$.

Also, to obtain the proceeds,

Express the rate decimally, and multiply the base by 1 *plus the rate in case of an increase, but by* 1 *minus the rate in case of a decrease.*

Ex. 2. The population of a town which contained 18,000 people, decreased 5% in a year; what was the population at the end of the year?

$$\begin{array}{r l} 18000 & \textit{Base} \\ .95 & \textit{Final rate per cent} \\ \hline 90000 & \\ 162000 & \\ \hline 17100.00 & \textit{Final population} \end{array}$$

If a number is diminished by 5% of itself, it becomes 95%, or .95 of itself. 95% of 18,000 is 17,100.

EXERCISE 61

Find:

1. 4% of $650. Of 1728 cu. in. Of 5280 ft.

2. 12% of $262.50. Of 1760 yd. Of 43560 sq. ft.

3. $23\frac{1}{2}\%$ of $4000. Of 2150.42 cu. in. Of 39.37 in.

4. 17.4% of 231 cu. in. Of 2240 lb. Of .30103.

5. If a boy had $36.50 in the savings bank on Jan. 1, and increased this by 24% during the year, what sum would he have in the bank at the end of the year?

6. If a girl had 85 chickens on March 1, and by Sept. 1 had increased these by 40%, how many chickens did she have on Sept. 1?

7. If 1 cu. ft. of water weighs 62.5 lb., and 1 cu. ft. of ice weighs 8% less, how much does a cubic foot of ice weigh?

8. Mr. X bought a farm for $4200, and by careful cultivation so improved it that he was able to sell it at an advance of 40%. What was his selling price?

9. Make up and work a similar example concerning a man who bought a horse and trained it so as to be able to sell it at an advance.

10. The profits of an unscientific farmer in one year were $1742. Later, by planting his crops in proper rotation and by the correct use of fertilizers, he increased his profits by 34%. What did he gain in one year by the new method?

11. If a certain kind of coal yields 65.9% coke, how many pounds of coke would be obtained from 5½ tons of coal?

12. If 12% of oats is protein (*i. e.* muscle-producing substance), how many pounds of protein are there in a bushel of oats? If 10% of corn is protein, how many pounds of protein in a bushel of corn?

13. If in building a macadam road the state pays 20% of the cost, the county 30%, and the township the rest, what per cent must the township pay?

14. Make up and work a similar example for yourself.

15. On squared paper draw a line ten spaces long. Mark off 10% of this line. 20%. 25%.

16. A man who had an income of $3750 gave 20% of it to support an invalid sister, and 8% to general benevolences. What per cent of his income did he have left for himself? How many dollars?

17. If a man receives $8500 for a crop of cranberries and the expense of raising the crop is 43% of this sum, what per cent of the crop is profit? How many dollars are profit?

18. Before he used antitoxin, a physician lost 80% of his patients who were sick with diphtheria. Since he has used antitoxin, he has lost only 5% of such patients. How many lives has he saved out of 620 cases treated by the new remedy? (What is the long and what the short way of solving this example?)

19. If a certain kind of copper ore is 13% copper, 30% iron, and the rest waste, what per cent is waste? In 6¼ tons of ore, how many pounds of copper are there and how many of waste?

20. If a certain quality of milk contains 3¾% of butter fat and 8½% of other solids, and the rest is water, what per cent of the milk is water? How many pounds of each of the three constituents are there in 3000 lb. of milk?

21. Practice the rapid addition of columns of figures.

EXERCISE 62

ORAL

Find:

1. 3% of 200
2. 7% of 500
3. 8% of 40
4. 20% of 65
5. 25% of 4
6. 30% of 5
7. 40% of 6
8. 25% of 90
9. 16% of 50
10. 18% of 30
11. 45% of 40
12. 7½% of $800

13. Of 75 examples, a boy solved 40%. How many did he solve?

14. A man 60 years old has a son 35% of his age. How old is the son?

15. A certain salary of $2500 is to be increased 12%. What will it then be?

16. My expenses last year were $1400, and this year they will be 5% more. What will they be this year?

17. The length of a field is 30% more than its width, which is 230 rd. What is its length?

18. The price of a carriage was 8% less than the price of a horse which cost $150. What was the price of the carriage?

19. I lost 5% of an investment of $8000 and then gained 30% of the remainder. What did I then have?

20. What is 25% of 10? Of 2? Of 5? Of 100?

21. Read Exs. 1–5, using hundredths instead of per cent.

22. What does the letter *b* stand for in percentage? The letter *r*? *p*? Show the value of *b*, *r*, and *p* in Ex. 1. Also in Exs. 5–8.

23. How many examples in Exercise 19 (p. 26) can you work at sight?

56. Rate Per Cent an Aliquot Part of 100. When a rate per cent is an aliquot part of 100, work may often be saved by using one of the following equivalents:

$6\frac{1}{4}\% = \frac{1}{16}$
$8\frac{1}{3}\% = \frac{1}{12}$
$12\frac{1}{2}\% = \frac{1}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$
$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$
$33\frac{1}{3}\% = \frac{1}{3}$
$37\frac{1}{2}\% = \frac{3}{8}$
$50\% = \frac{1}{2}$
$62\frac{1}{2}\% = \frac{5}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$
$87\frac{1}{2}\% = \frac{7}{8}$

For, by Art. 2, p. 41, $6\frac{1}{4}\% = .06\frac{1}{4} = \frac{1}{16}$, etc.

Ex. A man inherited $16\frac{2}{3}\%$ of \$45,000. How much money did he receive?

$16\frac{2}{3}\%$ of \$45,000 = $\frac{1}{6}$ of \$45,000 = \$7500 *Ans.*

57. Rates Exceptionally Large or Small. Special care should be exercised in computing a percentage where the rate is exceptionally large or small.

Ex. 1. Find 420% of \$2000.

\$2000 × 4.20 = \$8400 *Ans.*

Ex. 2. Find $\frac{3}{8}\%$ of \$2000.

\$2000 × $.00\frac{3}{8}$ = \$7.50 *Ans.*

EXERCISE 63

Find:

1. $12\frac{1}{2}\%$ of \$4872
2. 25% of 368 pupils
3. $33\frac{1}{3}\%$ of 291 acres
4. $37\frac{1}{2}\%$ of 5280 feet
5. 50% of \$2415
6. 20% of 7642 pounds
7. $87\frac{1}{2}\%$ of 512 inches
8. 315% of \$4160
9. 200% of 1760 yards
10. 100% of 520 men

11. ⅛% of $6240

12. ½% of $5246

13. ⅞% of $3550

14. 133⅓% of $663

15. 1.3% of 16

16. .06% of $200

17. Find .000015% of $3000.

18. On squared paper mark off a square each side of which is ten spaces. Now shade 16⅔% of this square. Also ⅙%. 33⅓%. ⅓%. 250%.

19. If 66⅔% of a certain ore is iron, how much pig iron can be made from 2400 lb. of the ore? From 12 tons? From 5¼ tons?

20. A certain quality of steel contains 98.1% of iron and 1.7% of carbon. What per cent of other elements does it contain? In 5000 lb. of steel, how many pounds of iron and of carbon are there?

21. If it cost $1800 to build a certain macadam road, of which sum the state paid 66⅔%, the county 16⅔%, and the township the remainder, what per cent did the township pay? What sum of money?

22. Find 300% of $15,000.

23. Find ⅛% of $42,000.

24. A city had a population of 8750 some years ago, but has now 240 per cent of its population at that time. What is its present population?

25. Which is greater: 87½% of $250, or ⅝% of $35,000?

26. Mr. Williamson owned ⅔ of a farm worth $8000, but sold 37½% of his share. What part does he still own, and what is its value?

27. Make up and work an example similar to Ex. 22. To Ex. 23. Ex. 24.

28. How many examples in this Exercise can you work at sight?

29. Work again Ex. 25 (p. 39) or similar examples dictated by the teacher.

58. II. To Find what Per Cent One Number is of Another (*i. e.* given b and p, to find r).

Ex. Out of a salary of \$2400 a man saved \$432. What per cent of his salary did he save?

To find how many hundredths \$432 is of \$2400, we divide \$432 by \$2400.

$$\begin{array}{r} .18 \text{ or } 18\% \ \textit{Ans.} \\ \$2400\overline{)\$432.00} \\ \underline{240\,0} \\ 192\,00 \\ \underline{192\,00} \end{array}$$

Hence, in general, to obtain the rate per cent,

Divide the percentage by the base.

Expressed in symbols, this rule is, $r = \frac{p}{b}$.

EXERCISE 64

1. On squared paper mark off a rectangle 30 spaces long and 10 spaces wide. How many small squares does the rectangle contain? Shade 15 of these. How many per hundred have you shaded? How many hundredths of the rectangle? What per cent of it?

2. Make up and work a similar example.

3. If, in a collection of small circles, you should shade 3 out of every 25, how many would this be per hundred? Answer the same for 3 out of every 20. Out of every 50.

4. Express each of the following as a per cent: $\frac{11}{50}$ $\frac{9}{20}$ $\frac{7}{25}$ $\frac{17}{20}$ $\frac{31}{50}$

5. If a girl receives 9 eggs out of every 50 that she finds, what per cent does she receive?

6. If a boy picks 40 qt. of blackberries and for his work receives 12 qt., what per cent does he receive?

7. Make up and work an example similar to Ex. 5. Also one similar to Ex. 6.

8. If a gold coin is 9 parts gold and 1 part copper, what per cent of the coin is gold? What per cent is copper?

Find the rate per cent when

9. Base is $45 and percentage is $9.

10. Base is 428 bushels and percentage is 160 bushels.

11. Percentage is 23 tons and base is 320 tons.

12. Percentage is $1.40 and base is $56.

13. Amount is $800 and base is $640.

14. Amount is $4420 and percentage is $420.

15. What per cent of $\frac{5}{6}$ is $\frac{1}{2}$?

16. If a certain kind of concrete is made of 1 part cement, 2 parts sand, and 4 parts crushed stone, what per cent of the whole is each of these constituents?

17. Lead is 11.35 times heavier than water. Gold is 19.256 times heavier than water. By what per cent is gold heavier than lead?

SUG. Subtract 11.35 from 19.256, and find what per cent the difference is of 11.35.

18. The area of Texas exceeds the area of the German empire (208,830 sq. mi.) by what per cent?

19. Of a farm of 240 acres, 80 acres are planted with corn, 60 with wheat, 40 with potatoes, and the remainder is used as pasture. What per cent of the land is pasture?

20. If a suit of clothes is marked down from \$30 to \$24, what per cent is deducted from its price?

21. A man bought a farm for \$3200 and, after improving it, sold it for \$6000. What per cent increase is this on the cost price?

22. A girl is offered her choice between 3 out of every 10 eggs that she finds, and 7 out of every 25. Which is the better offer? What per cent is each offer?

23. Make up and work a similar example concerning a boy picking cherries. Also a similar example wholly your own.

24. Which is the larger rate of income: \$48.50 received from \$810, or \$64.50 received from \$915?

25. A bankrupt paid 23 cents on a dollar. What per cent of his debts did he pay?

26. If a man has \$1500 and owes \$2700, how many cents can he pay on the dollar?

27. On an examination, a boy worked 15 out of 28 examples. Did he pass, if 60% is the passing mark?

28. $1\frac{1}{4}$ cu. ft. exceeds a bushel by what per cent?

29. If you spend 4 hr. 48 min. each day in school, what per cent of 24 hr. do you spend there?

30. One basketball team has won 16 games out of 21 played, and another has won 19 out of 24 games played. Find the difference between the per cents won by the two teams.

31. Make up and work a similar example concerning games played and won by three baseball nines.

32. Make up and work three examples similar to such of the examples in this Exercise as your teacher may indicate.

33. Work again Exercise 38 (p. 78).

EXERCISE 65

ORAL

Express as a per cent in the decimal form:

1. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ $\frac{2}{3}$ $\frac{3}{8}$ $\frac{1}{4}$ $\frac{3}{10}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{5}{12}$ $\frac{5}{8}$ $\frac{9}{10}$ $\frac{2}{5}$

2. $\frac{1}{7}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{7}{10}$ $\frac{5}{7}$ $\frac{4}{9}$ $\frac{7}{100}$ $\frac{19}{40}$ $\frac{7}{11}$ $\frac{11}{10}$ $\frac{12}{5}$ $\frac{25}{6}$

3. What per cent of 8 is 4? Is 2? 1? 5?

4. What per cent of 12 is 6? Is 4? 8? 2?

5. What per cent of 24 is 3? Is 6? 8? 9? 12? 15? 16?

6. 12 is what per cent of 18? Of 24? 30? 48?

7. 15 is what per cent of 25? Of 30? 35? 50? 75?

8. 18 is what per cent of 27? Of 36? 54? 90? 180?

9. 1 is what per cent of 100?

10. What per cent of a foot is 3 in.? 4 in.? 6 in.? 9 in.?

11. Make up a similar example concerning a pound and a number of ounces. Concerning parts of a bushel. Of a gallon.

12. 6 per score is what per cent? 11 per score? 8 per score?

13. If you work 8 hours out of every 24, what per cent of the time do you work?

14. A boy earned \$40 in one year and spent \$16 of it. What per cent did he save?

15. Which is easier to decide: the relative value of $\frac{4}{20}$ and $\frac{6}{25}$, or of 20% and 24%?

16. What, then, is one of the uses of a knowledge of percentage? Can you name any other uses of percentage?

17. Which is the greater rate of loss: 5 per dozen, or 7 per score?

18. Point out the value of p, b, and r in each of Exs. 5–8 in Exercise 64.

19. Work again Exercise 33 (p. 67).

59. III. Given the Rate and Percentage, to Find the Base (*i. e.* given r and p, to find b).

Ex. 1. Of a given farm, 28%, or 70 acres, are planted with corn. How many acres are there in the farm?

$$
\begin{array}{llcl}
 & 28\% \text{ of the farm} & = & 70 \text{ acres} \\
\text{Hence,} & 1\% \text{ `` `` ``} & = & \frac{1}{28} \text{ of } 70 \text{ acres} = 2.5 \text{ acres.} \\
 & 100\% \text{ `` `` ``} & = & 100 \times 2.5 \text{ acres} = 250 \text{ acres.}
\end{array}
$$

That is, the whole farm = 250 acres.

Or, more briefly, since $\frac{70}{28} \times 100 = \frac{70}{.28}$

$$
\begin{array}{r}
2\,50.\ \textit{Ans.} \\
.28.\overline{)70.00.} \\
\underline{56} \\
140 \\
\underline{140} \\
0
\end{array}
$$

Hence, in general, to find the base, either proceed by analysis, that is,

Obtain 1% *of the base by dividing the percentage by the per cent, and multiply the result by* 100, *in order to obtain the* 100% *of the base;* or,

Divide the percentage by the rate, expressed decimally.

In symbols, $b = \frac{p}{r}$

Ex. 2. A house is sold for $4800 at a gain of 20%. What was the cost of the house?

$$
\begin{array}{rcl}
\text{Let } 100\% & = & \text{Cost of the house} \\
\text{Then } 120\% & = & \$4800
\end{array}
$$

Hence, $\frac{\$4800}{1.20} = \$4000$ *Ans.*

Ex. 3. In a certain kind of iron ore, $66\frac{2}{3}\%$ is iron. How many tons of ore are needed in order to obtain 2460 tons of iron?

$66\frac{2}{3}\% = \frac{2}{3}$

Hence, $\frac{2}{3}$ of the ore = 2460 tons of ore
$\frac{1}{3}$ " " " = 1230 " " "
$\frac{3}{3}$, or all the ore = 3690 " " "

EXERCISE 66

Find the base, if

1. Percentage is $120 and rate is 6%.
2. Percentage is 760 bu. and rate is 5%.
3. Percentage is 115 gal. and rate is $2\frac{1}{2}\%$.
4. Rate is 4% and percentage is 550 da.
5. Rate is 23% and percentage is $27.83.
6. Rate is $1\frac{1}{2}\%$ and percentage is $225
7. 140 is 8% of what number?
8. 169 is $3\frac{1}{4}\%$ of what number?

Find the quantity of which

9. 48 bu. is 20%.
10. 75 da. is 25%.
11. 145 tons is 5%.
12. $1476 is 48%.
13. 97 is $12\frac{1}{2}\%$.
14. 123 is $66\frac{2}{3}\%$.
15. 429 is $37\frac{1}{2}\%$.
16. $31\frac{1}{4}$ is $\frac{3}{8}\%$.

What quantity increased by

17. 27% of itself is $439.42?
18. $33\frac{1}{3}\%$ of itself is $114.48?

What quantity diminished by

19. 25% of itself is 96?
20. $16\frac{2}{3}\%$ of itself is $76\frac{1}{2}$?
21. Find the cost, if the selling price is $50 and the gain is 25%.
22. Also if the selling price is $7.14 and the loss is $33\frac{1}{3}\%$.

23. If 7 per cent of a square lot is worth $21, what is the whole lot worth? On squared paper draw a diagram showing the meaning of this.

24. The office expenses for a year in a certain business were reduced $567, which was 7% of what the expenses had been. What were the office expenses during the preceding year?

25. If 5% of bronze is tin, 5% zinc, and the rest copper, what per cent is copper? 10 lb. of tin will be sufficient in making how many pounds of bronze?

26. If 2.75% of milk is butter fat, how many pounds of milk will be needed in order to get 55 lb. of butter fat?

27. Make up and solve a similar example concerning mortar and lime, taking mortar as 32% of lime.

28. After 3% is deducted from $3880, how much will be left?

29. A boy has gained 20% during the last year and now weighs 90 lb. What was his weight a year ago? What will be his weight a year hence at the same rate of growth?

30. A grocer sold 20 eggs for 55 cents, thereby gaining 10%. What did a dozen eggs cost him?

31. A certain kind of muslin will shrink $2\frac{1}{2}$% of its length. How many yards of this muslin must a housekeeper buy, in order that she may have 124 yd. after it is shrunk?

32. I sold 2 horses for $180 each; on the one I gained 20%, and on the other I lost 20%. Did I gain or lose on them both together? How much?

33. Make up and work three examples similar to such of the above examples as you think are most interesting or instructive.

34. Work again Exercise 40 (p. 83) or a similar Exercise.

EXERCISE 67

ORAL

1. If the percentage is $75 and the rate is 3%, what is the base?
2. What is the base, when the rate is 8% and the percentage is 24 bu.?
3. $24 is 20% more than what number of dollars?
4. 27 is 10% less than what number?
5. The gain is $40, or 5%. What was the cost?
6. The loss is $30, or 15%. What was the cost?
7. The selling price was $84, and loss was 30%. What was the cost?
8. I gained 20% when I sold a lot for $60. What was the cost?
9. A knife selling at 70 cents yielded 40% gain. Find the cost.
10. If the percentage is 63 bu. and the rate is 70%, find the base.
11. If the proceeds are 525 tons and the rate is 5% gain, find the base.
12. What was the base, if the rate was 16% and the percentage $16?
13. Work again Exercise 58 (p. 120) or a similar Exercise.

EXERCISE 68

REVIEW

Draw a diagram similar to the following, and fill in the given, or known, parts in each of Exs. 1–10 on page 142. Then compute the unknown part for each of these examples.

Ex.	1	2	3	4	5	6	7	8	9	10
b =										
p =										
r =										

The same method may be used with other groups of examples in Chapter V, at the option of the teacher.

1. 56 is what per cent of 240?

2. 42 is 6% of what number?

3. Find 7% of $351.70.

4. The tax rate is $1\frac{1}{5}\%$ and a man pays $36.60 in taxes. What is the valuation of his property?

5. A death rate of 17 per 1000 is what per cent?

6. $\frac{22}{7}$ exceeds 3.1416 by what per cent?

7. The area of its outlying possessions is what per cent of the area of the United States proper? (See p. 383.)

8. If a certain quality of limestone, when burned, yields 62% of lime, how much lime will be obtained from $2\frac{1}{2}$ tons of limestone?

9. If soft iron weighs 468 lb. per cubic feet and steel 492 lb. per cubic feet, steel is what per cent heavier than soft iron?

10. If a horse is bought for $160 and sold at a profit of 20%, what is the gain in dollars?

	Base	Percentage	Rate
11.	$236	$34.22	?
12.	?	$12.41	$21\frac{1}{4}\%$
13.	736 lb.	?	$2\frac{1}{3}\%$
14.	$58\frac{3}{5}$ bu.	8.76 bu.	?
15.	?	$355.	$\frac{1}{2}\%$
16.	$460	?	$1\frac{1}{4}\%$
17.	$427\frac{3}{5}$	$534\frac{1}{2}$	?
18.	?	750.527	$105\frac{1}{2}\%$
19.	$896	?	$96\frac{3}{4}\%$

20. If A has $3 and B has $4, B has what per cent more than A? A has what per cent less than B?

21. Which would you rather have: .1 of a coal mine, or 2% of

it? The greater of these shares is how many times as large as the smaller?

22. Find the sum of the areas of all states which differ from New York State by less than 30%.

23. Find the difference between 67% of 320 and 25% of 320 in the shortest way.

24. If the mean annual rainfall in Arizona is 8 in., in Colorado 12 in., in New York 36 in., and in Alabama 54 in., the first of these numbers is what per cent of the last? The second, of the last? The third, of the last?

25. A carpenter built a house at an expense of $5200, and sold it for $7098. What was the gain per cent?

26. A wholesale grocer buys coffee at 30¢ and sells it at 36¢ a pound. The local grocer buys at 36¢ and sells at 45¢. What per cent does each make? What per cent would the wholesale grocer make, if he sold directly to the consumer for 45¢?

27. Owning 30% of an office building, a man sold 25% of his share and then valued the remainder at $9000. What was the entire building worth at this valuation?

28. A motor cycle was sold for $190 at a loss of 24%. What should it have been sold for, to gain 5%?

29. If the cost of a certain article was $\frac{6}{7}$ of the selling price, what was the per cent of gain?

Prove your answer in a case where the cost is $240.

30. In a recent year, the farms and stock in the United States were valued at $28,508,000,000 and their products for a year at $7,500,000,000. What per cent is the latter number of the former?

31. If $\frac{1}{100}$, .01, and 1% have the same value, why do we have these three different forms to express this one value?

32. Find 60% of 2000. Also 6%. .6% .06% .000006% 6000%

33. The receiver for a bankrupt business paid the creditors at different times 72¢, 10¢, and 3¢ on a dollar. What per cent did he pay in all?

34. On squared paper show the meaning of 100% − 18%. Of 54% ÷ 3.

35. A boy has a garden plot 36 ft. long and 20 ft. wide. If he increases its length and width each by 50%, by what per cent will the area be increased?

36. Make up and work a similar example.

37. On squared paper show the meaning of $\frac{1}{8}$%. 80% 8% 800%

38. The interval from 1776 to the present year is what per cent of the interval from 1492 to the present year? From 1066 to the present year?

39. Make up and work a similar example.

40. By what per cent does the volume of a solid 5″ × 5″ × 10″ differ from a gallon?

41. The area of the United States (3,025,600 sq. mi.) differs from 3,000,000 sq. mi. by what per cent?

42. What is incorrect in the statement, "He increased his percentage from 18% to 23%"?

43. If dynamite is made up of 290 parts clay, 73 parts nitroglycerine, and 2 parts coal dust, what per cent of the whole is each of these constituents?

44. One kind of copper is 8.3 heavier than water. Another kind is 8.7 heavier than water. By what per cent is the second kind heavier than the first?

45. Make up and work an example in which $p = 400$, and $r = 20\%$. Also one in which $b = \$300$, and $r = 5\%$. Also one in which $p = \$60$, and $b = \$240$.

46. 90% of an object is how many times as large as the rest of the object?

47. 64% of an object is how many per cent less than $\frac{2}{3}$ of the object?

48. The English bushel (2218.192 cu. in.) exceeds the bushel in use in the United States by what per cent?

49. A death rate of 47 per 1000 is what per cent?

50. Express 3.2% entirely in the form of a decimal fraction.

SUG. $\frac{3.2}{100} = .032$

51. In like manner, express 5.4%. 15.3% .09% 1.09%

52. Write $\frac{1}{20}\%$ in the briefest decimal form.

53. Of the eggs put into a certain incubator, 50% hatched. Of the number of chickens hatched, 10% died. In all, 540 chickens were raised. How many eggs were put into the incubator?

54. Find the volume of a bin 24 ft. 9 in. long, 4 ft. 6 in. high, and 3 ft. 6 in. wide.

55. A boy who was recovering from typhoid fever gained 15 lb. 9 oz. in weight in 3 weeks. What was his gain per week? Per day?

56. In a scientific book I read that "pitchblende contains 48.5 millionths per cent of radium." Express this per cent as a decimal fraction. How many grains of radium are there in 5 tons of pitchblende?

57. If the Panama Canal is 49 miles long, and Gatum Lake is 23 miles long, what per cent of the length of the canal is the lake?

58. If the land surface of the world is 51,238,800 square miles, the area of the United States is what per cent of the land surface of the earth?

59. By what per cent does the area of Texas exceed that of a square each side of which is 500 miles long?

60. How many square yards are there in $1\frac{3}{4}$ yd. of cloth 28 in. wide?

61. Estimate the dimensions of your schoolroom, and then measure them. By what per cent is your estimate of each dimension incorrect?

62. The protein (or muscle-making substance) in oats is 12% of the whole, and in corn is 10%. How much more protein is there in 500 lb. of oats than in 500 lb. of corn?

63. Work at sight as many examples as you can in Exercise 21 (p. 29).

EXERCISE 69

ORAL REVIEW

In each of the following, give the missing number at sight:

	b	p	r
1.	$200	$50	x
2.	x	$200	4%
3.	800	x	5%
4.	x	$\frac{3}{4}$	$33\frac{1}{3}$%
5.	4.8	2.4	x
6.	x	8	10%
7.	1000	x	100%

8. Find $\frac{1}{2}$% of 40. 400 216 1000

9. Find $\frac{1}{8}$% of the numbers in Ex. 8. 1% 20% $\frac{3}{4}$%

10. What is the meaning of the word *per?* Of the word *centum?* Hence, of *per centum* or *per cent?*

11. 100% = how many hundredths? Find 100% of $250. Of $300. Of $100.

12. What is the difference between "rate per cent" and "percentage"? Show this by an example.

13. A full set of teeth consists of 8 incisors, 4 eyeteeth, 8 bicuspids, and 12 molars, or 32 teeth in all. What per cent of a complete set of teeth are the eyeteeth? Incisors? Bicuspids?

14. Find 5% of 200. 200 is 5% of what number? 5 is what per cent of 200?

15. $80 is 4% of what number? Find 4% of $80. 4 is what per cent of 80?

16. $60 is 10% of what number? $10 is what per cent of $60? Find 10% of $60.

17. A yellow wall paper reflects 45% of the light falling on it, while a red wall paper reflects 15%. The light reflected by the yellow paper is how many times as great as that reflected by the red paper?

18. A pupil has solved correctly 6 out of 11 examples. Is this more or less than 60%, the passing mark? Answer the same question for 7 out of 11 examples.

19. Find $33\frac{1}{3}$% of 60. 60 is $33\frac{1}{3}$% of what number?

20. If $37\frac{1}{2}$% of a certain number is 63, find $12\frac{1}{2}$% of the number. Also $87\frac{1}{2}$%.

EXERCISE 70

INDUSTRY REVIEW: FRUIT RAISING

1. If peach trees are planted 18 ft. apart each way, how many rows would there be in a field 1230 ft. long and 350 ft. wide? How many trees in each row? How many trees in all?

2. If trees were bought in an exact number of hundreds, how many hundreds were bought for this field? What did these trees cost at $9 per hundred?

3. Find to two decimal places the number of acres in the field.

4. The cost of plowing the field, and setting out and pruning the trees was $110. What was this per acre? Per tree?

5. 150 lb. of sulphate of potash, 100 lb. of ground bone, and 200 lb. of acid phosphate per acre were applied each year. Find the cost per acre of these fertilizers, if the cost per 100 lb. of each of the three fertilizers in order is $4, $3.50, and $3.75.

6. Find the cost of these fertilizers on the entire field for four years.

7. The second year ground limestone was applied at the rate of 2500 lb. per acre. What was the cost at $3.50 per ton?

8. The cost of cultivating the field during the first year was $7.25 per acre. During this year, 420 baskets of lima beans were raised on the field, and were sold at a profit of 37¢ a basket above the cost of planting and gathering them. By how much did the profit on the beans exceed the cost of cultivation?

9. If it cost $1.75 to spray one acre of the peach orchard,

what did it cost to spray the orchard three times a year for four years?

10. Four years after planting, the trees produced a crop of 924 baskets of peaches, which sold at a net profit of 72¢ a basket. What was the net profit on the crop?

By *net profit* in this case is meant the profit after the expenses of gathering and selling the crop have been deducted.

11. The next year the orchard produced 3172 baskets of fruit, which were sold while on the trees to a speculator for 80¢ per basket. By how much did the receipts for this crop exceed the entire expense of the first year in setting out, fertilizing, and cultivating the orchard?

12. A farmer who had been neglecting his apple orchard averaged, in a certain year, 1¼ bu. from each of 300 trees. The next year he pruned the orchard and sprayed it to protect it from insects. His crop was then 6½ bushels per tree. What was his gain, if the apples were worth 40¢ per bushel, the cost of pruning for the whole orchard was $36, and the cost of spraying $72.50?

13. If you live in the country (or visit the country), find out what kind of fruits are grown in the neighborhood, and make up a set of examples concerning that kind of fruit raising which is most profitable to the neighborhood.

CHAPTER VI

APPLICATIONS OF PERCENTAGE

60. Applications of Percentage. The method of reckoning with reference to 100 as a standard, or base, has so many advantages that it is widely used in many different departments of practical life.

In these applications of percentage, one or more of the terms *base, rate,* and *percentage* often receive some special name or are treated in some special way.

PROFIT AND LOSS

61. The subject of **Profit and Loss** differs very slightly from the general subject of percentage.

Profit or *loss* denotes the gain or loss in business transactions. It is expressed as a per cent.

Note that the *base is the money paid out or invested* (not the money received).

Ex. 1. A man sold his horse for \$120, which was a loss of 20%. What did the horse cost him?

Making the cost of the horse the base,

80% of the cost of the horse = \$120, the selling price.

Hence, by Art. 59, cost of the horse $= \frac{\$120}{.80} = \$150$ *Ans.*

Ex. 2. A man sold a house for \$2500 and afterward bought it back for \$2000. What was his rate of gain?

The base is \$2000, since this was the sum paid out.

Hence, by Art. 58, rate of gain $= \frac{\$500}{\$2000} = .25$ *Ans.*

EXERCISE 71

1. I bought a horse for $80 and after training it sold it for $100. What per cent did I gain?

2. I sold a wagon for $40 at a loss of 20%. What was the cost?

3. A man sold a horse at a gain of 70%, or $63. Find the cost and also the selling price.

4. Did I gain or lose by buying eggs at 18¢ a dozen and selling at 32¢ for 20? What per cent?

5. Who gained more money and who gained the greater per cent: John, who bought for $60 and sold for $65, or James, who sold for $240 at a gain of 20%?

6. I sold a horse for $396 and gained 12½%. What would have been my per cent of loss if I had sold the horse for $250?

7. I bought two farms for $4500 each, and sold one for $6300 and the other at a loss of 37%. Did I gain or lose on the transaction and how much?

8. By selling cloth for $1.20 per yard, a merchant lost 20%. At what price should he have sold it in order to gain 20%?

9. I sold two watches for $84 each. On one I gained 40% and on the other I lost 40%. Did I gain or lose on the whole? How much? What per cent?

10. A dealer buys carpet at 75¢ a yard. His expenses amount to 20¢ additional on each yard. What must be the selling price in order that the dealer may realize an advance of 20%?

11. One merchant bought goods at $250 and sold them for $302. Another bought goods at $360 and sold them at $445. Which merchant had the higher rate of gain?

12. If I buy a house for \$2000, spend \$1500 in repairing it, and then sell it for \$5000, what is the gain per cent?

13. If I sell a farm for \$5500 and buy it back for \$4000, what is my gain per cent?

14. If I buy corn at 75¢ a bushel, crack it at a cost of 1¢ per bushel, and sell it at the rate of \$1.53 per 100 pounds, what is the per cent of gain?

15. A merchant buys hoes at \$2.75 per dozen and sells them at 35¢ each. What is his rate per cent of gain?

16. Make up and solve a similar example concerning a merchant who buys hats by the dozen.

17. A stationer buys lead pencils at \$3 per gross and sells them for 5¢ each. Using these facts, make up and solve a problem which requires the gain to be found. The rate per cent of gain. The cost.

18. In making a sale, can a merchant gain 125%? Can he lose 125%? What is the largest per cent he can lose? Can gain?

19. If the selling price is twice the cost, what is the gain per cent?

20. How many examples in this Exercise can you work at sight?

21. Work again Exercise 58 (p. 120).

COMMERCIAL DISCOUNT

62. Commercial Discounts. It is the custom of manufacturers and dealers in merchandise to have a fixed or catalogue price of goods, and often to make deductions from the prices thus printed.

The **list price** of an article is the price of the article as printed in the catalogue.

A **commercial discount** is a deduction from the list price of goods. A commercial discount is sometimes termed a *trade discount.*

The **net price** of an article is the price of the article after the discount has been made.

The catalogues of goods and prices issued by business houses are frequently expensive, and when the prices of goods change, owing to cheapened processes of production or other reasons, it is more economical to print a brief list of discounts than to issue a new catalogue.

Merchants also sometimes make a reduction because they desire to close out their stock rapidly. Can you ascertain any other reasons why they make discounts?

63. In the **Computation of Successive Discounts,**

Deduct the first discount from the list price;

Compute the next discount on the remainder and deduct it from the remainder;

Proceed in like manner until all the discounts have been made.

Ex. Find the net amount of a bill of $450 for furniture on which there were discounts of 20%, 10%, with 2% off for cash.

20% of $450 = $90; $450 − $90 = $360
10% of $360 = $36; $360 − $36 = $324
2% of $324 = $6.48; $324 − $6.48 = $317.52 *Ans.*

Often in computing successive discounts, it is more convenient to

Subtract each rate, in turn, from 100 *and take the product of the base and all the remainders.*

Thus, in solving the preceding example, we may proceed as follows:

$450 × .80 × .90 × .98 = $317.52 *Ans.*

Business men often use books or tables which give the equivalents of given sets of successive discounts. For example, such a book would give 68.64% as the single discount which is equivalent to the successive discounts of 60%, 20%, and 2%.

EXERCISE 72

1. A music teacher, in buying a piano listed at $650, was allowed a discount of 40% with a further discount of 5% for cash. Find the net cost of the piano.

Find the net amount of

2. $850 with discounts of 30% and 2%.

3. $1020 with discounts of 40%, 10%, and 5%.

4. $372.50 with discounts of 20% and 3%.

5. $800 with $\frac{1}{4}$ and $\frac{1}{10}$ off.

6. 250 bbl. of apples @ $2.25 with 10% and 4% off.

7. 100 doz. forks @ $4.25 per dozen with 15% and 3% off.

8. Which is the greater discount on a bill of $15,600, and how much: a discount of 40% and then 8%, or one of 48%?

9. What single discount is equivalent to the discounts of 15% and 20%? Of 49%, 10%, and 5%?

10. Find the net amount of a bill of $675, subject to the three discounts of 20%, 16%, and 5%. Change the order of discounts in this example and ascertain whether or not there is a change in the final amount of the bill. Why is this?

11. In refitting a bathroom, I found that I could buy the required fixtures at one store for $180 with 20%, 15%, and 3% off, and at another store for $240 with 30%, 20%, and 2% off. At which of these stores was the net price smaller?

12. Make up and work a similar example for yourself.

13. Find out what the following means when printed on a bill: "Terms: 60 days net; 5% off 10 da." Also "Terms: 60 days net; 2% off 10 da.; cash 5%."

14. What is due on a bill for $650.42 marked in the first method of Ex. 13, if payment is made in 8 days? In 5 days? In 40 days?

15. What is due on a bill for $650.42 marked in the second method in Ex. 13, if payment is made on the day of purchase? 5 days after purchase? 50 days after?

16. Make up and work an example similar to Ex. 14. Also one similar to Ex. 15.

Make a bill for each of the following sets of items, find the net amount of the bill if he paid cash, and receipt the bill:

17. On June 2, 1914, Albert Morris bought of James Andrews and Co., Philadelphia, 2 doz. axes @ $7.50, 50 kegs of nails @ $4.30, 20 doz. forks @ $6.50, 8 lawn mowers @ $4.50. Trade discounts: 20%, 5%. 3% cash.

18. On Jan. 5, 1915, W. R. Appleton bought of the James Hadley Co., of New York, 200 boxes of oranges at $2.15; 150 bbl. of apples at $1.90; 20 boxes of grapefruit at $2.80 ; 50 boxes of lemons at $3.20. Trade discount 15%. 3% cash in 10 days.

19. The net amount of a bill on which discounts of 40% and 10% had been allowed was $216. What was the original amount of the bill?

20. A merchant bought flour at $6 a barrel. He fixed a selling price such that he might give a discount of 10% and still make 20% profit. What was the price?

21. How many examples in this Exercise can you work at sight?

22. Practice the oral addition and subtraction of two-figured numbers as in Exercise 5 (p. 13).

COMMISSION AND BROKERAGE

64. Agents and Commissions. It is often an advantage in various ways to buy or sell goods through an agent. For instance, an agent may be in a more favorable place in which to buy or sell goods. Also, an agent, by making a specialty of a certain line of goods, may be able to buy or sell to greater advantage because of his superior knowledge.

Thus, a farmer may receive a higher price for peaches by selling them through an agent in a city, than by selling them himself near his home.

An agent who buys or sells general merchandise is called a **commission merchant.** Goods sent to him to be sold are called a **consignment,** the person sending them being the **consignor,** and the person receiving them the **consignee.**

An agent at a distance is sometimes called a *correspondent;* the person employing a correspondent is called a *principal.*

The terms *commission broker* and *broker* are often used, instead of "commission merchant."

65. Commission or brokerage is the percentage paid a person who buys or sells goods or collects money for another person.

The **base** of a commission is the amount of money paid out or received by the agent in a given transaction.

The **net proceeds** is the sum left after the commission has been deducted from the amount received by an agent from the sale of goods.

Ex. 1. An agent sold an automobile for $3000 and received a commission of 20%. What was his commission?

$$\$3000 \times .20 = \$600 \textit{ Ans.}$$

Ex. 2. A man sent an agent $257.50 to expend in buying hay at $20 a ton. The agent charged 3% commission. How many tons of hay did he buy?

$$\begin{array}{r} \$2\,50. \\ 1_{\circ}03.\overline{)\$257_{\circ}50.} \\ \underline{206} \\ 515 \\ \underline{515} \\ 0 \end{array}$$

Since $257.50 includes both the cost of the hay and 3% commission, $257.50 is 103% of the cost of the hay. Hence, the cost of the hay = $257.50 ÷ 1.03 = $250.

If 1 ton of hay costs $20, the number of tons which can be bought for $250 is $250 ÷ $20, or $12\frac{1}{2}$.

$$\text{Hence, no. tons} = \frac{\$250}{\$20} = 12\tfrac{1}{2} \textit{ Ans.}$$

EXERCISE 73

1. A commission merchant sold a carload of fertilizer for $350 and received 3% commission. Find his commission. How much did he remit to his principal?

2. A farmer can sell 30 barrels of apples near his home for $1.60 a barrel. A commission merchant in the city can sell them for him at $2.20 per barrel, charging 3% commission. Freight to the city on 30 bbl. of apples will be $5.40. Which method of sale will bring the larger return?

3. I can sell a house and lot through an agent for $6000, the agent's commission being 4%. How much will I gain by this, instead of spending 3 days looking for a customer (my time being worth $10 per day) and selling the house for $5600?

4. An agent bought 500 tierces of lard at $24.50 per tierce. If his commission was 2½¢ per tierce, what was the entire cost? (What is a tierce?)

5. What is the commission on 1200 bales of cotton, at the rate of $5 per 100 bales?

6. What is the commission on 1250 bags of coffee, at the rate of $4 on 100 bags? (How many pounds of coffee are there in a "bag"?)

7. An agent sold 560 baskets of peaches at 90 cents a basket and charged $25.20 commission. What was his rate of commission?

8. After selling a property worth $8528, the agent sent to the former owner $8229.52. What rate of commission did the agent charge?

9. A real estate agent sold a house on 3% commission and sent the owner $7229.41. What commission did he retain?

10. A real estate agent sold four pieces of property worth $650, $1200, $780, and $672. What was his commission at 5%?

11. What is the cost of 1000 bushels of wheat bought at $1.02½, if the agent charges a commission of ⅛¢ per bushel?

12. A commission merchant bought for a customer 20 barrels of sweet potatoes at $2.25; 5 barrels of cranberries at $6.50; and 65 bushels of white potatoes at 65¢. What was his commission at 3½%? If freight was $17.40 and storage $8.25, what was the entire bill sent to the customer?

13. If $2388.33 includes commission at 2% and the amount to be invested in wool, how much was invested?

14. In a house costing $6000, what is the difference in cost between an architect's fee of 3¼% for plans and superintendence, and a fee of 1¾% for plans only?

15. Which is larger: a commission of 2½% on a sale of $30,000 worth of city property, or 4% on $22,000 worth of country property?

16. A salesman receives a salary of $12 per week and a commission of 2% on all goods sold. If he sells $30,000 worth of goods in the year, what is his income for the year?

17. A commission merchant secured a consignment of goods on which he paid $70.15 freight, $13.70 cartage, $12 storage. He then sold the goods, deducted the above charges and his commission of 5%, and found the net proceeds to be $3229.15. For how much did he sell the goods?

18. A farmer sold 840 lb. of poultry at 18¢ a pound through a commission merchant who charged him 3% commission, the bill for freight and cartage being $7.40. What did the farmer receive?

19. Ascertain the commission rates in your neighborhood and make up and work examples concerning them.

20. Make up and work three examples similar to such of the above examples as the teacher may indicate.

21. Work again Exercise 8 (p. 15).

TAXES

66. Revenues. Federal, state, town, and city governments need money in order to do the work delegated to them, such as maintaining schools, building roads, caring for the insane, etc.

The various governments collect their revenues in different ways. State and local governments generally collect their revenue as *taxes*.

67. Taxes. A **tax** is money assessed on property or persons by the government for public purposes.

A **property tax** is a tax assessed on property.

Property is of two kinds, *real estate* and *personal property*. Real estate is property not easily moved, as lands, buildings, etc.

Personal property is movable property, as money, stocks, bonds, household goods, cattle, etc.

A **poll tax** is a fixed sum, as $1, or $2, assessed on each voter in a community without regard to the amount of property he owns.

68. The **Method of Assessing Taxes** in a state, for instance, is usually as follows. The state legislature determines the amount of money to be expended. The amount of taxable property in the state is determined by local officers called *assessors*. The total amount of money to be collected is divided by the number of dollars of taxable property. This determines the amount of tax on one dollar. Hence, a *tax is a certain per cent of the property assessed.*

Thus, if a state desires to expend $1,200,000, and the amount of property in the state is $800,000,000, the tax rate will be $1,200,000 ÷ $800,000,000, or .0015.

The same general method is followed in assessing city, borough, and county taxes.

The different rates of taxes of each government are sent to an official, who calculates the amount of each kind of tax to be paid by each person, corporation, or piece of property, and tabulates the results in a book. The book is given to a collector, who collects each tax and returns it to the proper (county, city, or state) treasurer.

It is the custom in many localities to assess property for not more than $\frac{1}{2}$ or $\frac{2}{3}$ of its real value. After property has been assessed, the owner may appear before the proper official and make claim for such reductions or corrections as he thinks he is entitled to.

If taxes are not paid when due, a certain per cent is usually added to them as a fine.

Taxes are often stated as so many mills on a dollar.

Ex. What will be the county tax of Samuel Smith, the rate being $2\frac{1}{2}$ mills on a dollar, and his property being assessed at \$3500?

The tax on \$1 = .0025.
Hence, tax on \$3500 = \$3500 × .0025 = \$8.75 *Tax*

69. Computation of Taxes by Use of a Table. In computing taxes, time and work are saved by the use of a table like the following, which gives the tax on various sums at the rate of 3 mills on a dollar.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
1	.03	10	.30	100	\$3.00	1000	\$30	10,000	\$300
2	.06	20	.60	200	6.00	2000	60	20,000	600
3	.09	30	.90	300	9.00	3000	90	30,000	900
4	.12	40	1.20	400	12.00	4000	120	40,000	1200
5	.15	50	1.50	500	15.00	5000	150	50,000	1500
6	.18	60	1.80	600	18.00	6000	180	60,000	1800
7	.21	70	2.10	700	21.00	7000	210	70,000	2100
8	.24	80	2.40	800	24.00	8000	240	80,000	2400
9	.27	90	2.70	900	27.00	9000	270	90,000	2700

In this table, the columns headed "Prop." give the number of tens of dollars, not number of dollars which are taxed. That is, for the tax on a property of 5000, we refer to 500 in the table.

Ex. Compute by use of the table the tax on a property assessed at $5680, at the rate of .003.

Tax on $5000 = $15.00
" " 600 = 1.80
" " 80 = .24
$17.04 *Tax*

EXERCISE 74

1. What will be the tax on a farm assessed at $4500, if the rate is .007? If it is .0102? If it is .012?

2. Mr. Smith owns a house valued at $18,000 and assessed at $\frac{2}{3}$ of its value. The tax rate is .021. What will be his total tax, including a poll tax at $1.50? If he receives a discount of 2% for paying his tax within 30 days, what is the amount of his tax bill?

3. By the table, find the tax at the rate of .003 on the following amounts: $4175; $8925; $10,328; $27,030; $50,409; $66,666.

4. If my tax is $132.30 and my property is assessed at $7350, what is the rate?

5. My entire tax is $395.25, which includes a poll tax of $1.25. Find the tax rate on my property, if my property is assessed at $15,760.

6. One year a man paid $372.60 in taxes, when the rate of taxation was $1\frac{2}{3}$%. What was the assessed value of his property?

7. A certain town wishes to raise $30,500 by taxation. The real estate in the town is assessed at $2,150,000 and personal property at $1,350,000, and there are 500 polls taxed at $1.50 each. What will be the tax rate?

8. The assessed valuation in a certain town is $857,400, and a tax of $13,718.40 is to be collected. What will Mr. Clark have to pay, his property being worth $14,700, if he also pays a fine of 2% of his tax for not paying his tax before a certain date?

9. State the similarities and differences between the subject of taxes and that of percentage. Also between commercial discounts and percentage.

10. By use of the table on p. 160 find the taxes on amounts of Ex. 3, if the rate were 2 mills on the dollar instead of 3.

11. If the assessed value of the property in a state is $360,500,000 and the state university is given the income from a tax of $\frac{1}{20}$ mill, what will be the income of the state university from this source?

12. A borough plans to build a high school building to cost $90,000. The required money is to be raised by a special tax in three equal annual installments. If the assessed amount of property in the borough is $27,500,000, what will be the special tax rate for the building in each of the three years?

13. A borough needs to collect $132,000 in taxes and the collector is to receive 2% of the taxes for his services. What will be the tax rate, if the assessed value of the property in the borough is $3,516,000?

14. If, instead of paying an annual tax of $121.25, a man should pay $700 as salary to a teacher for his children and should pay $600 for a watchman to protect his property, how much would he lose each year? What are some of the benefits which a man gains from paying taxes?

15. Find out, if you can, the purposes for which the money raised by taxation is spent in your township, borough, or city, the rate of the various taxes, and the way in which these taxes are assessed and collected.

16. After thus ascertaining the various taxes and tax rates in your neighborhood, make up and work examples concerning them.

17. How many examples in Exercise 11 (p. 17) can you now work at sight?

CUSTOMS OR DUTIES

70. Revenues of the General Government. The general government of the United States obtains its revenue from two principal sources: (1) **Duties** (sometimes called *tariff* or *customs*), which are taxes imposed on goods imported into the country from foreign countries; and

(2) **Internal revenue**, or taxes charged on certain articles manufactured within the country, as spirituous liquors, articles made from tobacco, etc.

71. Duties. Duties on imports are authorized by laws called *tariffs*. A tariff is divided into two parts, the dutiable list and the free list. The former part enumerates all the articles subject to duty and the rates of duty thereon; the latter enumerates all the articles to be admitted free of duty.

Duties are of two kinds, *ad valorem* and *specific*.

An **ad valorem** duty is a certain percentage assessed on the cost of the goods in the foreign country from which they are shipped. This cost includes all charges for cases and packing. For example, the rate on books is a certain per cent of their cost in the foreign country where they are bought.

A **specific** duty is duty assessed on goods according to their quantity, weight, or number. For example, one class of goods pays a duty of $2.50 per ton; and another class, 40¢ per gallon.

A **compound** rate is a combination of both an ad valorem and a specific duty on the same article. For example, a duty of 44¢ per pound and 55% ad valorem on the same article would be a compound rate.

Ad valorem duties are more just, but they present greater chance for fraud by undervaluation. Hence, specific duties are assessed by most nations wherever possible.

In the United States, importations must be made at certain places called *ports of entry,* which are designated by law. Each port of entry has a building called a *customhouse,* where duties are collected by a government official called the Collector of Customs.

An *invoice* specifying the goods purchased, their cost, etc., is sent by the person selling the goods to the person or firm importing the goods. This invoice must be presented by the importer at the customhouse where the goods are received. In case the cost of the merchandise exceeds $100, the foreign shipper or seller is required to present to the nearest American consul an invoice of the merchandise, in triplicate. Before this official he must declare the invoice to be correct and true in all respects. The consul retains one copy of the invoice, returns one to the person making such declaration, and mails the third copy to the Collector of Customs at the port in the United States where the merchandise is to be entered.

Merchandise of certain kinds, imported but not intended for immediate use, may be placed in *bonded warehouses* provided by the United States, and may remain there not longer than three years. The owner is at liberty to withdraw merchandise from a bonded warehouse at any time, upon payment of the duties and charges.

EXERCISE 75

In the following examples, the rates of duty specified are in use, or have recently been in use; but the particular articles are not specified, because tariff rates are subject to change.

Find the duty on

1. 3000 tons at 15¢ a ton.

2. 3000 tons at $2.50 a ton.

3. 40 barrels which cost $6.50 per barrel, at 60%.

4. 500 lb. of goods invoiced at $2150, at 44¢ per lb. and 60%.

5. 1500 lb. invoiced at $3715.20 with $17.50 to be deducted, at 2¢ a pound and 15% ad valorem.

6. 1350 lb. invoiced at £80 15s., and subject to a duty of 44¢ a pound and 55%. (Use £1 = $4.8665.)

7. Goods invoiced at 10,500 francs at 45%. (Use 1 franc = $.193.)

8. Goods weighing 1200 lb. invoiced at $3720.18, at 1¢ per lb. and 35%.

9. In Ex. 3, at what price must the importer sell a barrel of the goods imported, in order to make 20%?

10. At the rate of 11¢ per pound, find the duty on 50 bales of goods weighing gross amount 21,906 kg. with tare of 7.5 kg. per bale. (Use kg. = 2.2046 lb.)

11. Find the duty on goods invoiced at 864.50 francs, including 13 francs of non-dutiable items, the rate being 35%.

12. A certain kind of goods pays a duty of $3\frac{1}{2}$¢ per square yard and is imported in rolls of 40 linear yards each. Find the duty on 233 rolls 36″ wide, and 42 rolls 27″ wide.

13. Find the duty on a number of articles in common use in your neighborhood and make up an example involving an ad valorem duty.

14. Also one involving a specific duty.

15. Also a compound rate.

16. Work again Exercise 9 (p. 15).

INSURANCE

72. Insurance is a system of business whereby a certain sum is payable in case of loss of property in a specified way, or in case of injury or death of a person.

There are three principal kinds of insurance:

(1) Fire Insurance.
(2) Life Insurance.
(3) Accident Insurance.

Beside these, there are many special kinds of insurance, as marine, tornado, plate glass, etc.

year, and the agent received 25% of the premium. What was the value of the building?

10. Find the difference between the annual premium on a wooden house for $3000 at 1¾%, and that on a brick house for the same amount at 1¼%. (What is the shortest way of getting this result?)

11. The owner of a factory who was insuring his property for $30,000 at an annual rate of 1½%, equipped his factory with automatic sprinklers at a cost of $2000. The fire insurance company then reduced the rate on his property to ¼% per year. In how many years would the savings on the premiums pay for the sprinklers? (Find out, if you can, what an automatic sprinkler is.)

12. A man erected an office building and, at an expense of $4000, had the chimneys lined with terra cotta, and the stairways and elevator shafts arranged to close with iron doors. By this means, he reduced the annual insurance premium on $80,000 from 1½% to 1%. In how many years would the extra cost of the building be paid for by the saving on the premium? State other advantages in making the building as thoroughly fireproof as possible.

13. Is the cost of fire insurance on a building affected by its nearness to other buildings? By its distance from a fire plug? Name some other causes which increase or decrease the cost of fire insurance.

14. Can you find out what a contract is? In an insurance policy, who are the contracting parties? Can you give examples of any other contracts?

15. A man took out a 20-year endowment life insurance policy for $10,000 at a rate of $62.15. If he died at the end of 11 years, during which time he had received $217.20 as rebate on the premiums paid, by how much did the receipts from the policy exceed the expenditure, no allowance being made for interest?

16. What are the advantages of fire insurance? Of life insurance?

17. Ascertain the fire insurance rates in your neighborhood on various kinds of buildings variously situated, and make up one example concerning each of three different rates.

18. Name as many different kinds of insurance as you can, and make up one example concerning each of three different kinds.

19. Work again Exercise 12 (p. 19).

STOCKS AND BONDS

75. Stock Companies (or Corporations) and Stocks. When a business enterprise, as the building and management of a railroad, requires more money than a single person possesses or cares to risk, it is customary for several persons to combine their resources and organize a stock company or corporation, for the purpose of carrying on the enterprise. The money thus invested is called the *capital stock*, or the **stock** of the company.

Stock is divided into a number of equal *shares*, each share being usually $100, but sometimes $50 or $25, or even a smaller sum.

The value of a share of stock as thus originally stated is termed its *face* or *par value*.

A **stockholder** is a person who holds stock in a corporation or stock company.

A **stock company** or **corporation** may be defined as a number of persons organized according to law so as to be able to do business as a single individual.

The stock company, or corporation, has a *charter* or certificate of incorporation secured by an act of a state legislature, or issued by a state officer in accordance with a general law. The charter specifies the name and purpose of the company, the amount of stock, the method in which the business is to be conducted, etc. A company is usually organized by electing a board of directors, each share of stock being allowed one vote. The year, and the agent received 25% of the premium. What was the value of the building?

10. Find the difference between the annual premium on a wooden house for $3000 at $1\frac{3}{4}\%$, and that on a brick house for the same amount at $1\frac{1}{4}\%$. (What is the shortest way of getting this result?)

11. The owner of a factory who was insuring his property for $30,000 at an annual rate of $1\frac{1}{2}\%$, equipped his factory with automatic sprinklers at a cost of $2000. The fire insurance company then reduced the rate on his property to $\frac{1}{4}\%$ per year. In how many years would the savings on the premiums pay for the sprinklers? (Find out, if you can, what an automatic sprinkler is.)

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Stock certificates are documents issued by a company, stating the number of shares of stock owned by each stockholder, respectively.

76. Dividends, Assessments. When the receipts of a company exceed its expenditures, it usually pays part or all of the net gains to the stockholders as a **dividend.** A dividend is paid out as a certain per cent of the face or par value of the stock.

When a company is losing money, it sometimes makes an **assessment** on its stockholders, to cover a deficit or extraordinary expense. An assessment also is made as a certain per cent of the par value of stock.

77. Par, Premium, and Discount. According as a company is paying large or small dividends, and the public has or has not confidence in it, the stock of the company may sell for more or less than its face value.

The **market value** of stock is the price at which it will sell. When stock sells for more than its face or par value, it is said to be *above par*, or *at a premium;* when for less, it is said to be *below par*, or *at a discount.*

The market value of stock is usually stated as a per cent.

Thus, when the statement is made that a given stock is selling at 112, it means that the stock is selling at 12 per cent above par, no matter what the par value of a share of the stock may be.

78. Stocks and Bonds. Some companies issue stock of two kinds: (1) *preferred,* (2) *common stock.* In dividing the gains of a company, the preferred stock receives a dividend first, up to a certain amount, as 5%, after which the remainder of the net income of the company, if there is any, is apportioned to the common stock. About one fifth of the railroad stock of the United States is preferred stock.

A **bond** is a note, or promise to pay, issued by a company to the person from whom the company or corporation borrows money. The bond specifies the amount, rate of interest, and the time when the face of the bond is to be paid to the owner of the bond.

Bonds issued by a company are secured by a mortgage on the property of the company; those issued by a city, county, state, or national government are simply promises to pay, without any such security.

Bonds are either *coupon* or *registered* bonds.

Coupon bonds have small certificates attached, which are cut off, as interest becomes due, and are cashed at the proper place, as at a bank, by any one who holds them.

The company which has issued **registered bonds,** keeps a record of the names and addresses of the owners of such bonds and, when interest is due, sends it to the owners of the bonds.

Bonds receive special names, indicating the year when they are due, or the number of years which they have to run, the rate of interest, etc. Thus, 4s of 1915 are bonds which mature in the year 1915 and pay 4% interest.

The subject of bonds will also be considered later with reference to the varying income derived from them. (See Art. 110, p. 226.)

79. The **Brokerage on Stocks and Bonds** is reckoned, not on their selling price, but on their face or par value. It is usually $\frac{1}{8}\%$, or $12\frac{1}{2}$ cents on a share of $100.

Ex. 1. A man bought 50 shares of stock at $112\frac{1}{2}$. What did the purchase cost him, if the rate of brokerage was $\frac{1}{8}\%$?

$$112\tfrac{1}{2} + \tfrac{1}{8} = 112\tfrac{5}{8}$$

$$50 \times \$112\tfrac{5}{8} = 50 \times \$112.625 = \$5631.25 \textit{ Ans.}$$

Ex. 2. Find the brokerage on 400 shares of stock (par value $50) bought at $62\frac{5}{8}$.

$$\tfrac{400}{2} \times .00\tfrac{1}{8} = 25. \quad \$25 \textit{ Ans.}$$

Ex. 3. If I buy 12 shares of stock at 120 (par value $100) and receive a semiannual dividend of 3½%, what is my annual income from the investment? What per cent does the investment pay?

$$\begin{array}{r} 12\\ \$100\\ \hline \$1200\\ .035\\ \hline 6000\\ 3600\\ \hline \$42.000\\ 2\\ \$1440\overline{)\$84.00}(.05\frac{5}{6}\\ 7200\\ \hline \frac{1200}{1440} = \frac{5}{6} \end{array}$$

The par value of 12 shares at $100 a share is $1200. Since the dividend is paid on the par value, the semiannual dividend on 12 shares will be 3½% of $1200, or $42. Hence, the annual income is $42 × 2, or $84.

The cost of 12 shares at $120 per share is $1440.

In order to determine the per cent paid by the investment, it is necessary to determine what per cent $84 is of $1440. This is $5\frac{5}{6}$%.

EXERCISE 77

Unless otherwise stated, in this Exercise the par value of a share of stock is $100.

In Exs. 1–6, the cost of brokerage is not to be considered.

1. A man bought 20 shares of stock at 108 and received 6% annual dividends on them. What was his annual income from the investment? What per cent was this of his investment?

2. I bought 25 shares of a certain stock at 60 (par is 50) and received semiannual dividends of 2½%. What is my annual income from the investment? What per cent do I receive on the investment?

3. Which will pay the larger per cent of income: stock which can be bought at 120 and pays 7% annual dividends, or stock which costs 108 and pays 6% annual dividends?

4. Which brings the greater income: $6438 invested in 6% stock selling at $111, or in 7% stock selling at 58 (par $50)? What is the per cent in each case?

5. What sum must be invested in 5% stock at 105 to yield an annual income of $1200?

6. What sum must be invested in 7% stock at 121½ to yield an annual income of $3500?

In Exs. 7–11, the cost of brokerage (⅛% per share of the $100) must be considered.

7. Find the cost of 26 shares of stock at 132. If this stock pays 5% dividend, what is the rate of income from the investment?

8. If I buy 50 shares of stock at 121½ and the stock pays a dividend of 7%, what per cent does the investment pay me?

9. How much money must I invest in this stock of Ex. 8 to get an annual income of $840?

10. If 150 shares of stock are bought at 121¾ and sold at 137½, what is the profit? (Point out the long way and the short way of working this example.)

11. If stock whose par value is $50 is selling at 72½, and pays a quarterly dividend of 1¾%, how much money must I invest in it to receive an annual income of $1400?

12. Get from the daily newspapers the price of eight or ten leading stocks; make up and solve examples concerning them.

13. Do you know or can you find out what is meant by "watering stock"?

14. Why is it that bonds issued by a city pay dividends as small as 3½%, while mining stocks are frequently offered for sale with the promise of 30%, 60%, or even higher dividends?

15. Make as large a list as you can of stock companies or corporations which do business in your neighborhood.

16. Can you find out some differences between a partnership and a corporation? Is each member of a partnership firm responsible for the whole amount of the debts of the firm? Is each stockholder in a corporation responsible for the entire debt of the corporation?

Do the acts of one partner bind all the other partners in a firm?

Do the acts of one stockholder in a corporation bind any other stockholder in the corporation?

17. Make up and work three examples similar to such of the examples in this Exercise as you consider most instructive.

18. How many examples in Exercise 15 (p. 22) can you now work at sight?

EXERCISE 78

REVIEW

1. If a single fare to the city is 60¢, what per cent do I save by buying 100 tickets for $50?

2. At $1\frac{4}{5}$%, I insured my house by payment of $94.32 premium. What was the face of the policy?

3. The assessable property in a borough is valued at $1,721,000. The borough is to build a schoolhouse at a cost of $17,484.25. If the tax collector is to receive a commission of 3%, what must be the rate of the tax by which money is collected for building the schoolhouse?

4. A man sold two horses for $75 each. On one he gained 20% and on the other he lost 20%. Did he gain or lose on the whole transaction? How much? What per cent?

5. An importer sold a line of goods for $15,048, thereby making a net profit of 20%. Previous to this, he had paid a duty of 10% on their cost. What was the cost?

6. A man bought a house for $12,000, and paid $230 tax and $1\frac{1}{3}\%$ insurance annually for 5 years. He then sold the house at a gain of 10% above all cost and expense. What was the selling price?

7. Mr. B bought stock at 72 ($50 par) and sold it at a loss of 20%. At what price was it sold?

8. The first cost of an importer's stock of goods was $13,200; duty was levied at 12%; insurance was computed upon this total value at $2\frac{1}{4}\%$. At what price must the goods be sold in order to return to the owner 15% above the entire cost?

9. If $7384.80 includes the price paid for a farm and the agent's commission at 2%, find his commission.

10. A coal dealer ordered through his agent 6000 tons of soft coal at $3.30 a ton, and paid 3% commission, $56.26 cartage, and $210.75 freight. The dealer sold the coal at $3.98 a ton. What was his gain per cent?

11. A stationer sold paper at 16 cents a quire, having paid $2.50 a ream for it. Find his gain per cent.

12. A drover bought 75 cows at $60 a head. Ten of them were killed by accident. He sold the rest so as to gain 10% on the transaction. At what price did he sell each cow?

13. A teacher spent 25% of her salary for board, 10% for clothes, $12\frac{1}{2}\%$ for traveling expenses, and 15% for books and other expenses, and saved the remainder, which was $450. What was her salary?

14. What number increased by $33\frac{1}{3}\%$ of itself is 900?

15. What per cent is gained by buying coal at $5 per long ton, and selling it at $5 per ordinary ton?

16. Name the principal kinds of taxable property and also of taxes.

17. Name the principal kinds of duties.

18. What single discount in addition to one of 20% will reduce a bill from \$150 to \$117.60?

19. A retail druggist buys goods at a discount of 30% from the list price and sells at the list price. What per cent does he make?

20. A salesman was offered pay in the three following ways: (1) a salary of \$3000 per year; (2) a salary of \$1500 a year, and a commission of $2\frac{1}{2}$% on all sales; (3) a commission of 6% on all sales. He accepted the second offer and sold \$50,000 worth of goods. Did he make the choice most profitable to himself?

21. A merchant wishes to mark some goods which cost \$1.20 a yard, so that he may reduce them 20% from the marked price and still gain 10%. At what price must he mark the goods?

22. I bought 320 shares of a certain stock (par \$10) when $3\frac{1}{2}$% below par, and sold them when $1\frac{1}{2}$% above par. Find my gain in dollars. Also my per cent of gain.

23. I sold 400 shares of one stock at $98\frac{1}{4}$ and, with the proceeds, bought 100 shares of another stock at $31\frac{5}{8}$. Allowing $\frac{1}{8}$% commission in each transaction, how much money did I have left from the transaction?

24. I sent my broker \$10,000, asking him to buy a certain stock under 45 and sell it over 60. He bought all he could with my remittance at $42\frac{3}{4}$ and sold it at $60\frac{1}{2}$ (commission at $\frac{1}{8}$%). Find the number of shares purchased; the commission; and my profit.

25. A merchant bought goods at successive discounts of 25%, 10%, and 1% from the list price, and sold them at successive discounts of 10% and 5% from the list price. What was his gain per cent?

26. Show how fire insurance in the long run is profitable both to the company and to the person taking out the insurance.

27. Show how commission (or brokerage) is profitable to both parties concerned. Illustrate by a numerical example.

28. How is taxation profitable to the person taxed? Give a numerical illustration.

29. Which is the better net discount on a bill: 40%, 20%, and 5%, or 50%, 10%, and 2%? Make up and work a similar example.

30. If the annual rate for insuring a certain property is 1½%, while the rate for a three-year policy is 3%, how much is saved by taking a three-year policy for $25,000, instead of insuring annually for three years?

31. On squared paper show the meaning of 27%. Of 2%. Of ½%. Of 250%.

32. Also show the meaning of the successive discounts of 40%, 20%, and 5%. Make up and work a similar example.

33. In dividing one fraction by another, why do we invert the divisor fraction? Explain by use of a simple example.

34. If a speaker keeps an audience of 2000 people waiting for 5 minutes, this loss will equal how many days of 8 hours each for one person?

35. In July, 1908, boy relay runners carried a message from New York to Chicago in 119 hr. 22 min. What was the rate per hour? Per mile?

36. 22 recitations of 55 minutes each equal how many recitations of 40 minutes each?

37. Compute each of the following in the shortest way:

(*1*) $1212 \times 33\frac{1}{3}$

(*2*) $728 \div 12\frac{1}{2}$

(*3*) $\dfrac{72 \times 6.4 \times .25}{60 \times 60}$

(*4*) $782 \times \frac{5}{6} - 782 \times \frac{1}{3}$

(*5*) $.625 \times \$1024 + .125 \times \$1024$

(*6*) $\dfrac{7\frac{1}{2} \times 24 \times 28}{21 \times 10}$

38. How many items of work does the zero do in each of the following numbers: 2.076; 3075; 17.208; 3750?

39. Express each of the following in the Roman notation: 1492; 1620; 1776; 1066. What important event occurred in each of these years?

40. If the record high jump is 6 ft. $5\frac{5}{8}$ in. and a boy can jump 5 ft. $6\frac{3}{4}$ in., how much will he have to improve in order to equal the record?

41. How many per cent are in the whole of anything? In one half of anything?

42. How many examples in Exercise 16 (p. 23) can you now work at sight?

EXERCISE 79

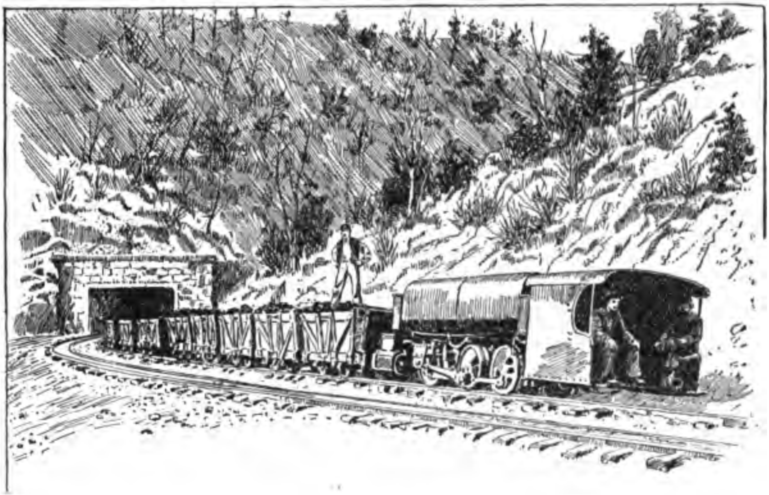
INDUSTRY REVIEW: MINING

1. If there are 400,000 square miles of coal lands in the United States, what fraction of the whole country is this?

2. If, in these coal lands, the coal on the average forms a layer 8 ft. thick and 40 cu. ft. of coal makes a ton, how many tons of coal are there in the United States? If the coal in Alaska is 10% of the coal in the United States proper, how much coal is there in the two countries?

3. If 30,000,000 acres of coal land in the United States belong to the government, and as much more in Alaska, how many acres is this all together? If the coal on this land is a layer 8 ft. thick, at 10¢ a ton, what would this coal be worth to the government?

4. If a vein of lignite coal 40,000,000 acres in extent is utilized by converting it into gas, what would the coal be worth at $3 per ton?



5. Steam engines on the average utilize only a small part of the heating power in coal. An invention which enabled them to double the amount of heating power utilized, would be worth how much per year, if we estimate the coal annually used by engines in the United States as 200,000,000 tons, and this coal to be worth $4 per ton?

SUG. One half of the coal now used by engines would be saved.

6. A certain city burns on the average 6000 tons of soft coal per day, 30% of which goes off unconsumed as smoke. If coal is worth $3 per ton, how much would be saved in a year by using grates and furnaces which would consume this smoke? What other advantages would there be to the city in this, beside the saving of money?

7. If there are 2,500,000,000,000 tons of unmined coal in the United States, and 30% of this should be wasted in the mining, how many tons would be wasted? How long would it supply the

country, at the rate of 400 million tons per year? What would its money value be at $5 per ton?

8. Before the time of James Watt, on the average 27 lb. of coal were burned per hour to produce 1 horse power from a steam engine. After Watt's improvements in the steam engine, only 6 lb. on the average were burned. How many tons of coal were thus saved in running a 100 horse power engine one year, if it was run 10 hours a day for 300 days?

9. Since Watt's inventions, other improvements have reduced the coal consumed per horse power per hour to $1\frac{1}{2}$ lb. How much is the coal worth which is now saved in the engine of Ex. 8 in a year, at $3 per ton?

10. Name as many uses of coal (and the products of coal, as coke, gas) as you can. Make up and work examples concerning these materials and their uses.

11. Coal can be mined and burned but once. What source of power have we, which cannot be exhausted, but continues to be available year after year?

Use
an
example in arithmetic,
this
has
a

12. If 140 cu. ft. of water per minute falling over a dam 10 ft. high will make in a year as much electricity or heat as the burning of 100 tons of coal, 5000 cu. ft. per minute falling over the same dam will equal how many tons?

quiet humor of its own,

13. A stream of water is 200 ft. wide and 8 ft. 6 in. deep on the average, and flows 2 ft. per second. How many cubic feet will it send over a dam in one minute?

especially as it is an example
un-
der

14. If the water power of the United States when fully utilized equals 200,000,000 horse power, and one horse power equals the annual consumption of 50 tons of coal, this water power is equiva-

mining. And as another partic-
ularly good example of mining,
consider ex. 13, which has to do
only with water, flowing over a dam.

lent to how many tons of coal burned annually? What is its value at $4 per ton?

15. If one horse power acting day and night is equivalent to the manual labor of 25 men, the water power of the United States is equivalent to the manual labor of how many men?

16. 33,000 lb. of water falling 1 ft. per minute produce one horse power. How many horse power are in the water power at Niagara Falls, if 222,400 cu. ft. of water fall every second a distance of 200 ft?

17. In a recent year, 49,237,000 tons of iron ore were dug in the United States. Out of this, 25,307,000 tons of pig iron were made. What percentage of the ore was iron?

18. If the iron mines of the United States contain 15,000,000,000 tons of iron ore, how long will this last, if 100,000,000 tons of ore are used yearly? How long, if 200,000,000 tons are used? How long, if 150,000,000 tons are used?

19. If the iron mines of the entire world contain 75,000,000,000 tons of ore, what fraction of this is the supply in the United States?

20. If 1,670,000 tons of pig iron were produced in the United States in the year 1870, and the population of the United States in that year was 38,558,000, how many pounds per capita were produced that year?

21. If 27,636,000 tons of pig iron were produced in the year 1910, what was the production per capita, the population being 91,972,000?

22. If a man by aid of a steam shovel can dig 1800 tons of iron ore per day, while a man without a steam shovel digs 7 tons of ore, how many times is a man's efficiency increased by the steam shovel?

23. The cost of unloading 10,000 tons of iron at Cleveland without the use of machinery was $3500. By use of machinery, this cost was reduced to $200. How much was saved in unloading one million tons by the improved method?

24. With every 10 tons of iron ore melted in a certain furnace, 6 tons of coke and 3 tons of limestone are used. How much coke and how much limestone would be used with 1000 tons of iron ore? How much iron ore and limestone, with 1 ton of coke?

25. If 27,636,000 tons of iron were made in the United States in the year 1910, and this was $\frac{11}{27}$ of the amount produced in the entire world, what was the world's production?

CHAPTER VII

INTEREST

EXERCISE 80

1. A business man paid $900 for the use of a store for a year. What is the $900 called?

2. Later, he borrowed money and built a store of his own. Ought he to pay for the use of the money thus borrowed?

3. If I pay $200 for the use of a barn for 1 year, what ought I to pay for the use of the barn for 2 years? For 5 years? For 6 months?

4. If I pay $200 for the use of borrowed money for 1 year, what ought I to pay for the use of the money for 2 years? For 5 years? For 6 months?

5. An investor borrowed $10,000, by the aid of which he developed an invention. What do we call the money which he pays for the use of the $10,000?

6. If the inventor pays $500 for the use of the borrowed money for 1 year, what ought he to pay for its use for 2 years? For 2 years and 6 months?

7. At the end of 2 years and 6 months, the inventor sold his invention for $50,000. After deducting the sum of the borrowed money and interest on the same, how much money did he have left from the $50,000?

8. Give another instance showing the advantage in being able to borrow money by paying interest.

9. Name some of the dangers connected with borrowing money.

10. Why do we now make definitions and rules?

80. **Interest** is money paid for the use of money.

Interest is reckoned as a certain per cent of the borrowed money. This per cent is paid each year, and proportionally for a part of a year.

Hence, interest differs from percentage, in general, in that the element of *time* must be carefully considered in every problem.

81. The **principal** is the sum of money on which interest is paid.

The **rate** is the per cent of the principal paid for the use of the principal for one year.

The **interest** is the sum of money paid for the use of the money for the entire time.

The **time** is the number of years for which interest is paid.

The **amount** is the sum obtained by adding the interest to the principal.

82. **Legal Interest** is interest determined according to a rate fixed by law. In business transactions, when no rate is specified, the legal rate is understood.

Usury is interest at a rate higher than that fixed by law. The exaction of usury is punishable by law.

Different states and nations have different legal rates of interest, according to their financial needs.

In 1911 the legal rate was 6% in all the states, with the following exceptions: 5% in Louisiana, Illinois, and Michigan; 7% in California, Georgia, Idaho, Minnesota, Nebraska, Nevada, North Dakota, South Carolina, and South Dakota; 8% in Alabama, Colorado, Florida, Montana, Utah, and Wyoming.

In many of the states, the law allows a higher rate than the legal one to be used by agreement between the borrower and the lender.

83. Uses of Interest. The process of lending money and receiving interest for it, as a rule, is beneficial to the lender, since it enables him to increase the return which he receives for work done in the past, by himself or others.

It is generally useful to the borrower also, since it enables him to get a larger return from the work which he will do later in life. (See Exs. 5, 6, 7, p. 183.)

84. Interest for Years and Months.

To compute the interest for a given number of years, or months, or years and months,

Express the time as years only;

Find the interest for 1 *year and multiply this by the number of years.*

If the principal, interest, rate, and time (in years) are denoted by p, i, r, and t, respectively, it follows that

$$i = p \times r \times t.$$

During the process of computing interest, the mills are retained. In the final result, 5 mills or more are taken as 1 cent; less than 5 mills is ordinarily neglected.

Ex. Find the interest on $350 for 2 years 3 months at 5%.

2 yr. 3 mo. = 2¼ yr.
$350 *Principal*
.05 *Rate*
$17.50 *Interest for 1 yr.*
2¼
437½
3500
$39.37½, or $39.38 *Interest for 2 yr. 3 mo.*

EXERCISE 81

Find the interest on

1. $340 for 4 yr. 6 mo. at 5%.
2. $275 for 3 yr. 8 mo. at 6%.
3. $515 for 5 yr. 4 mo. at 4½%.
4. $108.50 for 6 yr. at 3%.
5. $214.61 for 8 yr. 6 mo. at 5%.
6. $2075 for 10 yr. 2 mo. at 6%.
7. $489.30 for 2 yr. 5 mo. at 7%.
8. $307.15 for 9 mo. at 2½%.
9. $2560.60 for 7 yr. 11 mo. at 3½%.
10. $1971.40 for 10 mo. at 5½%.
11. $7327.50 for 3 yr. 8 mo. at 6%.
12. $956.70 for 5 yr. 6 mo. at 4½%.

13. Make up and solve an example similar to Exs. 1–12, in which the time is less than 1 year. More than 1 year. More than 3 years. More than 10 years.

14. A merchant, by borrowing $2000 for 3 months, was able to buy 1550 barrels of apples at $1.80 and sell them for $2.05. How much did he make by the transaction, if he paid interest at the rate of 6% on the borrowed money?

15. In order to complete his education, a young man borrowed $200 on Jan. 1, 1911; $150 on March 1, 1912; and $100 on Jan. 1, 1913. Computing interest at 5%, how much would he owe on July 1, 1913, just after his graduation?

16. If a man has $3000 and spends $500 a year, how many years will the money last him? If he puts the $3000 at interest at 5%,

and draws out and spends $500 at the end of each year, how long will the money last?

17. Make up and work a similar example.

18. Mr. X, who was paying $40 a month for house rent, borrowed $3000 and built a house of his own. If he paid 5% a year for interest on the borrowed money and $36 a year for taxes and insurance on his house, what did he gain per year by borrowing the money?

19. Make up and solve a similar example.

20. Make up and work another example, showing the utility of borrowing money and paying interest for it.

EXERCISE 82

ORAL

Find the interest on

1. $3 for 4 yr. at 6%. At 2%. At 5%.
2. $8 for 1 yr. 6 mo. at 5%. At 3%.
3. $9 for 2 yr. 4 mo. at 3%. At 4%.
4. $10 for 3 yr. 3 mo. at 4%. At 6%.
5. $15 for 5 yr. 8 mo. at 2%. At 5%.
6. $100 for 6 yr. 9 mo. at 5%. At 6%.

85. The **Six Per Cent Method** of finding interest is as follows:

(1) *Find the interest on* $1 *for the given time at* 6%;

(2) *Find the interest on the entire principal for the given time at* 6%;

(3) *Find the interest on the entire principal at any other desired rate.*

The interest on $1 for the required time at 6% is readily obtained by the use of the following:

Interest on $1 for 1 year = $.06
" " $1 " 1 mo. = .005 (= $\frac{1}{12}$ of .06)
" " $1 " 6 da. = .001 (= $\frac{1}{5}$ of .005)
" " $1 " 1 da. = $.000\frac{1}{6}$ (= $\frac{1}{6}$ of .001)

In computing interest according to this method, each month is considered as containing 30 days, and 1 year as containing 360 days.

Ex. 1. Find the interest on $312 for 2 yr. 7 mo. 15 da. at 6%.

Interest on $1 for 2 yr. = 2 × $.06 = $.12
" " $1 " 7 mo. = 7 × .005 = .035
" " $1 " 15 da. = 15 × $.000\frac{1}{6}$ = .0025
" " $1 " 2 yr. 7 mo. 15 da. = .1575
" " $312 " " " " = 312 × $.1575
= $49.14 *Ans.*

Ex. 2. Find the interest on $312 for 2 yr. 7 mo. 15 da. at 5%.

By Ex. 1, the interest on $312 for the given time at 6% = $49.14
Hence, " " " " " " " 1% = 8.19
" " " " " " " 5% = $40.95

Ex. 3. Find the interest on $317.25 from Apr. 1, 1913, to Nov. 19, 1917, at $4\frac{1}{2}$%.

From Apr. 1, '13, to Nov. 19, '17, is 4 yr. 7 mo. 18 da.
Interest on $1 for 4 yr. 7 mo. 18 da. at 6% = $.278
Interest on $317.25 for 4 yr. 7 mo. 18 da. at 6% = 317.25 × $.278 = $88.20
" " $317.25 " " " " $1\frac{1}{2}$% = $22.05
" " $317.25 " " " " $4\frac{1}{2}$% = $66.15 *Interest*

EXERCISE 83

By the 6% method, find the interest on

1. $260 for 2 yr. 8 mo. 24 da. at 6%.
2. $450 for 4 yr. 6 mo. 15 da. at 4%.

3. $846 for 7 yr. 10 mo. 12 da. at 5%.

4. $2350 for 5 yr. 9 mo. 6 da. at 3%.

5. $246.70 for 6 yr. 1 mo. 20 da. at 4½%.

6. $93.45 for 3 yr. 11 mo. 3 da. at 7%.

7. $928.50 for 8 yr. 3 mo. 25 da. at 8%.

8. $1250.40 for 10 yr. 10 da. at 3½%.

9. $760 from July 4, 1900, to Feb. 22, 1915, at 4%.

10. $45.50 from Oct. 19, 1910, to Mar. 11, 1919, at 5%.

11. $325.60 from Nov. 8, 1911, to June 23, 1920, at 3%.

12. $50.60 from Dec. 25, 1909, to May 1, 1921, at 6%.

13. $2500 from Aug. 17, 1903, to Nov. 8, 1930, at 5%.

14. $3280 from July 4, 1776, to Jan. 3, 1850, at 4½%.

15. $372 from Oct. 12, 1492, to to-day at 5%.

16. $9800 for 8 yr. 3 mo. 26 da. at 3½%.

17. $7384.80 for 9 yr. 2 mo. 10 da. at 5½%.

18. $12.75 for 7 yr. 18 da. at 6½%.

19. $5.64 for 3 yr. 8 mo. 20 da. at 3½%.

20. An inventor borrowed $4570 to enable him to put an invention on the market. 3 yr. 2 mo. 11 da. after borrowing this money, he sold his invention for $16,000. If he paid 6% interest on the borrowed money, how much did he make by the transaction, leaving out of account the labor of making the invention?

21. Make up and solve a similar example for yourself.

22. Find out the legal rate of interest in the place where you live, and using this rate, make up and work three examples in which the time involves years, months, and days.

23. Practice the use of small fractions, as in Exercise 20 (p. 28).

86. Interest for an Exact Number of Days. When the time does not exceed a few months, it is customary to reduce it to a fraction of a year, by counting the exact number of days and dividing this number by 360 or 365.

When the time is obtained by dividing the exact number of days by 365, the interest obtained is termed *exact interest.* Exact interest is computed by the United States government and by many business men.

Banks usually divide the number of days by 360, when *charging interest* on loans they have made; but divide the number of days by 365, when themselves *paying interest* on deposits.

Ex. Find the interest on $217.50 from Jan. 12 to Mar. 16, 1913, at 5%.

No. days left in January = 19
No. days in February = 28
No. days in March = 16
Time in days 63

$$\frac{217.50 \times \cancel{5} \times \overset{7}{\cancel{63}}}{100 \times \underset{\underset{8}{\cancel{40}}}{\cancel{360}}} = \$1.90 \quad \textit{Ans.}$$

EXERCISE 84

Use 360 days to the year in Exs. 1–9.

Find the interest on

1. $517 for 33 da. at 6%.
2. $208.75 for 63 da. at 6%.
3. $631.15 for 93 da. at 7%.
4. $1000 for 100 da. at 5%.
5. $720 from Apr. 17 to July 16 at 7%.
6. $5000 from May 21 to Oct. 5 at 6%.
7. $225 from Dec. 15 to Feb. 15 at 5%.
8. $127.75 from Sept. 25 to Mar. 25 at $4\frac{1}{2}$%.
9. A man bought a farm on Mar. 2 for $4500 and paid $3000. He agreed to pay the remainder with interest at 6% on Nov. 1,

after he had gathered his crops. What would he have to pay on Nov. 1?

10. Make up and work an example concerning the profits of a dealer who borrowed money and bought coal on Mar. 1 at $5.50 a ton and sold it Nov. 15 for $6.50 a ton, and then repaid the borrowed money.

11. Make up and work a similar example.

Use 365 days to the year in Exs. 12–18.

12. $400 for 146 da. at 7%.

13. $325 for 3 yr. 219 da. at 4%.

14. $136.40 for 4 yr. 150 da. at 4%.

15. $350 from Jan. 10, 1913, to Dec. 1, 1917, at 5%.

16. $90.50 from July 7, 1913, to Mar. 3, 1921, at 4%.

17. Find the difference between the exact interest and the interest obtained by taking 360 days as a year, on $7000 from Mar. 11 to Sept. 10, at 6%.

18. Find the difference between the exact interest and interest reckoned by taking 360 days as a year, on $5,678,000 from Jan. 10 to July 10 of same year, at 7%.

19. When the time is less than a year, exact interest is what fraction of interest obtained by regarding a year as containing 360 days?

20. Work again Exercise 19 (p. 26) or similar examples dictated by the teacher.

87. Interest Tables afford the most convenient way of computing interest. In some cases the tables are made on the basis of 360 days to a year; other tables give exact interest. In either case, the exact number of days is reckoned between two dates, in computing any problem in interest.

A part of a page from an interest table is printed below. It gives the exact interest on various sums from $10 to $150 at 1, 5, 6, 7% for 60 and 61 days.

	60 days				61 days			
Dols.	**7**	**6**	**5**	**1**	**7**	**6**	**5**	**1**
10	.1151	.0986	.0822	.0164	.1170	.1003	.0836	.0167
20	.2301	.1973	.1644	.0329	.2340	.2005	.1671	.0334
30	.3452	.2959	.2466	.0493	.3510	.3008	.2507	.0501
40	.4603	.3945	.3288	.0658	.4679	.4011	.3342	.0668
50	.5753	.4931	.4110	.0822	.5849	.5014	.4178	.0836
60	.6904	.5918	.4931	.0986	.7019	.6016	.5014	.1003
70	.8055	.6904	.5753	.1151	.8189	.7019	.5849	.1170
80	.9205	.7890	.6575	.1315	.9359	.8022	.6685	.1337
90	1.0356	.8877	.7397	.1479	1.0529	.9025	.7520	.1504
100	1.1507	.9863	.8219	.1644	1.1699	1.0027	.8356	.1671
110	1.2657	1.0849	.9041	.1808	1.2868	1.1030	.9192	.1838
120	1.3808	1.1836	.9863	.1973	1.4038	1.2033	1.0027	.2005
130	1.4959	1.2822	1.0685	.2137	1.5208	1.3035	1.0863	.2173
140	1.6109	1.3808	1.1507	.2301	1.6378	1.4038	1.1699	.2340
150	1.7260	1.4794	1.2329	.2466	1.7548	1.5041	1.2534	.2507

Ex. Find the exact interest on $146.50 at 5% for 60 days by the use of tables.

Interest on $140 for 60 days at 5% = $1.1507
" " $6 (= $\frac{1}{10}$ of that on $60) for 60 days at 5% = .0493
" " $.50 (= $\frac{1}{100}$ of that on $50) for 60 days at 5% = .0041
$1.2041
or $1.20 *Interest*

EXERCISE 85

By use of the tables, find the interest on

1. \$120 at 5% for 61 days.
2. \$140 at 3% for 60 days.
3. \$95 at 6% for 61 days.
4. \$105.50 at 3% for 120 days.

PROBLEMS IN INTEREST

To the Teacher. Articles 88–92 may be omitted at the discretion of the teacher.

88. General Principle. Five quantities are considered in interest, *viz.:* the *principal, rate, interest, time,* and *amount.* When any three of these are given, the other two may be found. The case when the principal, rate, and time are given, and the interest (or amount) is to be found, has been considered in Arts. 84–87.

89. I. Given the Principal, Interest (or Amount), and Time, to Find the Rate Per Cent.

Ex. At what rate per cent will \$360 produce \$66 interest in 3 yr. 8 mo?

Interest on \$360 for 1 yr. at 1% = \$3.60
" " \$360 " 3 yr. 8 mo. at 1% = $3.60 \times 3\frac{2}{3}$ = \$13.20

If the given principal produces \$13.20 interest in the given time at 1%,

$$\text{Rate required} = \frac{\$66}{\$13.20} = 5\% \textit{ Ans.}$$

Hence, in general,

Divide the given interest by the interest on the principal for the given time at 1 *per cent.*

Or *rate = interest ÷ no. years × principal.*

$$\text{Or} \quad r = \frac{i}{t \times p}$$

90. II. Given the Principal, Interest (or Amount), and Rate, to Find the Time.

Ex. In what time will the interest on \$424 at $2\frac{1}{2}\%$ amount to \$37.10?

Interest on \$424 at $2\frac{1}{2}\%$ for 1 year = \$10.60.
Hence, it will take as many years to produce \$37.10 interest as \$10.60 is contained times in \$37.10.

$$\text{Number of years} = \frac{\$37.10}{\$10.60} = 3\tfrac{1}{2}\text{ years} = 3\text{ yr. }6\text{ mo. } \textit{Ans.}$$

Hence, in general,

Divide the given interest by the interest of the principal at the given rate for one year.

$$\text{Hence, also,}\quad t = \frac{i}{p \times r}$$

91. III. Given the Interest (or Amount), Time, and Rate, to Find the Principal.

Ex. 1. What principal will produce \$33.75 interest in 2 yr. 3 mo. at 6%?

Interest on \$1 for $2\frac{1}{4}$ yr. at 6% = \$.135.
If \$1 produces \$.135 interest in the given time,

$$\text{Required principal} = \frac{\$33.75}{\$.135} = \$250\ \textit{Ans.}$$

Hence, in general,

Divide the given interest (*or amount*) *by the interest on* (*or amount of*) \$1 *for the given time and rate,*

Or *principal = interest ÷ (time × rate).*

$$\text{Or}\quad p = \frac{i}{r \times t}$$

Note that if the amount is given instead of the interest, it is necessary to *divide the given amount by the amount of* \$1 *for the given time and rate.*

Ex. 2. What principal will amount to $263.50 in 4 years at 6%?

Amount of $1 at 6% in 4 years = $1.24.

It will take as many dollars to amount to $263.50 as $1.24 is contained times in $263.50, or

$$\text{Principal} = \frac{\$263.50}{\$1.24} = \$212.50 \textit{ Ans.}$$

EXERCISE 86

Find the rate if

1. Interest on $420 for 3 yr. 6 mo. is $73.50.
2. Interest on $760 for 2 yr. 8 mo. is $121.60.
3. Interest on $900 for 2 yr. 4 mo. 20 da. is $96.75.
4. Interest on $370 for 6 yr. 6 mo. 6 da. is $72.335.
5. Amount of $360 in 3 yr. 10 mo. is $415.20.
6. Amount of $75 in 6 yr. 11 mo. 10 da. is $93.23.
7. Which pays the greater rate of income: an investment of $350 which in 2 yr. 6 mo. brings a return of $42, or an investment of $250 which brings in $24.75 in 2 yr. 3 mo.?

Find the time if

8. Interest on $75 at 6% is $15.75.
9. Interest on $475 at 4% is $33.25.
10. Interest on $570 at 7% is $332.50.
11. Amount of $8.25 at 6% is $13.134.

Find what principal will

12. Produce $156.40 interest in 5 yr. 8 mo. at 6%.
13. Produce $83.72 interest in 6 yr. 8 mo. 15 da. at 3%.
14. Amount to $99.22 in 3 yr. 6 mo. at 6%.
15. Amount to $327.384 in 7 yr. 4 mo. 12 da. at 7%.

Find:

16. Time for which the interest on $620 at 3% is $102.30.

17. Rate, if the interest on $1780 for 6 mo. 24 da. is $60.52.

18. Principal which produces $83.72 in 6 yr. 8 mo. at 6%.

19. Principal which will amount to $121.98 in 4 yr. 8 mo. 24 da. at 6%.

20. Time in which $420.75 will amount to $475.915 at 4%.

92. Present Worth and True Discount. If a man owes me $200, to be paid one year hence (without interest), the present worth of the $200 is that sum of money which, on being put at interest at the legal rate, will amount to $200 at the specified date.

In solving examples in present worth, the *amount*, *time*, and *rate* are given, and we have to find the *principal* (see Art. 91).

Thus, in finding the present worth of $200 to be paid one year hence,

$$\text{Principal} = \frac{\$200}{1.06} = \$188.68 \textit{ Present Worth}$$

Hence, in general, the **present worth** of a sum of money payable at a future date (without interest) is such a sum as, being put at interest, will amount to the given sum in the given time.

The **true discount** on a sum payable at a future date (without interest) is the difference between the sum due and its present worth.

Thus, in the above example,

$$\text{True discount} = \$200 - \$188.68 = \$11.32$$

EXERCISE 87

Find the present worth and true discount on

1. A debt of $378.75 due in 3 mo. without interest, the usual rate of interest being 4%.

2. A debt of $4377.80 due in 6 mo. 15 da. without interest, the usual rate being 6%.

3. A debt of $8255 due in 1 yr. 8 mo. without interest, when regular interest is at 3½%.

4. I owe you $1492 payable in 8 mo. 20 da. If I wish to pay the debt now, what sum should I pay you, interest being at 5%?

EXERCISE 88

REVIEW

1. A builder, by borrowing $10,000 on March 1, was able to buy a building lot for $1500 and erect a building upon it at a cost of $8500. On Nov. 12 of the same year, he sold the building and lot for $13,500. What was his profit on the transaction, if he paid 6% interest on the borrowed money?

2. If a young man borrows $1250 with which to educate himself, and pays interest on the sum borrowed for 5 yr. at 5%, what does his education cost him? If his education increases his earning power by $500 a year, in how many years will this increase in earnings pay for his education (the interest on the $500 increase not being considered)?

3. Make up and work a similar example for yourself.

4. If I buy 28½ tons of coal on Oct. 5 at $6.50 a ton, what is the cost? What would I have saved, if I had bought the coal on April 1 at $6 a ton, allowing 5% interest for the money thus invested?

5. Make up and work a similar example.

6. A man borrowed $500 for 2 yr. 6 mo. 17 da., giving his life insurance policy as security for the loan. If he paid 5% interest, what did the loan cost him? If the borrowed money enabled him

to save the life of his child by a surgical operation, can any one estimate the value to him of being able to borrow money?

7. Make up and work a similar example.

8. Compute the interest on \$367.45 from March 1 to June 3 at 5%, as a bank would do this. As the United States government would.

9. At what rate will \$360 yield \$66 interest in 3 yr. 8 mo.? In 7 yr. 4 mo.?

10. What principal will produce \$52.80 in 3 yr. 8 mo. at 4%?

11. If the rate is $4\frac{1}{2}$%, the time being 4 yr. 3 mo. and the amount \$123.89, find the principal.

12. In what time will a sum of money double itself at simple interest, the rate being 4%? 5%? 6%? 7%? 8%?

13. If 3 qt. 1 pt. of strawberries cost 42¢, what will 1 qt. 1 pt. cost?

14. If a floor $12\frac{3}{4}$ ft. wide is to be made from boards $\frac{5}{8}$ in. thick laid on edge, how many layers of boards will be needed?

15. If Texas is 212 times as large as Rhode Island, and New York is 39 times as large as Rhode Island, the area of Texas is how many times that of New York?

16. If a boy when 1 year old is 23 in. tall, and when 17 years old is 5 ft. 10 in. tall, what has been his average growth in height per year?

17. If a bushel of potatoes weighs 60 lb., what is the weight of the potatoes which fill a wagon body $12' \times 4' \times 2'$? ($1\frac{1}{4}$ cu. ft. = 1 bu.)

18. If 1 sq. rd. of cranberry bog yields 2 bu. 1 pk. 6 qt., at the same rate what will one acre yield?

19. The number of cubic inches in a gallon is what per cent of the number of cubic inches in a bushel?

20. Show that $2\frac{3}{4}$ equals $\frac{11}{4}$. Give an example, or case, where it is more advantageous to use $2\frac{3}{4}$ than $\frac{11}{4}$. $\frac{11}{4}$ than $2\frac{3}{4}$. Explain the advantage in each case.

21. The destruction of the quail and prairie chicken in Texas is said to be the cause of the boll weevil cotton pest in the United States. The annual loss due to this pest is estimated at $45,000,000 a year. This sum of money is the interest on what principal at 5%?

22. Work again Exercises 25 and 26 (pp. 39, 41) or similar groups of examples.

EXERCISE 89

INDUSTRY REVIEW: CORN

1. A field of corn, $\frac{1}{4}$ mile long and 75 rods wide, produced 1260 bu. of corn. How many bushels was this per acre?

2. The owner of the field in Ex. 1 selected the seed only from strong, vigorous stalks bearing two ears, and tested these ears before planting in the spring. By so doing, he later caused his field to produce 1820 bu. of corn in a year. What was his gain per acre, if 1 bu. of corn was worth 60¢ to him?

3. A certain gang plow travels 3½ miles an hour and has two plowshares, each of which throws a furrow 14 in. wide. How many acres of ground will it plow in 10 hours? A gang plow containing 8 shares will turn over how many acres in 10 hours?

4. What is the advantage in the use of a gang plow? Make up two examples showing the profit in the use of a gang plow.

5. A harrow 100 ft. wide moving at the rate of 3½ miles an hour goes over how many acres in 10 hours?

6. One man using a corn planter does 3 times as much work as when working without it, and the wages of a man are $1.50 per day. How much does the farmer save in wages, by having a man use a planter for 12 days?

7. A corn reaper and binder travels 3 miles an hour, and takes one row at a time; the rows are 3½ ft. apart. How many acres will it reap in 10 hours?

8. If the soil of the field in Ex. 1 was thoroughly pulverized before planting, at an extra cost of $2 an acre, and by this means 10 bu. per acre of the yield was saved in time of drought, what was the gain on 37½ acres?

9. If 1000 lb. of corn take 18 lb. of nitrogen from the soil, how many pounds does 1 bu. take? If red clover stubble adds 40 lb. of nitrogen per acre to the soil, this stubble will furnish nitrogen for how many bushels of corn per acre the next year?

SUG. No. lb. of nitrogen taken from the soil by 1 lb. of corn $= \frac{18}{1000}$. Hence, no. lb. of nitrogen taken by 1 bu. (or 56 lb.) of corn $= \frac{56 \times 18}{1000}$, etc.

10. A farmer has 1240 bu. of corn which he can sell for 78¢ a bushel. Instead of selling the corn, he feeds it to poultry and cattle which he afterward sells. He finds his returns are 3 times as great as they would have been if he had sold the corn. Allowing $200 as the cost of labor in caring for the poultry and cattle, what does he gain in money? Is this method of disposing of his corn profitable to the farmer in any other way?

11. If corn is planted in hills 3 ft. 6 in. apart each way, how many hills will there be per acre?

12. If each hill bears 3 ears of corn, the average weight per ear is 14 oz., and 70 lb. of corn in the ear make a bushel, how many bushels does the acre in Ex. 11 yield?

13. One acre has produced 255¾ bu. of corn. How much more was this than the yield in Ex. 12?

14. During a certain year, 100,000,000 acres were planted with corn in the United States, and yielded 2,200,000,000 bu. How much is this per acre? If this yield should be increased 5 bu. per acre, what would this increase be worth at 60¢ a bushel?

15. If 2,200,000,000 bu. of corn were raised in the United States in one year, and this was ⅞ of the crop in the entire world, what was the world crop that year?

16. In a certain year, the prize ear of corn for the whole country was sold at auction for $150. It contained 1092 grains. What was the price per grain? Why was this large sum paid for one ear of corn?

17. One year the average yield of corn per acre in a certain state was 15½ bu. On a demonstration farm in the same state, the yield was 37½ bu. If 4,000,000 acres were planted with corn in the state, what would the gain have been, if the yield had been equal to that on the demonstration farm?

18. In Ex. 17 the yield on the demonstration farm was what per cent greater than the average yield of the state?

19. Find out as many uses of corn as you can, and make up and work examples concerning its uses.

20. Find out, if you can, the number of acres planted with corn in your state, and the yield per acre, and make up examples concerning increased yields due to improved methods of cultivation.

CHAPTER VIII

APPLICATIONS OF INTEREST

PROMISSORY NOTES

EXERCISE 90

1. If Amos Campbell of Philadelphia should lend Walter Wilson $7500, what means would Mr. Campbell probably take to prevent Mr. Wilson from forgetting the obligation to repay the loan?

2. What means would he take to prevent any misunderstanding as to the amount of the obligation?

3. As to the date of repayment?

4. As to the place where the repayment is to be made?

5. As to the rate of interest to be paid on the loan?

6. If Mr. Campbell wants some one to guarantee that Mr. Wilson will pay the loan of $7500 at the specified time, where would this guaranty be most conveniently written, on a separate piece of paper, or on the back of Mr. Wilson's written promise to pay?

93. A **Promissory Note** is a written promise to pay a specified sum of money to a specified person (or to his order) at a given date.

A promissory note is often termed simply a "note."

$7500 Philadelphia, Pa., *Oct. 15*, 19*13*
Thirty days after date *I* promise to pay
Amos Campbell or order
Seventy-five hundred Dollars
Value received, with interest at *6%*
Walter Wilson

94. The **maker** of a note is the person who signs the note; as Walter Wilson in the above note of Art. 93.

The **payee** is the person to whom the note is made payable; as Amos Campbell in the above note.

The **face** (or principal) is the sum promised to be paid.

The face of the note is written in the body of the note in words, not figures, to avoid fraud or error. The number of cents, however, is usually expressed in figures, as the hundredths of a dollar.

The date of **maturity** is the date on which payment of the note is promised, or on which the note becomes due.

95. The **Essential Parts of a Note** are the *promise to pay*, the *sum* of money to be paid, the *name* of the person to be paid, the name of the person by whom the payment is to be made (*signature*), the *date* when the *promise* is made, the *date* when the *payment* is to be made, and the *place* where the payment is to be made.

Each of the essential statements in the note must be definite and precise. Thus, the exact sum of money to be paid must be specified; a statement that the "market value" of a bond, for instance, is to be paid is not sufficient.

If the place where the payment is to be made is not specified, the post-office residence of the maker of the note must be included in the note, and the note paid there.

It is desirable to have the words "value received" in the note. If these words are omitted, the holder of the note may be required to prove that the value of the note has been paid by him to the maker of the note.

If a note is not paid in whole or in part, and if interest is not paid on the note, within a certain number of years (the time varies from five years to twenty years in different states), the obligation to pay the note lapses by what is called the *statute of limitations*.

96. Time Classification of Notes. As to time, notes are classified as (1) **demand** (or **call**) notes, and (2) **time** notes.

A **demand note** is a note such that the payee may call for its payment at any time he chooses. Thus, in the note on p. 204, if the words "thirty days after date" are struck out, and the words "on demand" put in their place, the note becomes a demand note.

A **time note** is a note which is to run for a specified time, and the payment of which cannot be required until the specified time has expired. The note on p. 204 is a time note.

In some states, a note is not due until three days after the time specified in the note, the three days being called *days of grace*.

In the year 1910, days of grace were in use in the states of Mississippi, Nevada, Texas, and Wyoming, and in a partial or modified form (as for demand notes only, or in cases of special agreement between the maker and holder of a note) in Arizona, Arkansas, Kansas, Massachusetts, Maine, New Hampshire, Oklahoma, South Dakota, Vermont, and West Virginia.

In most states, when a note falls due on Sunday or on a legal holiday, payment is not legally due until the next succeeding legal business day. Thus, if a note falls due on Dec. 25 and Dec. 25 comes on Sunday, in those states where a legal holiday coming on Sunday is celebrated on Monday, such a note would not be due until Tuesday, Dec. 27.

97. Interest Classification of Notes. With respect to interest, notes are classified as (1) **interest-bearing** notes, and (2) **non-interest-bearing** notes.

If a note contain the words "with interest," interest is computed on the note from the date at which the note is given. The interest on the note on page 204 is computed beginning with Oct. 15, 1913.

If, however, the words "with interest" are omitted from a time note, interest is computed from the day on which the note becomes due. Thus, if in the above note the words "with interest at 6%" were omitted, interest would be computed beginning thirty days after Oct. 15, that is, beginning with Nov. 14.

If the words "with interest" are omitted from a demand note, the note bears interest from the date on which payment is demanded to the date when the note is paid.

Interest on notes is usually computed for the exact number of days (even when the note runs for a certain number of months), allowing either 360 or 365 days to the year.

Unless otherwise stated, interest on promissory notes, in this book, will be computed by taking the time as an exact number of days and regarding the year as 360 days.

Ex. Find the amount due on the following note at maturity:

$860.42. TRENTON, N. J., May 14, 1912.

Three months after date, I promise to pay Stephen Blake, Eight Hundred Sixty $\frac{42}{100}$ Dollars, value received, with interest at 5%.

JAMES EVANS

The note is due 3 mo. after May 14, that is, on Aug. 14.
From May 14 to Aug. 14, the number of days is as follows:

May,	17 days.
June,	30 "
July,	31 "
Aug.,	14 "
Total,	92 days

Interest on $860.42 for 92 da. at 5%

$$= \frac{\overset{\$430.21}{\cancel{\$860.42}} \times \cancel{5} \times \overset{23}{\cancel{92}}}{100 \times \underset{\underset{\underset{9}{\cancel{18}}}{\cancel{72}}}{\cancel{360}}} = \$10.99 \textit{ Ans.}$$

$$\begin{array}{r} \$860.42 \\ 10.99 \\ \hline \$871.41 \end{array} \textit{ Amount}$$

Amount due Aug. 14 = $871.41 *Ans.*

EXERCISE 91

1. Rewrite the note of Art. 93 so as to make it a demand interest-bearing note.

2. Rewrite so as to make it a demand non-interest-bearing note.

3. In the note of Art. 93, who is the maker? Payee? What is the principal? Date of maturity?

4. Why is a written promise to pay more satisfactory than an oral promise?

Find the date of maturity and the amount due then, on each of the following notes:

5. $360. ALBANY, N. Y., April 7, 1913.

Four months after date, I promise to pay Bradley Gould, Three Hundred Sixty Dollars, with interest at 5%, for value received.

JONATHAN SCUDDER

6. $935. SAVANNAH, GA., Dec. 5, 1912.

On demand, we, or either of us, promise to pay, with interest at $4\frac{1}{2}$%, to Charles R. Watson, Nine Hundred Thirty-five Dollars, without defalcation.

HORACE DAY

(Paid, 8 mo. from date.)

ERNEST F. KEIGWIN

7. $700. MOBILE, ALA., Oct. 8, 1913.

Ninety days after date, I promise to pay to the order of William Black, Seven Hundred Dollars, with interest at $3\frac{1}{2}$%, for value received.

ELLERY FRANKLIN

8. $520.60. CHICAGO, ILL., Dec. 16, 1914.

Six months after date, I promise to pay Sherman Thatcher, Five Hundred Twenty Dollars and $\frac{60}{100}$, with interest at 6%, for value received.

BENJAMIN CARROLL

9. $1400. LITTLE ROCK, ARK., Mar. 8, 1913.

On July 1, 1913, I promise to pay Robert Dolliver, or order, Fourteen Hundred Dollars, with interest at six per cent, for value received. ROGER SOMERVILLE

10. $250. ST. LOUIS, MO., Oct. 1, 1909.

On January 1, 1915, I promise to pay Dwight Ogden, Two Hundred Fifty Dollars, with interest at five per cent, payable semiannually. F. I. SHELL

SUG. Find the interest for the number of years and for the exact number of days in the rest of the time.

11. A 90-day note for $2800, with interest at 5½%, was paid at maturity. What amount was due?

12. A 3-month note for $6750 at 4¾% was paid when due. What amount was due?

13. Write a 60-day note with your teacher as payee and yourself as maker, for $100, interest at 5%. Compute its value when due.

14. Write a 6-month promissory note for $1670, bearing interest at 4½%, with Horace Mansfield as maker and John Douglass as payee. Compute its value at maturity.

15. Write a demand note for $3250, at 5½%, with yourself as payee and Richard Smith as maker. Compute its value 1 yr. 82 da. from date.

16. A note for $300 is given on March 1, with interest payable semiannually at 5%. Find the interest on the note from March 1 to Nov. 4 of the same year.

SUG. Six months after March 1, that is on Sept. 1, 2½% interest is due on the note. Find this interest, and to it add the interest from Sept. 1 to Nov. 4.

17. A note for $725.60 is given on Jan. 1, 1912, with interest

at 6% payable semiannually. Find the interest on the note from its date to Sept. 15, 1913.

18. Write a check in payment of the bill of goods on p. 44. Also write a promissory note which might be used instead of the check. Point out the differences between the check and the note.

19. Find out what a joint note is. Also what a joint and several note is.

20. Find out whether a note dated on Sunday is void. Whether one obtained by fraud or from an intoxicated person is void. Is a note given by a minor void?

21. If a note is lost or stolen, is the maker of the note released from payment?

22. Find out what different kinds of paper money are in use in the United States. Which of these are promissory notes?

23. Are the bonds issued by the government or by a corporation, in effect, promissory notes?

24. Point out the different reasons for which the following form of note is not allowable:

> After the next presidential election, if business revives, I promise to pay my partner in business the market value of the building in which we are now doing business.
>
> A FRIEND OF THE NEEDY

25. Make up and write a note which is defective in two and only two respects.

After the teacher has inspected the note, correct the defects in the note.

26. Ascertain the law of your state as to the rate of interest to be charged, if this rate is not specified in a note; also as to the place where the interest is payable. Also ascertain the custom of your local bank as to their method of computing interest. Make up and work three examples based on these facts.

27. Practice oral work with small decimal fractions, as in Exercise 29 (p. 49).

98. Transfer Classification of Notes. With respect to their transfer, notes are (1) **negotiable** and (2) **non-negotiable.**

A **negotiable** note is a note that is transferable, *i. e.* one which may be made payable to a person other than the person specified in the note.

To be negotiable, a note must be written as payable in one of the three following forms: (1) To a specified person "or bearer"; (2) "To the order of" a specified person; (3) "To the bearer."

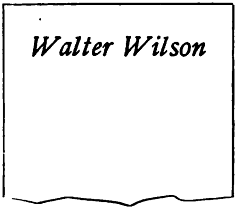
In some states, in order that a note may be negotiable, the law requires also that the place where the note is payable must be specified in the note.

A **non-negotiable** note is one drawn payable to a specified person only.

In general, the payee or holder of a note transfers the note by writing his name across the back of the note. This is called **indorsement.** The payee, in indorsing a note, becomes in effect a new maker of the note and becomes responsible for its payment.

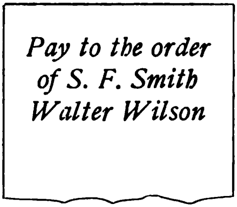
Indorsements are of several kinds, according as the indorser wishes to limit or modify his responsibility for the payment of the note.

Indorsement in Blank



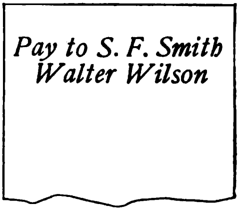
Walter Wilson

Indorsement in Full



Pay to the order of S. F. Smith
Walter Wilson

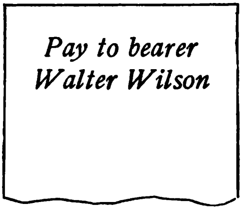
Restrictive Indorsement



Pay to S. F. Smith
Walter Wilson

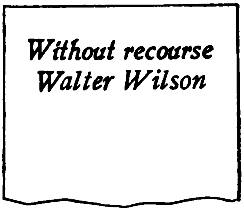
Another form of

INDORSEMENT IN BLANK



Pay to bearer
Walter Wilson

QUALIFIED INDORSEMENT



Without recourse
Walter Wilson

If the indorser writes the words "without recourse" above his name (*i. e.* gives a qualified indorsement), it means that he transfers the note, but is not responsible for its payment.

If the original note is made payable to bearer, or is indorsed as payable to bearer, it may be transferred without further indorsement.

99. Failure to Pay a Note. If the maker of a note fails to pay the note when it becomes due, every indorser of the note becomes liable for the payment of the entire note. If, however, an indorser has written the words "without recourse" above his name, he is not liable. Several indorsers may have an agreement to share the amount of the note between them.

In case of the failure of the maker of a note to pay the note, the usual method of the holder is to collect the amount of the note from the last indorser, who collects it in like manner from the preceding indorser, etc., up to the first liable indorser. The holder may, if he chooses, collect the amount of the note from any liable indorser; such collection releases all indorsers *subsequent* to the one from whom collection is made, though *prior* indorsers are still liable.

A **protest** is a written notice sent by a notary public at the request of the holder of a note to the indorsers of the note, stating that the note has not been paid when due.

If a protest is not sent to an indorser on the day on which the note matures, the indorser is released by law, unless the words "waiving demand and notice" have been written above the indorser's signature. If the notice has been properly made out and mailed by the notary, the law assumes that a protest has been made.

EXERCISE 92

1. What words must be struck out of the note of Art. 93 (p. 204) to make it non-negotiable?

2. Point out the negotiable notes in Exercise 91. Give reasons for your statement.

3. Rewrite the negotiable notes in Exercise 91, making them non-negotiable. Also rewrite the non-negotiable notes in the same Exercise, making them negotiable.

4. Let each pupil in the class write and sign a note for $600 negotiable on being indorsed, and payable to the pupil sitting to his right.

5. Let the payee of each of these notes indorse it in full in favor of the pupil to the indorser's right.

6. Suppose that one of these notes is not paid when due; what course must the holder follow in order to collect the amount due?

7. Which of the methods of indorsement on pp. 210–211 makes a note negotiable by the new payee, and which does not?

8. What is "paper" in business, *i. e.* what does a term like "John Wanamaker's paper" mean? What is a note broker?

9. Make as many different kinds of notes as you can, and give a list of the adjectives applicable to each.

10. What is a check, as used in drawing money from a bank?

11. Name the differences between a note and a check.

12. Pay the bill in Ex. 1 of Exercise 27 (p. 45) by a 60-day note negotiable without interest.

13. Pay the bill in Ex. 2 of Exercise 27 by a 3-months note non-negotiable with interest.

14. Pay the bill in Ex. 3 of Exercise 27 by a 30-day note non-negotiable and without interest.

15. How many examples can you now work at sight in Exercise 24 (p. 37)?

PARTIAL PAYMENTS

100. **Partial Payments.** It is frequently convenient for the maker of a note, instead of paying the note all at one time, to make *partial payments* on it, as money comes into his possession, until the entire note is paid. Payments can be made in this way by agreement between the maker and the payee of a note.

101. **Indorsements.** The partial payments are recorded on the back of the note, in a form known as *indorsements.* Each indorsement specifies the amount paid and the date of payment, and includes the signature of the holder of the note. It is thus, in effect, a receipt for each amount paid on the note.

Several kinds of indorsements occur in business, but all have reference to a writing of some sort on the back of a business paper, the word *indorsement* being obtained from the Latin word *dorsum*, meaning "back."

102. **United States Rule** for the settlement of a note on which partial payments have been made. The Supreme Court of the United States has decided that in the settlement of a note on which one or more partial payments have been made, no interest is to be allowed on a payment, or on interest due on the note. Hence, this court has fixed the following rule for the settlement of such notes:

Determine the earliest date at which the sum of the payments thus far made on the note equals or exceeds the interest due on the note;

Find the amount of the note at this date;

Subtract the sum of the payments from the amount, and use the remainder as a new principal;

Proceed in like manner until the amount due on the day of settlement is determined.

The time between dates in partial payments is usually obtained by finding the difference between the given dates by the method used in Exercise 34 (p. 68), and not by determining the exact number of days between dates.

Ex. Find the amount due on the following note on Jan. 1, 1914.

$800. NEW YORK, Jan. 1, 1912.

Two years from date I promise to pay James White, or order, Eight Hundred Dollars for value received, with interest at 6 per cent.

SAMUEL HILLMAN

Payments are recorded on the back of this note as follows:

Received on the within note:
April 1, 1912, $10
Jan. 1, 1913, $100
April 10, 1913, $200

As the interest on $800 for 3 months is $12, the first payment is less than the interest to date.

Hence, we have

Interest on $800 from Jan. 1, '12, to Jan. 1, '13	=	$48
Principal	=	800
Amount	=	848
Sum of payments to Jan. 1, '13	=	110
Remainder for new principal	=	$738

New principal = $738.00
Interest on $738 from Jan. 1, '13, to Apr. 10, '13 (*i. e.* for 3 mo. 9 da.) = 12.18
Amount = 750.18
Payment Apr. 10, '13 = 200.00

New principal = $550.18
Interest on $550.18 from Apr. 10, '13, to Jan. 1, '14 (*i. e.* for 8 mo. 21 da.) = 23.93
Amount due on Jan. 1, '14 = $574.11 *Ans.*

In working examples like that above, a diagram is helpful, since it makes clear at each point in the process what the next step should be. It facilitates comparison of different parts of the work and correction of errors.

	PRINCIPAL	TIME			INTEREST	AMOUNT	PAYMENT	NEW PRINCIPAL
		YEARS	MONTHS	DAYS				
First								
Second								
Third								
Fourth								

EXERCISE 93

1. $600. HARRISBURG, PA., Nov. 1, 1912.

Three years after date, I promise to pay George Morris, Six Hundred Dollars, with interest at 5%.

THEODORE JOHNSON

The following indorsements were made: Jan. 1, 1913, $60; July 1, 1914, $100; Dec. 10, 1914, $220. What remained due at maturity?

2. What is due July 1, 1915, on a note for $2100, dated Jan. 1, 1913, with interest at 4%, on which are indorsed the following payments:

Sept. 1, 1913, $450; Aug. 1, 1914, $620; Feb. 1, 1915, $500?

3. A note for $2400 dated July 15, 1912, drawing interest at 5%, bears the following indorsements:

Dec. 20, 1912, $740; Mar. 31, 1913, $250; June 18, 1914, $600. What is due Dec. 1, 1914?

4. How much is due Jan. 1, 1914, on a 6% note for $1500, dated Oct. 20, 1911, and indorsed as follows:

Dec. 30, 1911, $450; July 9, 1912, $25; Nov. 7, 1913, $600?

5. How much remains due Oct. 15, 1913, on a 5% note for $3600, dated June 23, 1910, and indorsed as follows:

Oct. 9, 1910, $100; Jan. 9, 1911, $200; Aug. 20, 1912, $300; Jan. 31, 1913, $1500?

6. Make up and work an example involving partial payments.

7. Work again Exercise 33 (p. 67).

103. Merchants' Rule. When a note is settled within a year of the date at which it is given, business men frequently use the following rule to determine the amount due when partial payments have been made.

Find the amount of the principal to the date of settlement;

Find the amount of each payment from the time it was made to the date of settlement;

Take the sum of the amounts of the payments, and subtract it from the amount of the principal; the remainder will be the balance due.

Since the periods of time are short in these notes, interest is usually calculated for the exact number of days. The method of exact interest is used in solving the following problem and those in Exercise 94.

Ex. Find the amount due on Dec. 31, 1913, on a note for $300, interest at 7%, dated Feb. 15, 1913, on which the following payments have been made: Mar. 25, $75; June 2, $37.50; Oct. 10, $50.

Amount of	$300	from	Feb. 15	to Dec. 31	(319 da.)	=		$318.35
"	75	"	Mar. 25	"	(281 da.)	=	$79.04	
"	37.50	"	June 2	"	(212 da.)	=	39.05	
"	50	"	Oct. 10	"	(82 da.)	=	50.79	
								168.88
				Balance due Dec. 31, 1913,				$149.47

EXERCISE 94

1. A note for $790 at 6%, dated Jan. 15, 1914, bears two indorsements: July 1, $400; Nov. 19, $60. What is due Dec. 31, 1914?

2. Find the amount due Dec. 15, 1914, on a 5% note for $2500, dated Mar. 12, 1914, and indorsed as follows: May 15, $800; Aug. 10, $350; Nov. 3, $90.

3. Find the amount due Nov. 29, 1913, on a 4% note of $1500, dated Feb. 6, 1913, and indorsed as follows: Apr. 25, $350; June 6, $240; Aug. 7, $120; and Nov. 3, $400.

Borrowing Money from a Bank; Bank Discount

EXERCISE 95

1. What is a bank? Name some of the uses of a bank.

2. Name some different kinds of banks.

3. In what respects is a trust company like a bank? How does a trust company differ from a bank?

4. While a bank is safeguarding the money of its depositors, how does it also use this money as a source of profit for itself?

5. If a man needs to borrow money, how can he often save himself the labor (and also the unpleasantness) of going from one of his acquaintances to another till he finds a person able and willing to lend him the money he wants?

6. If a man owns stocks or bonds, how can he obtain money through their use, without selling them?

7. Is a bank or an individual more likely to inconvenience a borrower by calling suddenly for the repayment of a loan?

8. As a rule, does a bank or an individual have the wider knowledge and experience by which to determine whether money can be safely loaned to carry on a given enterprise?

104. Bank Discount. Money may be borrowed from a bank, as from an individual, in return for a note; but a bank usually computes interest, not on the present worth of the note, but on the amount of the note at maturity. This interest is termed *bank discount.*

105. Kinds of Notes Used in Borrowing Money from a Bank.

(1) *A note held as payee by a person, indorsed by the person, and deposited in a bank.*

Such a note may be a note made out to a payee in return for a loan, or a note given a payee in settlement of an account, or a note transferred to the payee by some other payee.

If the holder of such a note wishes to obtain money on it immediately, he indorses it by writing his name on the back, and deposits the note in the bank. In return, the bank pays him the sum due at the maturity of the note, less the interest on this sum from the date of deposit at the bank to the date of maturity.

The sum paid out by the bank is called *proceeds* or *avails.*

(2) *A note made out by a person, payable to another person, indorsed by the other person, and deposited in the bank by the maker.*

The friend, by indorsing the note, makes it payable to the original maker, who thus is able to obtain money for it from the bank.

(3) *A note made by the borrower and payable to the bank.*

For instance, a person desiring to borrow money may make out a note payable to the bank, and get a friend to indorse it. Or, instead of securing a friend's signature, the maker of the note may deposit with the bank stocks or bonds (called "collateral") as security for the future payment of the note.

On a note made out in this last way, the maker pays to the bank interest as it becomes due, say every three months. Hence, this interest differs from the bank discount on an interest-bearing note, in that it is computed on the face of the note and not on the amount of the note when due.

Banks usually reckon discount on the exact number of days that the note runs, and usually count 360 days to the year.

Ex. 1. What are the proceeds on a note for $500 for 60 days discounted at a bank at 6%?

Interest on $500 for 60 da. = 500 × $.01 = $5 *Discount*
$500 − $5 = $495 *Proceeds*

Ex. 2. Find the proceeds of the following note discounted at 5%, on Jan. 25, 1914:

$650. PHILADELPHIA, PA., Jan. 6, 1914.

Ninety days after date I promise to pay to the order of Anthony Fisher, Six Hundred Fifty Dollars, value received, with interest.

ROBERT ALLEN

The note is an interest-bearing note. Hence, it is necessary first to find its amount at maturity.

Interest on $650 for 90 da. at 5%	=	$8.13
Amount of note when due (Apr. 6)	=	658.13
(From Jan. 25 to Apr. 6 = 71 da.)		
Discount on $658.13 at 5% for 71 da.	=	6.49

$658.13 − $6.49 = $651.64 *Proceeds*

EXERCISE 96

Before working each of the following examples, note carefully:

(1) Whether the note bears interest or not;

(2) The date of maturity;

(3) The date when the note is discounted.

1. Find the proceeds of a note for $1200 due in 90 days, discounted at 6%. At 5%.

2. Find the bank discount on a note for $870, due in 90 days at 6%.

3. A note for $800, due in 60 days, if interest is at 5%, is equivalent to how much cash?

4. A 6-months note for $650, dated June 1, was discounted July 15, at 6%. Find discount and proceeds.

5. A 3-months note for $1400, dated July 15, was discounted Aug. 10, at 5½%. Find the proceeds.

6. A 90-day note for $5000, with interest at 6%, dated May 4, was discounted June 8 at 5%. Find the proceeds.

In each of the following, determine the day of maturity, time to run, discount, and proceeds:

7. $1600. TRENTON, N. J., June 1, 1914.

Three months after date I promise to pay George Williams Sixteen Hundred Dollars for value received.

Discounted at 6%, July 15. WASHINGTON NORRIS

8. $2000. JACKSONVILLE, FLA., Feb. 20, 1913.

Ninety days after date I promise to pay Jacob Warren Two Thousand Dollars with interest at 6%.

Discounted at 7%, April 1. ANDREW FLEMING

9. $870. WHEELING, W. VA., July 10, 1913.

On Jan. 10, 1915, I promise to pay William Phillips Eight Hundred Seventy Dollars with interest at 5%.

Discounted Oct. 1, at 6½%. FREDERIC TOWNSEND

10. A note for $1200, bearing interest at 4½%, dated Oct. 25, 1913, due Feb. 28, 1914, and discounted Dec. 1, 1913, at 7%.

11. A 90-day note for $1800, given Dec. 16, 1914, was discounted Dec. 30, at 6%. If 3 days of grace are allowed, find the proceeds.

12. What is the difference between the true discount and the bank discount on $500 for 3 yr. 6 mo. at 5%?

13. Find the proceeds of a note for $460.30, dated Oct. 15, 1913, payable in 9 months, bearing interest at 6%, and discounted Feb. 25, 1914, at 5%.

14. A note for $2300, dated July 30, 1914, and payable in 90 days, with interest at 5%, was discounted Sept. 23, at 7%. Find the proceeds.

15. On Mar. 2, 1915, Andrew Walters of Chicago bought a horse of William Harris, and in part payment gave a note for $150 payable in 30 days without interest. Three days after the note was made out, William Harris took the note to a bank, indorsed it in blank, and had it discounted at 6%. Write this note, indorse it, and compute the amount received by Mr. Harris.

16. Make up a similar example concerning the sale of a building lot.

17. Make up another example entirely your own.

18. In Ex. 15, let us suppose that Mr. Walters (instead of giving his own note for $150) had in his possession a 6-months note for $150 with interest at 6%, drawn on Feb. 1, 1915, payable

to Mr. Walters, signed by James Silvers. Mr. Walters by indorsement makes this note payable to Mr. Harris. Make out this note with indorsements and compute its proceeds.

19. Make up and solve a similar example.

20. Work again Exercise 69 (p. 146) or work at sight similar examples concerning percentage.

106. Proceeds to be a Certain Sum. If a person wishes to obtain from a bank a certain sum, as $200 on a 3-months note at 6%, it will be necessary to determine the face of the note that will yield that sum.

Thus, on a note for $1 the bank would pay, in the above case, $1 less the interest on $1 for 3 months; *i. e.* $1 – $.015, or $.985. Hence, to obtain $200, the face of the note must be as many dollars as $.985 is contained times in $200, or $203.05.

Verify this result by obtaining the proceeds of $203.05 for 3 months at 6%.

Hence, in general,

Divide the given proceeds by the proceeds of $1 *for the given time and rate.*

EXERCISE 97

Find the face of the note which

1. Is to run 60 days, and when discounted at 6% will realize $891.

2. Is to run 6 months, and when discounted at 5% will realize $3493.30.

3. When discounted at the bank for 4 mo. 9 da. at 6% will give $72,850.

4. I owe a man $800 and wish to pay him by a 6-months note. What sum must the note demand, so that when discounted at the bank at 5%, the debt is exactly paid?

Compound Interest

107. Compound Interest. If interest is not paid when it becomes due, under some circumstances interest on the unpaid interest, as well as on the principal, is computed for the next period of time. Such interest is called *compound interest.*

Ex. Compute the compound interest on $400 for 3 years at 5%, interest being payable annually.

Principal	= $400	(1st principal)
Interest for the first year	= 20	
Amount at end of first year	= 420	(2d principal)
	.05	
Interest for second year	= 21.00	
	420	
Amount at end of 2d year	= 441.00	(3d principal)
	.05	
Interest for 3d year	= 22.05	
	441	
Amount at end of 3d year	= 463.05	
	400	
	$63.05	*Compound Interest*

If the above example had called for the compound interest on $400 for 3 yr. 2 mo. 15 da., instead of 3 years merely, after finding the compound amount for 3 years as above, or $463.05, we find the simple interest on this sum for 2 mo. and 15 da., or $4.82. We then add the $4.82 to the compound interest already found, $63.05, and obtain $67.87 as the compound interest for 3 yr. 2 mo. 15 da.

Savings banks usually compute interest on deposits in this way, adding the interest to the principal at the end of a stated period, as six months. Compound interest is not, however, allowed by law on ordinary debts.

When the annual rate of interest is 5%, for instance, and the interest is compounded semiannually, the compound interest is obtained by computing the interest for twice as many periods of time as there are years, at half the annual rate, $2\frac{1}{2}\%$.

Thus, to find the semiannual compound interest on $500 for 4 years at 7%, find the compound interest on $500 at $3\frac{1}{2}$% for 8 periods of time.

In computing compound interest, it is often useful to use tables giving the amount of $1 for various rates for different periods of time.

Such tables may be found, for instance, in the almanacs published by leading daily newspapers.

108. Details with respect to **Savings Banks**. (1) In savings banks interest is usually payable twice a year, on January 1 and July 1.

(2) If interest is not drawn when it is due, it is added to the amount on deposit, that is, interest is compounded by the bank.

(3) Interest is not computed on the fractional part of a dollar.

Thus, if a person has a deposit of $121.63 in a savings bank, interest is computed only on $121.

(4) In an interest credit, a fractional part of a cent is rejected.

Thus, if $7.958 interest is due a depositor, he receives, or is credited with, only $7.95.

EXERCISE 98

1. Find the amount of $600 in 4 years at 4 per cent, interest being compounded semiannually.

2. $100 left in the bank for 5 years will amount to how much at 4 per cent, if interest is added semiannually?

3. A hospital fund of $6000 is put in savings banks. What is the value of the fund in 4 years, if interest is at 4 per cent and is compounded semiannually?

4. If the same amount were put out at 4 per cent on a first mortgage, how much would it earn in the same time, interest being payable semiannually?

5. Find the difference in the income from the two methods of investment stated in Exs. 3 and 4.

Find the compound interest on

6. \$1800 for 8 yr. at 4%. Also find the amount.

7. \$900 for 3 yr. at 6% compounded semiannually.

8. \$1600 for 5 yr. at $4\frac{1}{2}$%.

9. \$3000 for 4 yr. 6 mo. at 4%.

10. \$2500 for 5 yr. 3 mo. at 6%. Find the amount also.

11. What is the difference between simple and compound interest on \$1500 for 6 yr. at 5%?

12. What sum at 4% compound interest will amount to \$1000 in 4 yr.? In 5 yr.? In 3 yr. 9 mo.?

13. If on Jan. 1, 1914, a boy had \$32 in a savings bank and he deposited \$12 on Apr. 1, and \$22.50 on Sept. 15, what balance did he have at the bank on Jan. 1, 1915, interest being at 4 per cent? (Reckon time in mo. and $\frac{1}{2}$ mo.)

14. Make up and work a similar example involving two deposits.

15. The United States postal savings bank pays 2% interest annually. If a woman has \$200 deposited in this bank for 3 years, what would be her income from this, at compound interest? The government will convert her deposit of \$200 into a bond from which she will receive $2\frac{1}{2}$% payable semiannually. Find the simple interest on this bond for three years.

16. Work again Exercise 40 (p. 83) or a similar Exercise.

ANNUAL INTEREST

109. Annual Interest. Sometimes, in addition to simple interest on the principal, simple interest is paid on each unpaid installment of interest. This is called *annual interest.*

In some states, if a note or bond contains the words "with interest payable annually," annual interest can be collected.

Ex. Find the annual interest on a note for \$500 for 4 years at 6%.

In this case, simple interest on the principal for 1 yr. is \$30. By annual interest, simple interest will be computed for 3 yr. on the first \$30 due, for 2 yr. on the second, and for 1 yr. on the third.

Hence,

Interest on \$500 for 1 year at 6% = \$30
" \$500 " 4 " 6% = 120
" \$30 " 3 + 2 + 1 (or 6) years = 10.80
\$120 + \$10.80 = \$130.80 *Annual Interest*

EXERCISE 99

Find the annual interest and amount of

1. \$700 at 5% for 3 yr.
2. \$1200 at 4% for 5 yr.
3. \$90 at $3\frac{1}{2}\%$ for 4 yr.
4. \$125 at 6% for 3 yr. 6 mo.
5. \$750 at 4% for 4 yr. 4 mo.
6. Find the difference between annual interest and compound interest of \$360 at 5% for 5 yr.
7. Find the simple, the exact, the compound, and the annual interest on \$4800 at 5% for 6 yr. 6 mo.
8. How many examples in Exercise 44 (p. 92) can you now work at sight?

BONDS AND TIME

110. Bonds and Time. Bonds are almost always issued to run a specified number of years. At the end of the specified period of time the company retires a bond by paying its face or par value. Hence, if a bond is selling above or below par, a person in buying it for an investment should carefully consider not only the rate of interest paid by a bond, but the number of years the bond has to run before it is retired.

Thus, if a bond is selling at 110, pays 5%, and has 6 years to run, the person who buys it and holds it till it is retired will receive considerably less than an annual return of 5% on his investment, since the bond will be retired at 100, or par. The exact rate of income which he will receive cannot be computed by the arithmetical processes treated in this book. Brokers have tables, computed for their use by higher mathematics, which give the rates of income in such cases.

In like manner, the net return from life insurance policies (and in certain cases from other insurance policies) cannot be accurately determined by the application of either simple or compound interest.

EXERCISE 100

If brokerage is $\frac{1}{8}$%, find the cost of the following bonds, and the average annual income from them.

1. Ten one thousand first mortgage 5s, at 102, which mature July 1, 1935, and which yield according to the bankers' tables about 4.8% income on the market value, 102.

2. Five one thousand first mortgage 4s at 92$\frac{1}{2}$, which mature Jan. 1, 1957, and yield about 4.4%.

3. Seven one thousand first mortgage 5s at 104$\frac{1}{2}$, which mature May 1, 1929, and yield 4.65%.

4. If bonds of a certain kind are selling at 103, mature Feb. 1, 1939, and hence yield 4.8% annually on the average, find the number of one thousand dollar bonds which must be bought to produce an income of not less than $960.

5. Have an oral review concerning practical measurements, similar to that in Exercise 58 (p. 120).

EXERCISE 101

REVIEW

1. A 60-day note for $700 without interest was paid in 90 days. What was the amount paid, if interest at 6% was charged after the note became due?

2. A note for $1000 is given Jan. 1, 1913, with interest from date, on which the following payments are made: July 8, 1913, $200; Oct. 1, 1913, $10; Sept. 25, 1914, $100; Mar. 18, 1915, $400. On Jan. 1, 1917, what is due on the note, the rate of interest being 6%?

3. $745.85. NEW YORK, Nov. 12, 1913.

Four months after date, for value received, I promise to pay Walter Harris, or order, Seven Hundred Forty-five $\frac{85}{100}$ Dollars.

OLIVER WALTERS

This note was discounted Nov. 29, 1913, at 6%. What were the proceeds?

How many terms can you apply to this note in describing it?

4. Benjamin Franklin in his will left $5000 to be kept at interest until it amounted to a certain sum. In 118 years it amounted to $408,000. How much more is this than it would have amounted to at simple interest in the given time at 5%? Why did it increase to so large a sum?

5. A note for $530 was given on July 1, 1912, with interest at 5% payable semiannually. Find the interest on this note from its date to Dec. 2, 1912.

6. Find the interest on the note in Ex. 5 from July 1, 1912, to May 5, 1914.

7. What gives value to paper money?

8. What must be the face of a 90-day note that will amount to $527.80, if interest is computed at 6%?

9. If a tank is 2′ 4″ × 1′ 10″ × 1′ 3″, how many gallons will it hold?

10. In digging the Panama Canal, 2,400,000 cu. yd. were excavated in one month. If the earth removed were spread over a square mile, how deep would it be?

11. If a bushel of corn weighs 56 lb., how many bushels are there in $2\frac{1}{2}$ tons?

12. If a family use $\frac{3}{4}$ pt. of kerosene each day on the average, how long will 5 gal. last them?

13. If gunpowder is made up of 2 parts sulphur, 3 parts charcoal, and 15 parts niter, what per cent of the whole is each of these?

14. The area of a lot 120 ft. × 300 ft. is what per cent of an acre?

15. In writing a check, is it necessary to number the check? Why do men usually number their checks?

16. In plowing one acre with a plow which turns a 12-inch furrow, how far does a boy walk? How far does he ride, when plowing one acre with a 6-plow gang plow?

17. If a certain kind of steel is 98.1 parts iron, 1.7 carbon, .2 other elements, how many pounds of carbon are there in a ton of steel?

18. A desk was sold for $60 at a gain of 20%. What would have been the loss per cent had it been sold for $40?

19. If 500 shares of railroad stock were bought at $28\frac{1}{2}$ and sold at $45\frac{3}{4}$, what was the gain, allowance being made for $\frac{1}{8}$% broker's charge on each transaction?

20. A grocer mixes equal amounts of 36¢ tea and 60¢ tea and sells the mixture at 56¢ a pound. What is his per cent of profit?

21. Write as rate per cent: $\frac{1}{6}$ $\frac{7}{10}$ $\frac{7}{16}$ $\frac{9}{40}$ $\frac{1}{300}$ $\frac{13}{40}$ $\frac{5}{8}$ $\frac{27}{6}$

22. A house worth $6400 was insured for $\frac{3}{4}$ of its value at $\frac{2}{3}$%. It was destroyed after the second premium had been paid. What was the owner's actual loss? The company's?

23. State the reasons why it is more advantageous, as a rule, to use decimal fractions than common fractions, as .37 rather than $\frac{37}{100}$. Illustrate by the use of examples.

24. Give three common fractions which are simpler than their equivalent decimal fractions.

25. Find the average height of five brothers whose heights are 5′ 10″, 5′ 7½″, 5′ 4″, 4′ 10½″, and 4′ 2″. By what per cent does the tallest exceed the average?

Make up a similar example concerning the average length of four long jumps.

26. It is estimated that if the common house fly could be kept from food, deaths in cities in summer could be diminished by $\frac{1}{3}$. How many lives would be saved thus in a city of 1,000,000 inhabitants, if the summer death rate in the city is .9%?

27. On a field of 5½ acres, a farmer had successive cuttings of alfalfa as follows: 9¾ tons, 6½ tons, 5¾ tons, and 3⅛ tons. What was the yield per acre for the season?

28. If one book contains 328 pages each 9¼″ × 12″ and another book 395 pages each 8″ × 10½″, by how many square inches does the page surface of one book exceed the other? By what per cent?

29. If 7 acres of town lots in a city were divided into 58 equal garden plots for the poor, each plot being 50 ft. wide, how long would each be?

30. Show that 19 ft. 8 in. equals 236 in. State a case where it is more advantageous to use 19 ft. 8 in. than 236 in. and give the reason. Also where it is more advantageous to use 236 in.

31. Make up a similar example concerning pints, quarts, and bushels.

32. If 27% of a piece of ground sold for $540, what is the rest of the ground worth at the same rate?

33. 67,200 lb. of corn put into a crib in November have lost 20% in weight by the next May. How many bushels of 56 lb. each will the corn make in May? At what price per bushel should this corn be sold in May, if the price in November was 60¢ per bushel, the shrinkage being the only reason for the difference in profit?

34. By what decimal of a mile does the height of Mt. Everest exceed 5 miles? Make up and work a similar example (see p. 386) concerning the height of Mt. Washington. Of Pike's Peak.

35. If the area of the United States in 1800 was 827,944 square miles, by what per cent has its area increased since then?

36. The volume of a cellar 32 ft. × 18 ft. × 6 ft. is how many times that of a cellar 21 ft. × 12 ft. × 4 ft.?

EXERCISE 102

INDUSTRY REVIEW: FARM ANIMALS

1. The feed for a cow for 75 days was 10 bu. of oats at 40¢, and 10 bu. of corn at 72¢, and 1 ton of clover hay at $15.50. Find the cost of keeping a cow for 6 months at these rates.

2. During the remainder of the year it cost $1.75 per month to pasture a cow. What was the entire cost of feeding the cow for a year?

3. During the year the cow produced 2600 qt. of milk worth 5¢ a quart. What was the profit from the cow, allowing $10 for

interest on her value and for depreciation and insurance? What would be the profit on 28 cows at this rate?

4. If 1 qt. of milk weighs 2.3 lb., how many pounds of milk did the cow produce in the year?

5. If 3.5% of the milk was butter fat, how many pounds of butter fat might have been obtained from this milk? What would this butter be worth at 26¢ a pound?

6. A Holstein cow gave per day 13 qt. of milk containing 3.5% butter fat, and a Jersey cow gave 9 qt. containing 8.5% butter fat. Which of the two yields was more profitable, when the milk was sold at 7¢ a quart? Which, when the butter fat was sold at 27¢ a pound?

7. If milk is 28% cream, and the cream is 35% butter fat, and the butter fat will make $1\frac{1}{8}$ times its own weight in butter, 100 lb. of milk will make how many pounds of butter?

8. If a separator gathers all the cream in Ex. 7, while the old skimming method gathers only 80% of the cream, how many

more pounds of butter will be made by use of the separator than by skimming?

9. A boy milked cows by hand 3 hours each morning and evening. Later he used a milking machine which enabled him to milk cows three times as fast as by hand. How many hours were saved for him each day by this means?

10. One cow has produced 27,432 lb. of milk in one year. How many quarts is this per day (see Ex. 4)? 1164.6 lb. of butter in one year is how many pounds per day? (Find to three decimal places.)

11. If in building a barn 1000 cu. ft. of space should be allowed for each horse, and 400 cu. ft. for each cow, how many cubic feet should there be in a barn which is to hold 6 horses and 12 cows?

12. If a farmer who has a large crop of corn buys 100 steers in November for $4692, feeds them with corn and hay worth $1080, and sells them in March for $6927.50, what is his gain, allowing $125 as the cost of labor in caring for them?

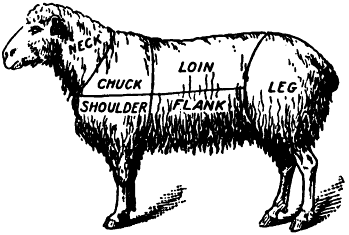
What other advantage is there in having corn fed to stock in this way on the farm, rather than in selling it at a distance?

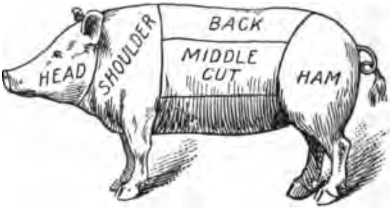
13. If instead of buying cattle, as in Ex. 12, the farmer had bought 200 hogs for $1400, fed them with $560 worth of feed, and sold them for $2352, allowing $75 for labor, would his rate of gain have been greater or less than the gain on the cattle?

14. If 252 sheep produce 8½ pounds of wool on the average, which sells at 32¢ a pound, what is the value of the wool?

15. The dressed carcass of a beef was cut up into the following parts: neck 40 lb., chuck 23 lb., prime ribs 116 lb., porterhouse steak 103 lb., sirloin 87 lb., rump 36 lb., round 184 lb., shank 30 lb., flank 52 lb., ribs 40 lb., and plate 138 lb. Draw an outline of a beef and on it mark the parts given above. Find the weight of one hind quarter.

16. Here are diagrams showing the different parts into which a sheep and a hog are cut by the butcher. Make up and work examples concerning these.





17. Find out the facts concerning the most important animal industry in your neighborhood, as dairying, raising of cattle, or the keeping of horses, mules, hogs, sheep, chickens, ducks, turkeys, pigeons, or bees. Make up and work examples concerning one of these.

18. Treat in the same way any form of fishing (including catching of oysters, clams, lobsters) that is practiced in your neighborhood.

CHAPTER IX

TRADE; EXCHANGE; COLLECTIONS

EXERCISE 103

1. If it costs a fruit grower in Michigan $1.10 to raise a barrel of apples and he sells the same for $1.50, how much does he make per barrel?

2. If it costs a wheat farmer in Dakota $4 to raise a barrel of apples and he can buy for $2.20 the barrel sold by the Michigan farmer, how much would the Dakota wheat farmer save by the purchase? How much is the combined profit of the two farmers by the transaction?

3. If the Dakota wheat farmer can raise a bushel of wheat for 40 cents a bushel and sell it for 90 cents, how much does he make?

4. If it costs the Michigan farmer $1.80 to raise a bushel of wheat and he can buy for $1.10 the bushel of wheat sold by the Dakota farmer, how much would he save by this purchase? How much is the combined profit of the two farmers by this transaction in wheat?

5. How much would they make together by the transactions in both apples and wheat? How much would each make by his part in the two transactions?

6. Make up and work a similar group of examples illustrating the benefit to both parties by an exchange, say of grapes raised in New York for sweet potatoes raised in New Jersey, or any other two agricultural products raised in different parts of the country,

7. Also make up and work examples concerning a farmer and a manufacturer who exchange products with mutual profit.

111. **Trade** is an exchange of commodities. It is based on the principle that by taking advantage of the natural resources of different places, each party to an exchange may profit by the exchange.

Trade is carried on almost altogether in terms of money (see Exs. 1–5 on page 235). Arithmetical operations greatly facilitate trade by making clear where exchanges are profitable, and also by making the processes of exchange easier.

112. **Transportation of Goods.** Trade is facilitated also by using arithmetic to make a careful study of the cost of different methods of transporting goods. For instance, it is found that the cost of transportation by water is only about one sixth of the cost by railroad.

113. **Classes of Railroad Freight.** Different freight rates are charged for the different classes of objects, because of differences in weight in proportion to bulk, differences in difficulty of handling, liability to breakage, value, etc. In general, there are six classes of railroad freight, known as classes 1, 2, 3, 4, 5, and 6. In actual practice, however, there are many other classes of freight.

Thus, in a freight office, D 1 means "Double First Class," $2\frac{1}{2}$ means "Twice and a Half First Class," etc. Also very heavy loose articles like sand and iron ore, are arranged in classes A, B, C, D.

If goods are shipped in carloads, the rates are much smaller than the rates for a part of a carload.

If a shipment weighs less than 100 lb., the full charge for 100 lb. is usually made.

114. A Bill of Lading is a receipt given by the railroad (or other carrier) to the shipper of articles of freight. It includes a list of items sent; number of pounds in each item; a statement of the class of freight to which each item belongs; and the freight rate from shipping point to destination. On the back of the bill of lading is the contract of shipment.

Let the teacher or a pupil obtain a bill of lading, so that the class may examine it.

The **invoice** is another list of items shipped. It is given by the shipper to the railroad company (or other carrier), and accompanies the freight on its journey.

EXERCISE 104

The following is the freight classification, on a certain system of railroads, for the articles named. (When a second rate is given, it means the rate for a carload.)

Bananas $1\frac{1}{2}$
Baskets 3 t 1 (*i. e.* 3 times first class)
Bone (ground) 4, 6
Coal (carload) A
Crockery 2
Eggs 2
Feathers D 1
Furniture (set up) 1
Furniture (not set up) 2
Glassware 1, 4
Harness 1, 3
Peas (green) 1, 4
Potatoes 4, 5
Reaping machines D 1
Sand (carload) 6
Wheat 5, A

In working the following examples, regard the freight rate per 100 pounds between two cities on the different classes of freight as follows:

1st class, $1.52
2d “ $1.36
3d “ $1.20
4th class, 98¢
5th “ 83¢
6th “ 65¢
A class, 52¢
B “ 55¢
C “ 53¢

Treat a carload as 30,000 lb.

Find the freight on

1. 200 lb. of eggs.
2. 400 lb. of bananas.
3. 150 lb. of feathers.
4. 150 tons of coal.
5. 250 lb. of baskets.

6. How much cheaper is it to ship 90,000 lb. of wheat in carload lots than in less than carload lots? Also 25,000 lb. of potatoes?

7. How much cheaper is it to ship 2000 lb. of furniture knockdown than to ship the same set up?

8. Make up and work three similar examples.

9. If the rate on second-class freight between two places is 47¢ per 100 pounds, and on fourth-class is 23¢ per 100 pounds, how much is saved by having 10,000 lb. of freight between the two places classified as fourth-class instead of second-class?

10. If it costs 38¢ per 100 pounds to ship a certain kind of goods from Pittsburg to Louisville by rail, and only 7¢ to ship the same by boat, how much is saved by shipping $2\frac{1}{2}$ tons by boat?

11. Practice the rapid addition of columns of figures.

Transferring Money without the Aid of a Bank

115. Transfer of Money. Trade is made more exact, swift, and extensive by the use of money. In the processes of trade, money must often be sent from place to place. People frequently exchange obligations to send money (as checks, notes, etc.) and thus avoid the risk and expense of actually sending the money itself.

116. Postal Money Orders. Often the transfer of money (with the risks and cost of such transfer) may conveniently be avoided by the use of a postal money order. The money to be sent is paid to the home postmaster, who gives in return a written order upon the distant postmaster to pay the sum named to the person specified in the order. This written money order is then mailed to the specified person, who by means of it obtains the specified sum from his home post office.

For making out and cashing money orders in this way a small charge is made at the post office where the postal order is made out. Thus:

For a postal money order

Sum	Charge
Less than $2.50	3 cents
Over $10 and less than $20	10 cents
Over $75 and less than $100	30 cents

Ascertain the charge for different sums between $20 and $75. Can money orders be bought at all post offices?

To send a sum larger than $100, more than one postal order must be bought.

A postal money order may be transferred by the payee to one other person, but the second payee cannot transfer it again. Hence, a postal money order has a *limited negotiability.*

If a postal money order is lost in the mail, a duplicate order may be obtained. This will contain the words "original unpaid," to prevent a payment of both orders.

117. Express Money Orders. The offices of an express company sell and pay money orders in the same way and at the same rate as post offices. Money orders sold by an express company have the advantage that they are payable not only at the offices of the company, but at the offices of any other express company, or at any bank, upon identification of the holder of the order. (What is meant by identifying a person?) An express money order also has the advantage of *unlimited negotiability,* like that of a check or bank draft; that is, it can be repeatedly transferred to another holder by indorsement.

118. Sending Money by Telegraph. Telegraph offices also sell orders for the payment of money in distant places. In this case, the order to pay is telegraphed both to the payee and to the paying office. The cost for the order is therefore greater, since the buyer of the order pays the cost of the telegrams. The usual charge for telegraphing money is 25¢ for $25 (or any sum less than $25), and 1% on any sum greater than $25, together with the cost of the telegrams (usually two 10-word messages) sent in each instance.

Before paying out the money telegraphed, the representative of the telegraph company requires the payee to be identified.

119. Utility of Money Orders. Since money orders are being sent back and forth between post offices all over the country, on the average each post office, in cashing orders, will pay out about as much money as it receives for the orders it writes. Hence, very little money need be actually transferred. The same is true with respect to express money orders and telegraphic orders. It is because of this fact that the cost and dangers of transmitting actual money are avoided.

120. Exchange is a term for methods of paying or collecting money at a distance without the actual transfer of money itself.

EXERCISE 105

Compute the cost of a postal money order for each of the following:

1. $2.75
2. $9.50
3. $19.20
4. $ 6.72
5. $62.80
6. $34.60
7. $99
8. $101
9. $320
10. $500

11. Compute the cost of an express money order for each of Exs. 1–10.

12. Make up and work three examples similar to Exs. 2, 7, 9.

13. What will it cost to telegraph $150 from St. Louis to a shipwrecked person at Long Branch, N. J., if the cost of a 10-word telegram between the two places is 50¢?

14. Using the telegraph rates from your home office to different places, make up and work two similar examples.

15. How much more will it cost to telegraph $120 to a distant city than to send the money by postal money order, the cost of a telegram of 10 words being 50 cents?

16. Make up and work a similar example.

17. In a recent year, express companies and the United States government charged 50¢ per $100 for transferring money from New York to Chicago, St. Louis, and the West; 75¢, to New Orleans and the South; and 25¢, to neighboring Eastern cities. What was the cost of sending $5,650,000 from New York to Chicago to aid in moving the grain crop in the West? Of sending $3,275,000 to New Orleans to aid in moving the cotton crop?

18. How many examples in this Exercise can you work at sight?

19. Work again Exercises 62 and 65 (pp. 131, 137) or similar examples dictated by the teacher or pupil.

121. Checks. If a person has money on deposit at a bank, he draws out the money by use of a check. A **check** is a written order to a bank directing the bank to pay a specified sum of money to a specified person.

A *certified check* contains a statement by the bank that the sum named in the check has been set aside by the bank, from the deposits of the maker of the check, for the payment of the check when presented.

TRANSFERRING MONEY BY THE AID OF A BANK

122. Sending Money by a Personal Check is a method frequently used when the place to which money is sent is not far distant.

The check is made out as usual and sent by mail to the person to whom money is to be paid. The person receiving the check indorses it, deposits it in a local bank, and receives either cash or credit for it. The bank which receives the check on deposit, collects its value, either directly or indirectly, from the bank on which it is drawn. For this service the bank often charges a small fee, which varies according to the amount of the check and which is called *cost of exchange*, or simply *exchange*.

Let the pupil point out the advantage of sending money by personal check. The disadvantage is that a check may be lost in the mail and much delay and inconvenience may result. Another disadvantage is that the payee of the check has no certain knowledge whether the sender has deposits equal to the face value of the check in the bank on which the check is drawn. This difficulty may, however, be met by the use of a certified check.

123. Sending Money by Bank Check or Draft. If a large sum of money is to be paid at a considerable distance, a bank check or draft is generally used. The process involved may be illustrated thus:

James Smith, of Chattanooga, owes Daniel Compton, of New York City, $2250. To send the money in the mail would involve risk of loss; to send it by express would be expensive. A direct check on a Chattanooga bank might be expensive for Compton to collect. Hence, Smith goes to a bank in Chattanooga (the Farmers'), which has money on deposit in a New York bank (the Chemical), and buys a draft, or order, in which the Farmers' Bank directs the Chemical Bank to pay the required sum to Daniel Compton, thus:

FARMERS' BANK,
$2250. CHATTANOOGA, TENN., *Jan. 22, 1914.*

At sight, pay to the order of *Daniel Compton, Two Thousand Two Hundred Fifty Dollars,* value received.

THOMAS FORSYTH
To the CHEMICAL NATIONAL BANK, Cashier
NEW YORK, N. Y.

If, in the above illustration, Daniel Compton lives in Boston instead of New York, he takes the draft to his bank in Boston, which cashes it for him. The Boston bank then sends the draft to its correspondent in New York, which receives payment from the Chemical National Bank through the New York Clearing House.

This business process is made possible by the fact that over nine tenths of the banks in the United States outside of New York City keep a considerable deposit in some New York bank, called a *correspondent* bank.

The above draft might have been drawn in several other different forms. It might have been drawn to the order of James Smith and indorsed by him to the order of Daniel Compton (banks prefer this, since it shows who bought the draft). Or, instead of "pay to the order of Daniel Compton," the following might have been used: "pay to New York Exchange for Daniel Compton," or "pay to order of Daniel Compton on New York Exchange."

124. A **draft** is a written order made in one place for the payment of money in another place.

Drafts are of two principal kinds: (1) **bank drafts** and (2) **commercial drafts.**

Commercial drafts will be considered later (see Art. 127).

A **bank draft** is a draft in which a bank directs a bank in another place to pay a specified sum to a specified person or party.

A bank draft is, in effect, a check drawn by one bank upon another. A bank draft is often called a banker's check, or cashier's check, or a bank check.

The **maker** or **drawer** of a draft is the person who signs the draft.

The **drawee** is the person directed to pay the sum.

The **payee** is the person to whom the money is to be paid.

In the above draft, the maker or drawer is the Farmers' Bank of Chattanooga, of which Thomas Forsyth is the cashier. Point out the drawee and the payee.

The fee charged by a bank for making a draft varies with the premium or discount of exchange.

125. Par, Premium, Discount. If, in buying a draft, a person has to pay the exact face of the draft, exchange is said to be *at par;* if more than the face, exchange is said to be *at a premium;* if less than the face, exchange is said to be *at a discount.*

Premium and *discount* of exchange (apart from paying for the labor involved in the exchange) arise from the fact that the banks of one large money center, as San Francisco, may owe the banks of another large center, as Chicago, a considerable sum. The San Francisco banks must either go to the expense of sending this money to Chicago, or must pay interest on it. In this case, a person in San Francisco, buying a draft on Chicago, would have to pay a considerable premium, since his draft would increase the balance at Chicago against San Francisco. On the other hand, a person at Chicago, buying a draft on San Francisco for a large amount, might get the draft at a discount, since it would diminish the balance against San Francisco.

Drafts or bills of exchange are negotiable, and when indorsed may be bought and sold. Certain brokers make it their main business to buy and sell these drafts and other commercial paper.

Exchange is sometimes reckoned as a fraction of 1% of the face of the draft, and sometimes as so many cents per $1000. (See Ex. 15, p. 246.)

126. Sight and Time Drafts. A **sight draft** is one which is to be paid immediately on presentation.

A **time draft** is one payable after a specified time.

A time draft is to be presented immediately to the drawee; if he agrees to pay it, he writes the word "accepted" across the face, with the date and his signature. This is called an acceptance, is equivalent to a promise to pay, and, in effect, makes the draft a promissory note.

It is evident that, since a time draft is not payable until its maturity, its value at any given date, apart from the cost of exchange, is the face value less the bank discount on it up to the time of maturity.

Ex. 1. Find the cost of a draft on New York for \$500 at $\frac{1}{2}\%$ premium.

Premium = $\frac{1}{2}\%$ of \$500 = $500 \times .005$ = \$ 2.50
Face of draft = 500.00
Cost of draft = \$502.50 *Ans.*

Ex. 2. What must be paid in Boston for a draft on St. Louis, at 30 days, for \$1200, exchange being at $\frac{1}{4}\%$ premium?

Discount on \$1200 for 30 days (at 6%) = \$ 6.
Proceeds of \$1200 = \$1200 − \$6 = 1194.
Premium on \$1200 at $\frac{1}{4}\%$ = 3.
Cost of draft = \$1194 + \$3 = \$1197. *Ans.*

EXERCISE 106

Find the cost of a draft for

1. \$900 at $\frac{1}{4}\%$ premium.
2. \$2500 at $\frac{1}{8}\%$ discount.
3. \$7600 at $\frac{1}{4}\%$ premium.
4. \$100,000 at $\frac{1}{8}\%$ discount.
5. \$570, payable in 30 days, exchange being at $\frac{1}{4}\%$ premium, and interest at 6%.
6. \$3000, payable in 60 days, exchange being at $\frac{1}{8}\%$ discount, and interest at 5%.
7. \$2400, given by a Boston merchant to a Chicago manufacturer, payable in 60 days, exchange at $\frac{3}{4}\%$ premium.

8. What is the difference between a check and a draft? Between a negotiable note and a draft?

9. How large a sight draft can be bought for $2500, exchange being at $\frac{3}{8}$% premium?

10. What will be the cost of a draft for $1680, payable in 60 days, exchange being $\frac{1}{2}$% premium, and interest being at 6%?

11. Find the face of a draft on New York, at 90 days' sight, bought for $450, exchange being 1$\frac{1}{2}$% premium, and interest at 5%.

12. Write a personal check drawn on the Marine Bank of Philadelphia which James Bowers might use in paying a bill of $207.30 which he owes E. C. Githens of Richmond, Va.

13. The following item appeared in a daily newspaper:

> *New York, Apr. 16.* — Exchange on New York at Boston was 15¢ discount. Chicago, 20¢ premium. St. Louis, 35¢ premium. New Orleans, 90¢ premium. St. Paul, 55¢ premium. Savannah, buying, 75¢ discount; selling, 75¢ premium. Montreal, 15$\frac{5}{8}$¢ premium.

At these rates find the cost in Boston of a New York draft for $3500. Also find the cost of such a draft on New York in each of the other cities named. In which of these cities is money plentiful and in which scarce, as compared with New York?

14. Cut a similar clipping from a daily newspaper and make up and work two examples, using the exchange rates given in it.

15. Henry Otis of Minneapolis owes Charles Treat of Providence $2500 and wishes to pay the debt. Mr. Otis goes to the Mechanics Bank of Minneapolis and buys a draft on the National City Bank of New York, and sends it to Mr. Treat, who deposits it in the Traders Bank of Providence. Write the draft and its indorsements as far as you can, and describe the steps of the transaction in full. What is the cost of the draft at 55¢ premium?

16. Make up and solve a similar example.

17. 500 bbl. of cranberries are shipped by Wilkinson and Co. to a commission house of Denver, Col. The commission house sold 300 bbl. at $6.75 and the rest of the cranberries at $6.50 per barrel. They deducted $2\frac{1}{2}\%$ for commission, $128.50 for freight, $14.75 for storage, $\frac{1}{8}\%$ of $2000 for insurance, and sent Wilkinson a sight draft for the balance, after deducting the cost of exchange, which was at a premium of 65¢. How much did they remit?

18. A hardware jobber in New Orleans buys $3200 worth of goods in Chicago at 30%, 10%, and 2% off, and sells them at an advance of 22% on the cash price paid by him. If he pays for the goods with a New York draft at 5¢ premium, how much does he make?

19. Sometimes in telegraphing money from one part of the country to another, the cost of exchange must be allowed for. What is the cost of telegraphing $3500 from San Francisco to New York, if the premium of exchange is $8\frac{1}{2}$¢ and the cost of one telegram is $1.35?

20. Make up and work a similar example.

21. Which will cost more to send: $85.40 from New York to Chicago by postal money order, or by a bank draft with exchange at 15¢ premium?

22. Work again Exercise 67 (p. 141).

COLLECTIONS OF COMMERCIAL DRAFTS

127. The Meaning and Use of Commercial Drafts may be illustrated as follows:

A. L. Smith and Co. of Philadelphia have a creditor, R. W. Gates and Co. of Pittsburg, who owes them $1200. A. L. Smith and Co. either need money at once or they desire to bring pressure to bear on their creditor, who has been slow in payment. A. L. Smith and Co. make out a draft as follows:

$1200.00 Philadelphia, Pa., *Aug. 6*, 19*14*.

Thirty days after date pay to the order of

First National Bank of Philadelphia

Twelve hundred $\frac{00}{100}$ ∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼∼ Dollars

Value received and charge to account of

To *R. W. Gates & Co.* *A. L. Smith & Co.*
63 Margin St., Pittsburg, Pa.

A. L. Smith and Co. take this draft to the First National Bank of Philadelphia and deposit it with the bank. The First National Bank of Philadelphia sends the draft to some Pittsburg bank. The Pittsburg bank presents the draft (say on Aug. 10) to R. W. Gates and Co., who, if they desire to pay the debt, write (in red ink) across its face "Accepted Aug. 10, 1914." The draft now becomes equivalent to a time promissory note. (If it had been a sight draft, it would have become equivalent to a demand promissory note.)

This accepted draft is now returned to A. L. Smith and Co. and they may have it discounted, if they have credit or desire to have it discounted.

If R. W. Gates and Co. decline to write "Accepted" on the draft, A. L. Smith and Co. must try in some other way to collect the money due them.

If A. L. Smith and Co. are of good standing, the First National Bank of Philadelphia may credit them with the value of the draft when it is first presented to the bank; that is, may discount the draft for them.

A **commercial draft** is an order written by one party directing another party to pay a specified sum to a specified bank on or before a specified date.

128. The Process of Shipping Goods and Collecting Payment for them by Commercial Draft may be illustrated as follows:

The American Fork and Hoe Co. of Cleveland, Ohio, sell Hart and Co. of Atlanta a carload of hoes worth $600. The Am. F. & H. Co. wish to secure either a quick collection of the cash value of the goods, or a promise from Hart and Co. to pay for the goods on a specified date. To accomplish this purpose, the Am. F. & H. Co. use a *bill of lading* and a *commercial draft.* From the freight office in Cleveland where the goods are shipped, the Am. F. & H. Co. receive the bill of lading, or written statement that the goods have been received for shipment. (Hart and Co. cannot take the goods from the freight car at Atlanta till this bill of lading has been received by them and presented at the Atlanta freight office.) The Am. F. & H. Co. take this bill of lading and deposit it in a Cleveland bank (say the Second National) together with a draft like the following:

No Protest

$600.00 Cleveland, O., *Mar. 6,* 19*14*.

At sight pay to the order of

Second National Bank of Cleveland

Six hundred and $\frac{00}{100}$ Dollars

Value Rec'd and charge to acc't of

To *Hart & Co.* *American Fork and Hoe Co.*

Atlanta, Ga.

The Second National Bank of Cleveland sends the bill of lading and draft to a bank in Atlanta, which presents the draft to Hart and Co. for payment. If Hart and Co. pay the draft (or accept it, if it is a time draft), they receive the bill of lading and can take the hoes from the car in the Atlanta freight station. If Hart and Co. decline to pay the draft, the Atlanta bank notifies the Second National Bank of Cleveland. It, in turn, notifies the Am. F. & H. Co., who must then dispose of the carload of hoes in some other way.

If the draft is a time draft and is accepted by Hart and Co., the draft is returned to the Am. F. & H. Co., who can then discount it in their home bank; or the Am. F. & H. Co. might ask the Atlanta bank to discount the draft and send them the proceeds, allowing for exchange.

In discounting a time draft, the discount is computed from the date of discount, not from the date of the draft.

If the shipper has the confidence of the bank, often the home bank will discount a draft (whether sight or time draft) before forwarding it for collection.

Observe carefully the advantages of thus collecting the value of goods, as compared with the apparently simpler method of sending a bill to a purchaser and waiting for a personal check in payment.

1. By the method of bill of lading and commercial draft, the customer cannot get possession of the goods until he has paid for them.
2. He cannot make the false claim that he has not received a given bill.
3. He cannot claim that he has sent in payment for the goods a check which has been lost in the mails.
4. He cannot send a worthless check in payment and thus put the American Fork and Hoe Co. to great inconvenience.
5. If the American Fork and Hoe Co. are in immediate need of money, they can get the cash for their goods from their local bank (the Second National) without waiting for their customer to remit.

EXERCISE 107

1. Walters and Co. of Minneapolis sell Hyde and Cable of Chicago, on Jan. 1, 1913, 1800 bushels of wheat at $1.02, and draw on them at sight through the City National Bank of Minneapolis. Write the draft which they would send with the bill of lading.

2. Instead of a sight draft, Walters and Co. might send a 60-day draft. Write this draft in a form which shows its acceptance by Hyde and Cable.

If this draft were bought by a Chicago bank at 6% discount and exchange were $\frac{1}{8}$%, find the proceeds remitted to Walters and Co.

3. Make up and work for yourself an example similar to Ex. 1 concerning 20,000 feet of white pine lumber sold by Henry and Sons of Concord, N. H., at $62 per M, to James Altman of Boston.

4. Write a draft similar to that called for in Ex. 2, about the facts of Ex. 3.

5. Make up and work an example entirely your own similar to Ex. 1.

6. Also one similar to Ex. 2.

7. A commercial draft for $2200 payable 60 days after date was discounted 12 days after date, the rate of discount being 6%. If the cost of exchange was $\frac{1}{8}$%, find the net proceeds of the draft.

8. Work again Exercise 69 (p. 146), or similar examples concerning percentage.

FOREIGN EXCHANGE

129. **Foreign exchange** is exchange between persons (or parties) living in different countries.

Exchange between two persons (or parties) living in the same country is called **domestic exchange**.

In foreign exchange, payments (and collections) of money are made in the same general way as in domestic exchange. However, the names or terms used in foreign exchange often differ from those used in domestic exchange. Thus, a foreign postal order is frequently called a *post remittance;* an express order, a *travelers' check;* a telegraph money order, a *cable transfer;* and a draft, a *bill of exchange.*

The cost of foreign exchange is also naturally greater than the cost of domestic exchange. For instance, for foreign postal money orders, the cost is about three times as great as that for domestic orders.

Also premium and discount of exchange (see Art. 124) are more prominent. Thus, the premium or discount of exchange must be reckoned as a part of a post remittance to foreign countries, or of a cable transfer of money.

130. A Bill of Exchange is usually drawn in duplicate and the two copies are sent by different routes to avoid delay or loss. Each of the two copies of the draft contains a clause specifying that if the other draft has been paid, the draft in hand is void.

131. Par of Exchange and Quotations. Foreign exchange is based on the par of exchange, or the standard value of the currency of one country in terms of that of another country. For instance, the par value of a pound sterling is $4.8665.

In foreign exchange, instead of being quoted as a certain per cent of the face of the draft, a premium is added to (or a discount subtracted from) the value of the fundamental money unit of one of the countries.

Exchange on the following countries is usually quoted as follows:

Great Britain, Ireland. — United States money (in dollars) that must be paid for £1.

Thus, London 4.87$\frac{3}{4}$ means that $4.8775 must be paid in New York for a draft for £1 payable in London.

Germany. — United States money (in cents) that must be paid for 4 marks of German money.

Thus, Berlin 95½ means that $.955 must be paid in New York for a draft for 4 marks payable in Berlin.

France (and all countries using the franc or its equivalent). — French money (in francs) or lire, etc., that may be bought for $1.

Thus, Paris 5.15½ means that $1 in New York will buy a draft of 5.15½ francs payable in Paris.

Ex. What is the cost in Philadelphia of a bill of exchange on London for £250, when exchange is at $4.875 for the pound sterling?

$$\begin{array}{r} \$4.875 \\ 250 \\ \hline 243750 \\ 9750 \\ \hline \$1218.75 \end{array} \text{ \textit{Ans.}}$$

132. Travelers' Checks. Many persons, in traveling either in the United States or in other countries, carry money orders in the shape of travelers' checks. These are small booklets of printed checks (in denominations of $10, $20, $50, $100, $200) issued by express companies and banks. These checks may be detached and used one at a time.

The purchaser signs the checks when buying them, and must sign each check again at the time when it is detached and used. The comparison of the two signatures is a means of preventing fraud. Forgery of signatures on these checks is almost unknown, because of (1) the speed at which a signature must be made and (2) the small amount gained by such forgery.

The express company or bank which issues such checks usually charges ½% of the face value of the check for so doing. Travelers' checks are readily accepted in all parts of the United States and Europe at hotels and various business places.

EXERCISE 108

1. Give the meaning of each of the following exchange quotations:

(*1*) London, 4.87¾
(*2*) Paris, 5.16½
(*3*) Leipsic, 95¼
(*4*) Dublin, 4.86½
(*5*) Rome, 5.18
(*6*) Berlin, 95½
(*7*) Florence, 5.14½
(*8*) Brussels, 5.15

2. At 95¾, find the cost of a bill of exchange on Berlin for 5000 marks.

3. What is the cost in New York of a bill of exchange on London for £6300 at $4.865 per pound sterling?

4. What is the face of a bill of exchange on London that was purchased for $13,406.25, exchange being quoted at $4.875 per pound sterling?

5. When exchange is quoted at 5.18⅜ francs to $1, what will be the face of a bill of exchange on Paris that is bought for $2300?

6. When exchange on London is quoted at 4.85, what will be the face of a draft that can be bought for $7779.40?

7. Find the cost of a bill of exchange on Antwerp for 14,176.75 francs, exchange being at 5.17¼.

8. How large a bill of exchange on Paris can be bought for $8000, exchange being at 5.21?

9. When the quotations for London exchange are "sight 4.87¾; 60 da. 4.841," what is the difference between the cost of a sight draft and a 60-day draft for £560?

10. The following items appeared in a daily newspaper:

> Sterling exchange was firm, with demand quoted at $4.8765 @ 4.8775, 60 days at $4.8410 @ 4.8420, and cables at $4.8795 @ 4.8805.
>
> Continental bills were quoted as follows:
>
> Francs, 5.17 @ 5.17 plus 3–64 for cables and 5.17 less 1–32 @ 5.17

plus 1–64 for sight; reichmarks, 95 @ 95 5–16 less 1–32 for cables, 96 3–16 @ 95 plus 1–64 for sight and 94 @ 94 for 60 days.

Find out the meaning of the various statements contained in it. Why does a sight draft cost more than a time draft?

11. A Chicago merchant sold 10,000 bushels of wheat in London and drew on the buyer for £2160. If the merchant sold his draft to a broker at $4.84\frac{1}{2}$, brokerage $\frac{1}{64}$%, what did he receive? Would he have received more or less by selling to another broker at $4.84\frac{3}{4}$ with the brokerage at the rate of $1 per £1000?

12. What is meant by a letter of credit?

13. If a tourist buys a letter of credit in London for £250 at $4.875 and 1% commission, what does the letter cost?

14. If he had bought the same amount in travelers' checks at $\frac{1}{2}$%, what would the cost have been?

15. Find the cost of 200 lire sent to Naples by postal money order, if exchange is at $5.17\frac{1}{2}$ and the postal fee is 70¢.

16. Make up and work two examples, using the rates given in Ex. 1.

17. Work again Exercise 82 (p. 187) or similar examples concerning interest.

EXERCISE 109

REVIEW

1. What is the difference between a personal check and a bank check?

2. State the advantages in collecting the value of goods by means of a commercial draft rather than by sending a bill for the goods by mail and waiting for a personal check from the buyer of the goods in payment for the same.

3. What is meant by the statement "We will draw on you at sight"?

4. What is meant by "putting a note in bank"?

5. State the advantages and disadvantages in sending money by post-office money orders. By express money orders. By telegraph. By personal check. By bank check.

6. What is the cost of sending $78.50 by postal money order? By express money order? By personal check, if the cost of collection is $\frac{1}{2}\%$?

7. If the exchange on a bank draft was $41 and the rate was $\frac{1}{4}\%$ premium, what was the face of the draft?

8. Albert H. Howard of Camden, N. J., draws on Warren Adams of Wheeling, W. Va., for $3600 at 60 days' sight, through the First National Bank of Camden. Write the draft. If the draft is accepted and discounted at 6% on the day of acceptance, find the proceeds, if the charge for collection was $\frac{1}{12}\%$.

9. What is the cost of a draft on Paris for 4139 francs when exchange is quoted at $5.17\frac{3}{4}$?

10. If a young woman can write 17 words a minute with the pen, and 52 words with the typewriter, how many minutes will she save by using the typewriter in copying 20,000 words? Which result will probably be easier to read?

11. What is the short way of telling whether a number is divisible by 3? Write a number containing five figures which is divisible by 3. Also one divisible by 5. By 2. By 9. By 4. By 8. Write a number which is divisible by all of these at the same time.

12. The office expenses in a certain business were $17,920.50. By introducing a card and loose-ledger system of keeping books, these charges were reduced to $13,540. What per cent of the office charges was saved?

13. In a recent year the exchanges at the New York Clearing House amounted to $103,704,000,000, while those in all the clearing houses in the United States amounted to $157,741,000,000. What per cent of the total were those in New York?

14. A farmer has 30,000 bushels of wheat stored in elevators. Instead of selling the wheat, he borrows $7000 for three months at 6% on his elevator certificates and thus is able later to sell his wheat for 12 cents a bushel more than he otherwise would. How much does he gain? How does this illustrate the utility of banks?

15. A room is 16.5 ft. long and 10.5 ft. wide. Find the length of a pattern on a wall-paper border which shall fit each side and end of the room an exact number of times.

16. If a visual impression lasts $\frac{1}{8}$ sec. on the human retina, how many pictures must be run through a moving picture machine in 1 min. 12 sec. to make a continuous impression?

17. 32 dry quarts exceed 32 liquid quarts by how many cubic inches? By what per cent?

18. If Mt. Mitchell in North Carolina is 6703 ft. above the sea level, by what per cent is it higher than Mt. Washington? (See p. 386.)

19. If sound travels 1090 ft. per second, how many hours does it take a sound wave to travel 1200 miles (as at the time of the Krakatoa eruption)?

20. A steamer in crossing the Atlantic Ocean had 2891 miles to go. After she had gone 504, 492, 511, and 501 miles on four successive days, what per cent of her journey did she still have to make?

21. In 1875 there were 10,803,030 acres planted with cotton in the United States, and they produced 4,632,313 bales of cotton. In 1898 the acreage was 24,967,295, and the yield was 11,189,205

bales. Compute, to the nearest whole number, the average number of bales produced per 100 acres in each of these years.

22. A New York bank bought £75,000 in Alabama in bills of exchange for cotton at 4.81¾, and sent the same to its London correspondent. The London correspondent collected these bills for a commission of ½%. If the London correspondent remitted the proceeds to the New York bank at the rate of 4.86½, how much did the New York bank make by the transaction?

23. A box car 40′ × 8′ 6″ × 8′ 9″ will hold what per cent more than a box car 36′ × 8′ × 8′? How many more tons of freight will it carry, if 40 cu. ft. represent 1 ton?

24. If a given grade of coal makes an amount of coke equal to 56% of its weight, how many tons of coal must be used to make 1000 tons of coke?

25. By careful selection, the per cent of sugar in the sugar beet was increased from 11% in 1870 to 17% in 1908. What is the increase of sugar in 3000 tons of sugar beets worth to the farmer, taking the sugar as worth 3¢ a pound to him?

26. It is estimated that 6000 stars are visible to the naked eye, and 200,000,000 through the telescope. The latter number is how many times the former?

27. The rainfall on a flat roof 20 ft. long and 12 ft. wide is collected in a covered rectangular cistern 5 ft. long and 4 ft. wide. If after a certain rain the water in the cistern has risen 2 ft., how deep was the rainfall on the roof?

28. If the Simplon Tunnel is 12¼ miles long and it was dug at the rate of 15 ft. per day, how many years did it take to dig it, allowing 300 work days per year? (Use cancellation.)

29. 98.7° is the normal temperature of the human body and a temperature of 107.3° is fatal. What per cent is 107.3° above 98.7°?

30. If the United States paid $7,200,000 for Alaska, and Alaska contains 590,884 square miles, what was the cost per acre?

31. What is the cost of making a macadam road $\frac{1}{2}$ mile long and 9 ft. wide at 92¢ per square yard?

32. A merchant bought goods at a discount of 40% from the list price and sold them at a discount of 30% from the list price. What was his per cent of profit? What was his profit on goods whose list price was $2000?

33. The death rate in London in the year 1815 was 29 per 1000, and in 1908, 13.8 per 1000. The population of London in 1908 was 4,795,700. How many lives were saved in the year 1908 at the diminished rate? Can you give any reasons for the diminished death rate?

34. Sensation travels along the nerves of the human body at the rate of 120 ft. per second. What fraction of this rate is the velocity of a runner who passes over 100 yd. in 10 sec.? What fraction of a second will it take a sensation, as of a burn, to travel from the finger tip to the brain, if this distance is 5 ft.?

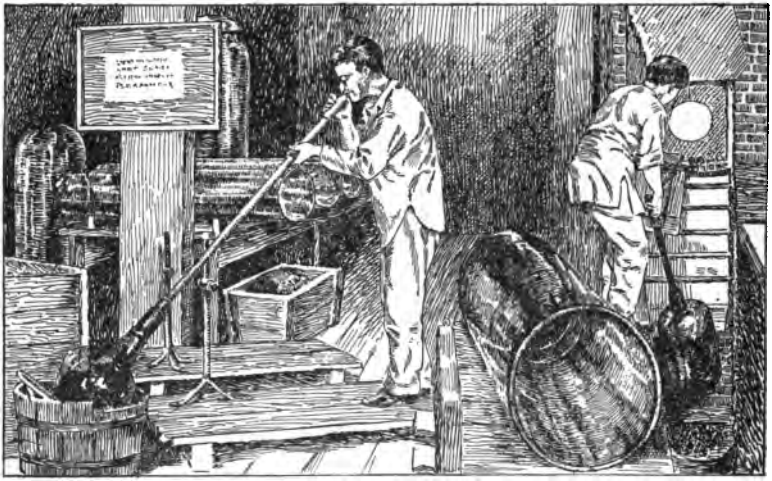
35. If wheat loses 18% on being ground into flour, how many bushels of wheat are needed to make a barrel of flour?

36. If a steam shovel handles 6 cu. yd. of dirt at one stroke, how many common shovelfuls is this, estimating a shovelful as 8 in. × 6 in. × 3 in.?

37. In a certain trade school 190 pupils were educated for a certain time at a cost of $13 each. As a result, their average wages per week were increased from $5.39 to $7.52. If each pupil works 50 weeks in the year, what was the increase in wages of all the pupils in a year? The gain for each pupil was what per cent per year on the cost of his education?

EXERCISE 110

Industry Review: Manufacture of Glass



Blowing Large Cylinders of Glass in a Glass Factory

1. In a recent year the value of the glass manufactured in the United States was $56,539,712. The cost of the materials used was $16,731,009, and of the wages paid $27,084,710. What was the net income of the factories?

2. If the capital employed amounted to $61,423,900, what per cent income did this capital pay?

3. If 355 factories in all were engaged in the manufacture of glass, what was the average capital invested in each factory? If 49,999 men were employed in the factories, what was the average number of employees in each factory? What was the average cost for materials, for wages, and average net income in each factory?

4. If in order to make window glass 7000 lb. of sand, 1260 lb. of lime, 2100 lb. of soda, and 180 lb. of alumina are melted together, what per cent of each is used?

5. $200,000 capital was invested in a glass factory and the weekly running expenses were $3050. Later the running expenses were reduced 7% by the use of a melting tank and a regenerative gas furnace. How much was saved in a year in this way, if the factory was in operation 40 weeks during the year? The sum saved was what per cent of the capital?

6. How many 10 in. × 12 in. panes of glass can be made from a sheet of glass 6 ft. long and 3 ft. 7 in. wide, if 15% is allowed for breakage and unused margins?

7. If the import duty on a certain kind of glass is 55%, what is the invoice value of glass on which the duty was $1655.50?

8. The following are the stock or standard sizes in which window glass is sold:

6″ × 7″
6″ × 8″
6″ × 9″
7″ × 9″
8″ × 10″
8″ × 12″
9″ × 12″
9″ × 13″
10″ × 12″
10″ × 14″
11″ × 14″
11″ × 15″
11″ × 17″
12″ × 18″
12″ × 20″
12″ × 24″
13½″ × 26″
13½″ × 28″
15″ × 30″
16″ × 30″
16″ × 34″
16″ × 36″
18″ × 34″
18″ × 36″
18″ × 38″
24″ × 26″
26″ × 27″

From which standard size would a piece of glass 10 in. × 17½ in. be cut with the least waste? What would be the waste?

Sug. In the column of widths, find the first number equal to or larger than 10″; in the column of lengths, find the same for 17½″. Use the standard size containing the latter of the two numbers found.

9. Answer the same for a piece of glass 10½ in. × 16 in.

10. Also for a piece 10½ in. × 22¾ in.

11. Name as many different uses of glass as you can. Five hundred years ago oiled paper was used instead of window glass. In what respects is window glass superior to oiled paper?

12. Why cannot a private individual of average means make glass to supply his own needs?

13. Obtain the numerical facts concerning some branch of manufacturing carried on in your neighborhood and make up a set of examples concerning them. These facts should cover:

- A. Cost
  - I. Of Material
    - 1. Interest on cost of buildings, and machinery (including insurance, taxes, repairs).
    - 2. Raw material (including fuel, freight, etc.).
    - 3. Interest on floating capital.
  - II. Of Labor
    - 1. Manufacture.
    - 2. Selling (agents, advertising, catalogues).
    - 3. Office and superintendence expenses.
- B. Returns.

CHAPTER X

RATIO; PROPORTION; PARTNERSHIP

Ratio

EXERCISE 111

1. In finding the cost of 6500 lb. of oats at 48 cents a bushel (32 lb.) we first write: no. bushels of oats = $\frac{6500}{32}$.

Why do we not then divide 6500 by 32 and obtain $203\frac{1}{8}$, and multiply 48¢ by this result?

Give the complete solution of this problem in the shortest way.

2. Of what advantage is it to indicate a division of one number by another and not to perform the division for the time being?

3. Point out two examples on p. 19 which illustrate the advantage of indicating a division instead of immediately performing it.

4. Make up an example similar to the two examples mentioned in Ex. 3.

133. A **ratio** is an indicated quotient of one number divided by another.

The **terms** of a ratio are the quantities whose quotient is indicated. The first of these (the indicated dividend) is called the **antecedent**; the second (the indicated divisor) is called the **consequent**.

Thus, in the ratio 12 to 8, 12 is the antecedent, 8 the consequent.

A ratio is usually indicated by the sign, :, between the numbers compared. This sign is probably an abbreviation of ÷, the sign of division.

Thus, the ratio of 12 to 8 is denoted by 12 : 8, which is read "12 is to 8," or "12 divided by 8." A ratio may also be indicated in the fractional form, as $\frac{12}{8}$.

The numbers which are compared in a ratio must be expressed in terms of a common unit. Thus, to express the ratio of 3 qt. to 2 bu., either the quarts must be expressed as bushels or the bushels as quarts.

If two quantities, as 5 in. and 2 bu., cannot be expressed in terms of the same unit, no ratio between them is possible.

134. A **direct ratio** of two numbers is the ratio indicated by dividing the first of the given numbers by the second. Thus, the direct ratio of 5 and 12 is $\frac{5}{12}$ or 5 : 12.

An **inverse ratio** of two numbers is the ratio indicated by dividing the second of the given numbers by the first. Thus, the inverse ratio of 5 and 12 is $\frac{12}{5}$ or 12 : 5.

The word *ratio* means *direct ratio,* unless the contrary is stated.

135. Utility of Ratios. One advantage in the use of ratios is that they often open the way to the use of cancellation and the saving of labor.

Ratios also have many other uses which will be realized later.

Ratios in which the first term is unity are of especial importance in making mental estimates and comparisons, even though the second term is only approximate.

See Ex. 2 under Art. 136.

136. Transformation of Ratios. In studying fractions, we have found that if both the numerator and the denominator of a fraction are multiplied or divided by the same number, the value of the fraction is not changed.

In like manner,

If both the antecedent and the consequent of a ratio are multiplied or divided by the same number, the ratio is not changed in value.

Ex. 1. The ratio $12:8 = \frac{12}{8} = \frac{3}{2} = 3:2$.

Ex. 2. A woman sold 7 chickens out of 29 which she owned. Express the ratio 7 : 29 as an approximately equal ratio having 1 for its first term.

Dividing both terms of 7 : 29 by 7, we obtain 1 : 4 approx. *Ans.*
Hence, the woman sold about $\frac{1}{4}$ of the chickens that she owned.

EXERCISE 112

1. What is the ratio of 1 qt. 1 pt. to 1 gal.?

SUG. $\frac{1\text{ qt. }1\text{ pt.}}{1\text{ gal.}} = \frac{3\text{ pt.}}{8\text{ pt.}} = \frac{3}{8} = 3:8.$

2. What is the ratio of 1 ft. 3 in. to 1 yd.? Of 5.2 to 7.28?

3. Make up and work a similar example concerning pecks, quarts, and bushels.

4. Concerning units of time.

5. What is the ratio of 25% to $37\frac{1}{2}\%$? To $87\frac{1}{2}\%$? To $62\frac{1}{2}\%$?

6. Make up and work a similar example involving $16\frac{2}{3}\%$.

7. What is the ratio of $37\frac{1}{2}\%$ to $\frac{9}{8}$?

8. Reduce the following to ratios having 1 for the first term. (Where an exact result cannot be obtained, find an approximate one, as in Ex. 2, of Art. 136.)

(*1*) 7 : 35
(*2*) 5 : 32
(*3*) $\frac{1}{2}$: 4
(*4*) .3 : 6
(*5*) .01 : 4
(*6*) 2.1 : 85
(*7*) 117 : 324
(*8*) 10.9 : 650
(*9*) .0002 : .0725

9. What is the approximate ratio of the area of New York to that of Texas? Of Rhode Island to Texas? (See p. 383.)

10. Make up and work two similar examples involving the area of your own state.

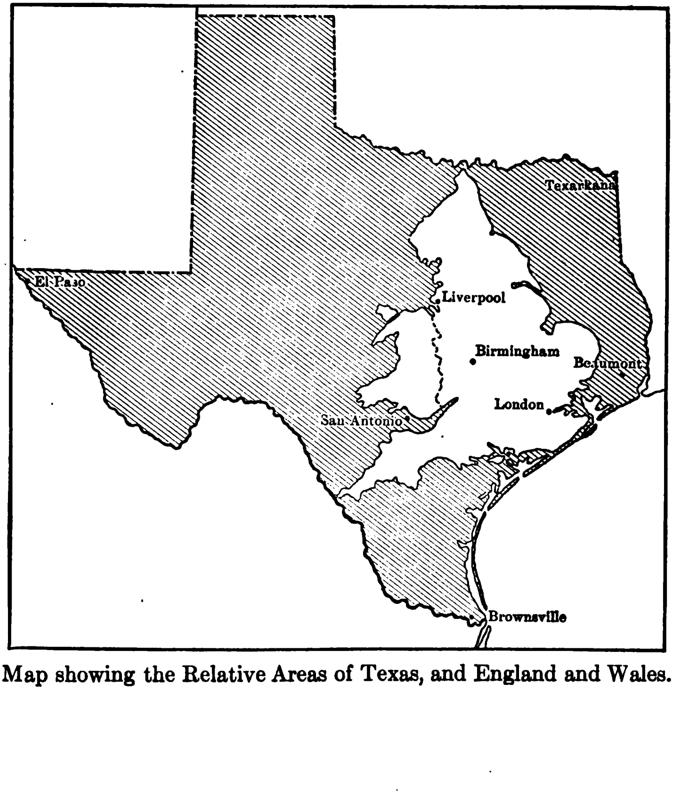
11. Also two involving the lengths of important rivers.

12. Find the number of years between the Declaration of Independence and the present year. Also between the discovery of America and the Declaration of Independence. Find the ratio of these two numbers.

13. A horse can pull 2 tons on a macadam road, 15 tons on a level iron track, and 70 tons on a canal. Find the ratio of each pair of these loads.

14. Find the ratio of 12 hats to 8 apples. Of 12 hats to 8 hats.

15. The area of England and Wales is 58,324 sq. mi. Find the ratio of this area to that of Texas.



Map showing the Relative Areas of Texas, and England and Wales.

16. The area of Great Britain and Ireland is 121,391 sq. mi. Find the ratio of this number to the area of the United States. Draw a diagram similar to that given with Ex. 15, showing, as accurately as you can, the meaning of your result.

SUG. Draw the outline of the United States and of Great Britain and Ireland on the same scale, as, for instance, 300 miles to the inch.

17. In a city of 320,000 inhabitants, 4760 people died in one year. How many people was this per 1000?

18. On squared paper show the ratios of 3 square units to 25 square units. Of 3 linear units to 25 linear units.

19. What is the ratio which is called specific gravity? (See Ex. 10, p. 112.) Give three examples to show what it is.

20. If the specific gravity of brick is 2.3, what is the weight of the bricks which fill a wagon body $1\frac{1}{2}' \times 4' \times 6'$?

21. Make up and work an example showing the advantage of knowing that the specific gravity of a given kind of stone is 2.5.

22. Make up and work three other examples showing the advantage of knowing other specific gravities.

23. How many examples in this Exercise can you work at sight?

24. Work at sight as many examples in Exercise 21 (p. 29) concerning telegraph and telephone rates as you can, or work similar examples dictated by the teacher.

To the Teacher. Art. 137 and Exercise 113 may be omitted, at the discretion of the teacher.

137. Nutritive Ratios. The food of animals, as of horses, cows, pigs, etc., should vary with the kind of life these animals are required to lead.

Thus, if a horse is doing work, as in drawing loads, it should have a food which contains a large amount of muscle-producing substance (called protein).

A knowledge of what are termed *nutritive ratios* is useful in determining the food ration most useful for an animal under given conditions.

The same principles apply to human diet, to a certain extent.

$$\textit{Nutritive ratio} = \frac{\textit{protein}}{\textit{carbohydrates} + 2.5 \times \textit{fat}},$$

only the digestible parts of food being considered. By carbohydrates are meant starch, sugar, etc.

Ex. If 100 lb. of clover hay contain 7.4 lb. of protein, 28.4 lb. of carbohydrates, and 1.9 lb. of oil or fat, what is the nutritive ratio of clover hay?

$$\text{Nutritive ratio of clover hay} = \frac{7.4}{28.4 + 2.5 \times 1.9} = \frac{7.4}{33.15} = 1 : 4.48$$

Animals doing muscular work or producing milk need a ration with a small nutritive ratio (called a narrow ration). Animals which are being fattened need a wide ration.

The digestible elements in different foods vary with circumstances. The following table gives the average number of pounds of each nutrient in 100 pounds of the food named.

	Protein	Carbohydrates	Fats
Corn	7.8	66.5	4.4
Corn stover	1.8	32.5	0.7
Corn silage	0.9	11.2	0.7
Cottonseed meal	37.4	17.0	12.1
Hay (alfalfa)	11.1	39.7	1.2
Hay (clover)	7.4	28.4	1.9
Hay (cow peas)	10.4	38.7	1.1
Hay (timothy)	2.8	43.6	1.4
Milk (skimmed)	2.9	3.5	0.3
Oats	9.2	47.4	4.2
Wheat bran	12.4	39.4	2.6

EXERCISE 113

1. Find the nutritive ratio of corn. Also of oats. Which of these is an appropriate food for a work horse? For pigs being fattened? For cows giving milk?

2. Find the nutritive ratio of timothy hay. Should timothy hay or clover hay predominate in a mixed hay for a work horse? For cattle being fattened?

3. Find the nutritive ratio of a combination of 100 lb. of timothy hay and 100 lb. of clover hay.

SUG. In the required nutritive ratio, the numerator will be the sum of the protein in clover hay and that in timothy hay, or 7.4 + 2.8, etc.

4. Make up and work a similar example by use of the above table.

Find the nutritive ratio

5. Of 10 lb. of corn and 20 lb. of timothy hay.

6. Of the following ration used for a work horse: 6 lb. corn, 8 lb. oats, 8 lb. timothy hay, 8 lb. clover hay.

7. Of the following ration for a milch cow: 36 lb. corn silage, 10 lb. corn stover, 5 lb. corn, 5 lb. wheat bran.

8. Of 150 lb. cow pea hay, 30 lb. corn silage, 4 lb. cottonseed meal, 3 lb. wheat bran, and 3 lb. corn meal.

9. Make up and work a similar example.

10. Practice the oral addition and subtraction of small two-figured numbers, as in Exercise 5 (p. 13).

PROPORTION

138. A **proportion** is an equality between ratios.

A proportion is indicated by placing the symbol =, or : :, between the two equal ratios.

Thus, 2 : 3 = 8 : 12, is a proportion.

It may be read in several ways, as "2 is to 3 as 8 is to 12"; or "2 over 3 equals 8 over 12"; or "the ratio of 2 to 3 equals the ratio of 8 to 12," etc.

A proportion contains four terms. The first and last terms are called the **extremes**; the second and third terms are called the **means.**

139. Useful Property of Proportions. In the proportion 2 : 5 = 4 : 10, the product of the means is 20, and the product of the extremes is also 20. Hence, the product of the means equals the product of the extremes.

Show that the same principle holds true in the following proportions:

(1) 3 : 6 = 4 : 8 *(3)* 5 : 3 = 20 : 12

(2) $\frac{1}{2} : 4 = 1 : 8$ *(4)* $\frac{2}{3} : \frac{1}{5} = \frac{4}{3} : \frac{2}{5}$

Hence, we obtain the following principle:

In any proportion, the product of the extremes equals the product of the means.

This principle can be proved in a general way, but this proof lies beyond the scope of this book.

This principle enables us to solve a proportion, that is, to determine the value of an unknown term in the proportion. It is often useful to denote an unknown number by x.

140. Solution of a Direct Proportion.

Ex. 1. Find the value of x in the proportion $12 : 8 = 15 : x$.

By Art. 139, $12x = 8 \times 15$

or, $12x = 120$

Dividing both of the two equal numbers, $12x$ and 120, by 12,

$x = 10$ *Ans.*

Ex. 2. If 12 boys receive \$15 wages for a day's work, what will 8 boys receive?

We let x = no. of dollars 8 boys will receive.

$$12 \text{ boys} : 8 \text{ boys} = \$15 : \$x$$
$$\text{or } 12 : 8 = 15 : x$$

Hence, by Art. 139, $12x = 8 \times 15$

Dividing by 12, $x = 10$, or wages of 8 boys = \$10 *Ans.*

141. Solution of an Inverse Proportion.

Ex. If it takes 30,000 4-inch shingles to build a certain house, how many 5-inch shingles would it require?

Let x = no. of shingles required.
Since the wider the shingles, the smaller the number, we have the inverse ratio of width = direct ratio of number.

$$5 : 4 = 30{,}000 : x$$
$$5x = 120{,}000$$
$$x = 24{,}000 \text{ } Ans.$$

This example might also be worked by the method of analysis (p. 19), Try to work it by analysis. You will find the solution much longer and more difficult than the solution by proportion.

The proportion used in the above example is an *inverse proportion.*

142. To Solve a Problem by Use of a Proportion,

Denote the required quantity by x, and write x as the last term of the proportion;

Use the quantity with which x is compared as the third term;

Find out, by study of the problem, whether the answer should be greater or less than the third term;

Write the first and second terms of the proportion accordingly;

Take the product of the extremes = the product of the means and find the value of x.

EXERCISE 114

Find the missing term, x, in each of the following proportions:

1. $3 : 8 = 6 : x$
2. $12 : 5 = x : 40$
3. $5 \text{ ft.} : 30 \text{ ft.} = \$8 : \$x$
4. $16 \text{ T.} : 24 \text{ T.} = \$x : \$15$
5. $3\frac{1}{2} : 4\frac{2}{3} = 9\frac{3}{7} : x$
6. $41\frac{1}{3} : x = 196\frac{1}{3} : 232\frac{1}{2}$
7. $x : 9.75 = \$13.25 : \$10.40$
8. $2.76 : 3.45 = 2.28 : x$

9. If a factory produces 3000 pairs of shoes in 12 days, how many will it produce in 18 days?

10. If an aeroplane goes 95.5 miles in 87 minutes, how far will it go in 210 minutes, at the same rate?

11. If a certain steamer burns 47 tons of coal in 3 days, how long will 85 tons last the steamer?

12. If the ratio of the length of the forefinger to the height of the human body is 3 : 22, and the Statue of Liberty in New York Harbor is 151 ft. high, how long should the forefinger on the statue be?

13. If $3\frac{2}{3}$ bushels of grain are used to plant $13\frac{3}{4}$ acres, how many bushels will be required to plant 100 acres?

14. Make up and work a similar example concerning numbers of barrels of apples yielded by numbers of trees.

15. If a railroad track has a rise of 1 inch in 100 feet, what will be the rise in 1 mile at the same rate?

16. If I pay $27.30 tax on property assessed at $3000, at what figure must my neighbor's property be assessed, if he pays a tax of $22.75?

17. Make up and work a similar example.

18. On a certain map, 7 inches represent 119 miles. On the same map, 9 inches represent how many miles?

19. Denoting the distance from the earth to the sun by $\frac{1}{2}$ inch, on this scale draw a diagram showing the distance of each planet from the sun. (Use table of distances, p. 385.)

20. If 432 lb. of water can be converted into steam by burning 56 lb. of coke, how many pounds of coke are necessary to convert 729 lb. of water into steam?

21. If one man raising wheat by aid of machinery can now do in 3 hours what formerly required 61 hours of hand labor, and 20 men were formerly required to farm a square mile of wheat land, how many men (approximately) are now required to raise wheat on a square mile?

22. The back and front wheel of a carriage have diameters of 4 ft. and $3\frac{1}{2}$ ft. respectively. When the back wheel has made 259 revolutions, how many revolutions has the front wheel made? (Observe that an inverse proportion is here involved.)

23. If 27,000 3-inch shingles are required in building a house, how many 4-inch shingles would be required?

24. If a train going at the rate of 30 miles per hour takes 2 hr. 12 min. to travel between two cities, how long will it take a train going 40 miles an hour to travel the same distance?

Also solve this example by analysis. Which of the two solutions is simpler?

25. If 14 men can pave a certain road in 36 days, how long will it take 21 men to pave it?

26. If the hay in a certain field will last 9 horses 6 months, how long will it last 12 horses?

27. If in testing seed corn 16 grains of corn out of 20 sprout, how many out of 1020 should sprout?

28. If 15 yd. of carpet 36 in. wide are required for a certain room, how many yards 27 in. wide would be required?

29. If 7 barrels of flour will last a hotel 5 weeks, how long will 15 barrels last?

30. If a loaf of bread weighs 6 oz. when flour is $6 a barrel, what ought a loaf, sold at the same price, to weigh when flour is $8 a barrel?

31. If a loaf of bread of given weight costs 5¢ when flour is $6 a barrel, what ought the loaf to cost when flour is $7.20 a barrel?

32. If sediment is deposited on an ocean floor at the rate of 3 inches in 100 years, how long would it take to deposit a layer 26 ft. 6 in. thick?

33. If a town is provisioned to last 2000 men during a 6-months siege, and a reinforcement of 1000 men arrives, how long a siege can the town then endure?

34. Make up and work two examples in direct proportion.

35. Also two illustrating inverse proportion.

36. How many examples in this Exercise can you work at sight?

37. Work again Exercise 9 (p. 15) or similar examples.

PARTITIVE PROPORTION

143. Partitive Proportion is the process of dividing a given number into parts which shall be proportional to a given series of numbers.

Ex. Three men working in a mine agree to divide the profits in the proportion of 2, 3, and 4. They make $2700. What is the share of each?

Since $2 + 3 + 4 = 9$, we may regard the profits as divided into 9 equal shares (units), of which the miners will receive 2 shares, 3 shares, and 4 shares respectively.

Let $x = \frac{1}{9}$ of profits.
Then $2x$, $3x$, $4x$ = respective shares of the miners.

$$2x + 3x + 4x = \$2700$$

Hence, $$9x = \$2700$$

And $$x = \tfrac{1}{9} \text{ of } \$2700, \text{ or } x = \$300$$

$$2x = \$600,\ 3x = \$900,\ 4x = \$1200 \ \textit{Ans.}$$

Problems in partitive proportion may be solved by proportion (see p. 270), but they may usually be solved to better advantage by the method used in solving the preceding example.

EXERCISE 115

1. Divide 20,000 into four parts proportional to 5, 7, 8, 12.

2. Divide 3864 into parts proportional to 2, 3, 4, 5.

3. Three boys whose ages are 12, 15, and 16, farm a garden together and make $98.60 during the season. The 15-year-old boy receives twice as much profits as the 12-year-old boy. The 16-year-old boy, who furnishes the garden and tools, receives 3 times as much as the youngest boy. How much of the profits does each boy receive?

4. A man who had four children bequeathed a double share to a daughter who had taken care of him in his old age, and also to a son who had managed his property for him. If his property amounted to $60,600, how much would each child receive?

5. If 100 pounds of a given fertilizer contain 2.7 pounds of nitrate of soda, 3.1 pounds of ground bone, and 4.2 pounds of potash, how many pounds of each does a ton of the fertilizer contain? 3650 lb. of fertilizer?

6. The receipts per mile for one year on a certain railroad were for passengers $1511 and for freight $1575. If the total receipts on the entire road for the year were $351,804, what were the total receipts for passengers? For freight?

7. The yield from a ton of coal of a certain kind is:

	coal-gas	9500 cu. ft.
	coke	1500 lb.
By-products	tar	70 lb.
	ammonia	80 lb.

How much coal must be used to make by-products which weigh 2310 lb.?

Find also how much gas, coke, tar, and ammonia will be produced from the coal thus used.

Sug. The weight of the by-products from 1 ton of coal = 150 lb.

Hence, to obtain 2310 lb. of by-products, the number of tons (x) of coal required is obtained by solving the following proportion:

$$150 : 2310 = 1 : x, \text{ etc.}$$

8. Make up and work two examples in partitive proportion.

Partnership

144. Partnership. When two or more persons think that they can carry on a business to better advantage together than singly, they may unite and form a partnership.

A partnership differs, in general, from a corporation (see Art. 75) in that each partner is liable for all the obligations of a firm, while usually a stockholder in a corporation is liable only for the amount of his stock. Also each partner usually helps manage the business directly, while a board of directors elected by the stockholders manages the affairs of a corporation. In law, a corporation is regarded as a unit or individual (person), while a partnership is regarded as a number of individuals.

145. The **Capital** is the money invested by the different partners in a business.

In *simple partnership,* all the partners invest their capital for the same length of time.

Rights in a patent, mechanical skill, or business experience are often regarded as capital and receive a share of the profits of a partnership.

In *compound partnership,* the partners invest their capital for different periods of time.

Ex. 1. Three men, A, B, and C, are in business together, and gain \$3600. A's capital is \$2000; B's is \$4000; and C's is \$6000. What is each one's share of the profits?

$$\text{Entire capital} = \$2000 + \$4000 + \$6000 = \$12000.$$

$$\begin{array}{llll} & \text{A's share of the capital} & = \frac{2000}{12000} & = \frac{1}{6} \\ & \text{B's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{4000}{12000} & = \frac{1}{3} \\ & \text{C's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{6000}{12000} & = \frac{1}{2} \\ \text{Hence,} & \text{A's share of the gain} & = \frac{1}{6} \text{ of } \$3600 & = \$600 \\ & \text{B's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{1}{3} \text{ of } \$3600 & = \$1200 \\ & \text{C's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{1}{2} \text{ of } \$3600 & = \$1800 \end{array}$$

Ex. 2. A, B, and C form a partnership. A puts in \$500 for 8 months, B \$600 for 9 months, and C \$800 for 12 months. They gain \$3040. What share of this belongs to each partner?

$$\begin{array}{lllr} \$500 \text{ invested for} & 8 \text{ mo. is the same as} & \$4000 & \text{for 1 mo.} \\ \$600\ \ \ \text{“}\ \ \ \ \ \ \text{“} & 9\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \text{“}\ \ \text{“} & 5400 & \text{“}\ \ 1\ \text{“} \\ \$800\ \ \ \text{“}\ \ \ \ \ \ \text{“} & 12\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \text{“}\ \ \text{“} & \underline{9600} & \underline{\text{“}\ \ 1\ \text{“}} \\ & \text{Entire capital} = & \$19000 & \text{for 1 mo.} \end{array}$$

$$\begin{array}{lllll} \text{Hence,} & \text{A's share of the gain} & = \frac{4000}{19000} \text{ or } \frac{20}{95} & \text{of } \$3040 & = \$640 \\ & \text{B's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{5400}{19000} \text{ or } \frac{27}{95} & \text{“}\ \ \ \ \ \ \text{“} & = \$864 \\ & \text{C's}\ \text{“}\ \ \text{“}\ \ \text{“}\ \ \ \ \text{“} & = \frac{9600}{19000} \text{ or } \frac{48}{95} & \text{“}\ \ \ \ \ \ \text{“} & = \$1536 \end{array}$$

EXERCISE 116

1. Three men rent a pasture, into which A puts 7 horses, B puts 9, and C 13. Of the rent, which was \$17.40, how much ought each man to pay?

2. In a partnership A places $3200, B $4100, and C $1700. How should a gain of $4500 be divided?

3. Three men gain $500 after investing $560, $640, and $800 respectively. How do they share the profit?

4. By a will, a man left to his widow $12,000, to a son $9000, and to a daughter $4000. But upon investigation the estate produced only $20,000. How should it be divided equitably?

5. A firm lost in a year $3300. A's stock was $3200, B's was $7100, and C's was $6200. How is the loss to be distributed?

6. A, B, and C went into business with a capital of $12,000. Of the gain in one year, A's share was $1250, B's $1000, and C's $750. What was each man's capital?

7. Three persons enter partnership. A puts into it $1600 for 3 months; B, $800 for 5 months; and C, $900 for 3 months. How should they justly share the profits of $575? The losses of $1035?

8. A pasture is rented by 3 persons for $760. A puts in 7 cows for 5 months; B, 8 cows for 3 months; and C, 9 cows for 4 months. What rent should each pay?

9. Three laborers contracted to dig a trench for $49.50. The first worked 8 days of 7 hours each; the second 10 days of 8 hours each; and the third 14 days of 6 hours each. What should each receive?

10. A entered business with $5000, and after 3 months took in B with $4000. After 2 months more, C entered the firm with $12,000. At the end of the year they had gained $8100. How should it be divided equitably?

11. State some of the relative advantages and disadvantages in conducting a business as a partnership and as a corporation.

12. How many examples in this Exercise can you work at sight?

13. How many examples in Exercise 11 (p. 17) can you now work at sight?

EXERCISE 117

REVIEW

1. If the number of Indians in the United States is 270,000, and the population of New York City is 4,000,000, find the ratio of the two numbers.

2. Find the ratio of atmospheric pressure (15 lb. per square inch) to the pressure on the foundation of a large office building (200 tons to the square foot).

3. If the heart of a frog beats 10 times per minute, that of a man 72 times, and that of a bird 120 times, find the ratio of each pair of these numbers.

4. Taking the weight of a man as 150 lb., of a horse as 1000 lb., of an elephant as 2 tons, and of a whale as 60 tons, find the ratio of each pair of these numbers.

5. In an aeroplane, a 45-horse-power gas engine weighing only 44 lb. has been used. Find the ratio of 44 lb. to the weight of the 45 horses, each weighing 1200 lb., which would be needed to do an amount of work equivalent to that of the engine.

6. If the English language in the year 1800 was spoken by 21,000,000 people and in the year 1900 by 130,000,000, and if the world's population in 1800 was 1,400,000,000 people and in 1900 1,500,000,000, what fraction of the world's population spoke English on each of the dates named?

7. If each heartbeat sends 6 oz. of blood through the heart, what fraction of a pint is this? If the heart beats 70 times per minute, how many gallons of blood pass through the heart in a day?

8. If an engine consumes 420 lb. of coal in $1\frac{1}{2}$ hours, how much will it consume in 10 hours?

9. If 7 men can dig a ditch in 6 hours, how long will it take 14 men to dig it? 21 men? 10 men?

10. In Exs. 8 and 9, which proportion is inverse?

11. If Mt. McKinley is 20,464 ft. above the sea level, by what per cent is it higher than Pike's Peak?

12. A balloon has been observed to ascend 97,700 ft. How many miles is this?

13. State some of the advantages of being able to borrow money by paying interest on the sum borrowed. State some possible disadvantages. Give illustrations. How are the borrowing and handling of money facilitated by percentage?

14. If a boy grows $3\frac{1}{4}$ in. in 1 yr. 6 mo., at the same rate how much will he grow in 2 yr. 9 mo.?

15. Astronomers sometimes divide the day decimally. What hour of the day would April 18.5 be, if the day were made to begin at midnight?

16. If a watch gains 1 min. 22 sec. in 2 months and 3 days, what is the average gain per day?

17. If milk is 3.75 parts fat, 8.5 parts other solids, 87.75 parts water, find the ratio of each pair of these constituents.

18. Make up and solve an example in partnership. (Let the example state why the different partners receive different shares in the profits.)

19. Before the use of principles of modern sanitation in armies, the deaths from wounds were to the deaths from disease as 1 : 4. In the Russo-Japanese War, the Japanese employed sanitary methods and the above ratio was changed to 2 : 1. If in the

Japanese armies 59,000 died from wounds, how many lives were saved by the sanitary methods employed?

SUG. By former sanitary methods, the number of deaths from disease in the Japanese army would have been the number (x) obtained by solving the following proportion:

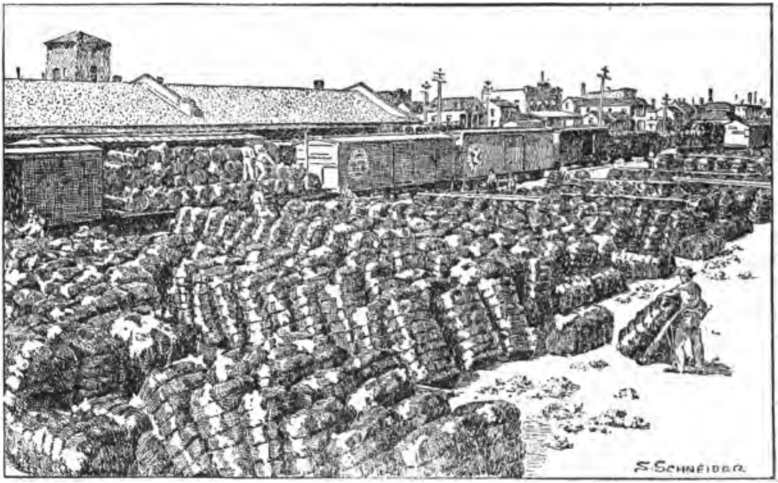
$$1 : 4 = 59{,}000 : x.$$

The actual number of deaths from disease (y) in the Japanese army is obtained by solving

$$2 : 1 = 59{,}000 : y.$$

Find the difference between x and y.

20. Work again Exercise 20 (p. 28) or a similar Exercise concerning small fractions.



EXERCISE 118

INDUSTRY REVIEW: COTTON

1. 215 acres planted with cotton yielded 354,750 lb. of seed cotton. How many pounds was this per acre?

2. By ginning, seed cotton is separated into the seed proper and the fiber (or lint). One third of the seed cotton of Ex. 1 was lint which sold for 14½¢ a pound, and the rest was the seed which sold for $26 per ton. What were the proceeds per acre?

3. It cost $12.25 per acre to cultivate, fertilize, pick, gin, bale, and market the cotton as raised in Ex. 1. What was the cost on the entire 215 acres?

4. What was the net income from the cotton raised on the 215 acres?

5. If by using a machine to plant cotton, and using two mules to a plow instead of one, the cost of planting and cultivating had been diminished $1.76 per acre, what would have been the gain from this source? (How does the use of two mules together as a team save expense?)

6. If by sowing cow peas on the land in the fall and plowing them under in the spring, $2.27 per acre had been saved in the cost of fertilizer, what would have been the gain from this source?

7. If by the use of selected seed the amount of seed cotton produced per acre had been increased by 470 lb., what would have been the increase in receipts from the crop?

8. By keeping a system of accounts, the owner of the plantation was enabled to prevent the loss of $500 by waste, and gained $700 by more intelligent planting and cultivation. What was the gain per acre?

9. What was the total gain per acre by the methods of Exs. 5–8? The total gain on the 215 acres?

10. A modern gin and compress require only half as many men to operate them, and handle ten times as much cotton. The expense in the former method of ginning and compressing cotton is how many times that in the modern method?

11. A box car which holds 35 soft bales of cotton will hold 72 compressed bales. How many more compressed bales than soft bales will be carried by a train of 75 box cars?

12. Would the proceeds from the seed, as obtained in Ex. 2, pay the cost of picking, if the pickers were paid at the rate of 90¢ per 100 pounds of seed cotton?

13. If the proceeds from the seed were all expended for fertilizer, what would be the cost of the fertilizer used per acre for one year?

14. If the cost of picking seed cotton is 90¢ per 100 pounds, what is the cost per pound of picking seed cotton?

15. If cow pea stubble, when plowed under, increases the cotton crop the next year by 20%, how much will it increase the crop of lint cotton on an acre which has been producing 350 lb. of lint cotton? What is the increase worth at 14½¢ a pound?

16. A farmer in Texas raised 38,570 lb. of lint cotton, which he sold for 14½¢ a pound. He estimated that one half his crop had been destroyed by the boll weevil. If by use of selected seed and early cultivation, and by scientific methods for the destruction of the boll weevil, he diminished the loss by one half, how much would his receipts from lint cotton be increased in 1 year?

17. If seed cotton is ⅓ lint, and 13,500,000 bales of lint cotton of 500 lb. each were produced in the United States in a given year, how many pounds of seed cotton were produced? At 90¢ per 100 lb., what was the cost of picking the seed cotton?

If a machine to pick cotton could be invented which would save 35% of the expense, what would be the saving on each bale of lint cotton?

18. In a recent year the yield of lint cotton per acre of a certain state was 169 lb., while at the same time the yield on the

demonstration farm was 428 lb. per acre. What would have been the gain, if the yield from 2,500,000 acres of cotton in the state had been brought up to the standard of the demonstration farm?

19. If the cotton crop is on the average $\frac{1}{2}$ bale of lint cotton per acre, how many acres are now used in cultivating the cotton crop of the United States (see Ex. 17, for size of crop)? If, by the use of selected seed and better methods of cultivation, the crop of lint cotton could be increased 150 lb. per acre, by how many bales would the crop of the country be increased?

20. In one year the crop of cotton in the United States was 12,500,000 bales, and this was $\frac{3}{4}$ of the crop of the entire world. What was the crop of the entire world?

21. The cotton crop of the United States given in Ex. 20 was worth how much at 12¢ a pound? If 70% of this cotton crop was sold to other countries, how much per day did other countries pay the United States for cotton during a year?

22. If, by the use of levees and proper drainage, 20,000,000 acres of unused land along the Mississippi River could be cultivated and made to grow $1\frac{1}{2}$ bales of lint cotton per acre per year, the cotton thus raised would be how many times as great as the present average crop in the United States?

23. Find out as many uses as you can of cotton lint, and also of cotton seed. Point out some ways in which a knowledge of arithmetic aids in the cultivation of cotton and extends the uses of cotton.

CHAPTER XI

INVOLUTION AND EVOLUTION

INVOLUTION

EXERCISE 119

1. In $3 \times 2 = 6$, 3 and 2 are called the factors of 6. Give a definition of *factors*.

2. Give the factors of 10. Of 15. Of 9.

3. State how many signs are used in $2 \times 2 \times 2 \times 2$. In 2^4.

4. 2^4 is a short way of writing $2 \times 2 \times 2 \times 2$. (The 4 indicates the number of times 2 is used as a factor.) Write $5 \times 5 \times 5$ in a short way. Also $5 \times 5 \times 5 \times 5 \times 5 \times 5$. Also 5×5.

146. The **square** of a number is the product obtained by using the given number as a factor twice.

Thus, 16 is the square of 4.

The **cube** of a number is the product obtained by using the given number as a factor three times.

Thus, 64 is the cube of 4.

147. A **power** of a number is the product obtained by using the given number as a factor a certain number of times.

Thus, the fifth power of 3 is $3 \times 3 \times 3 \times 3 \times 3$, or 243.

What is the special name for a second power? For a third power?

148. An **exponent** is a small figure placed above and to the right of a number to show how many times the number is taken as a factor.

Thus, 2^4 means $2 \times 2 \times 2 \times 2$.

149. **Involution** is the process of computing any required power of a given number.

The *base* is the number which is used repeatedly as a factor in a power.

Thus, in 2^4, the base is 2.

150. **Uses of Involution.** Involution is often useful as a means of shortening multiplications.

A knowledge of involution is also useful as a means of visualizing large objects or numbers.

A knowledge of the processes of involution also leads to the discovery of methods of performing evolution. (See Arts. 152, 158.)

151. **Methods of Involution.** The powers of numbers may be obtained (1) by actual multiplication, or (2) by the use of tables, or (3) by the use of logarithms (see Art. 163).

EXERCISE 120

Find the value of

1. 26^2
2. 28^3
3. 512^2
4. 205^3
5. 1.75^2
6. 23.1^3
7. $.038^3$
8. $.66^3$
9. $(1\frac{1}{3})^3$
10. $(3\frac{1}{2})^2$
11. $(4\frac{1}{4})^3$
12. $(7\frac{1}{7})^3$
13. 15^4
14. 23^4
15. $(3.1\frac{2}{3})^3$
16. $(1.02)^2$.
17. $3^3 \times 4^2$
18. $(\frac{1}{2})^2 + 5^3$
19. $2^3 \times 3^4 \div 6^2$
20. $(2.5)^2 \times (3.5)^2$
21. $15^3 - 45^2$
22. $7.2^2 \times .5^3$

23. Find the area of a square each of whose sides is

(*1*) 9 in.
(*2*) 4 yd.
(*3*) 17 in.
(*4*) 2 ft. 3 in.
(*5*) 1 yd. 2 ft.
(*6*) .05 ft.
(*7*) 5 mi.
(*8*) .12 mi.

24. Show the meaning of 3^2 on squared paper. Also of 4^2. Of 6^2.

25. Find the volume of a cube each of whose edges is

(*1*) 4 in.
(*2*) $1' \, 2''$
(*3*) 2 yd.
(*4*) 23 in.

26. What number raised to the fourth power equals 16? 256? 81?

27. What number raised to the fifth power equals 32? 243? 1024?

28. Compute $(1.04)^5$. Do you know of any use to which your result may be put, in connection with the subject of interest? Do you know of any tables from which you might have obtained your result, thus saving the labor of multiplying to obtain the power?

29. Is $(\frac{2}{3})^2$ larger or smaller than $\frac{2}{3}$? Why?

30. Is the square of any proper fraction larger or smaller than the fraction itself?

31. $7{,}000{,}000 = 7 \times 1{,}000{,}000 = 7 \times 10^6$.

In like manner, express the following numbers in a short way:

(*1*) 186,000 miles per second (the velocity of light in round numbers).

(*2*) 93,000,000 miles (the distance of the earth from the sun in round numbers).

(*3*) 18,000,000,000,000 miles (approximate distance of nearest fixed star).

32. Show that .00000007 may be expressed as $\frac{7}{10^8}$.

33. Calculate the following and write the results in a convenient place (as in your notebook or on the fly leaf of this book) and commit them to memory:

(*1*) *Square* of each number from 1 to 25.
(*2*) *Cube* of each number from 1 to 9.
(*3*) *Fourth* power of each number from 1 to 6.
(*4*) *Fifth* power of each from 1 to 5.
(*5*) *Sixth* power of each from 1 to 4.
(*6*) *Seventh, eighth, ninth,* and *tenth* powers of 1 and 2.

34. By use of your table of squares, tell the nearest square number to each of the following: 290, 123, 319, 217, 627, 363, 194.

35. By use of this table, visualize 200 square miles of land.

SUG. The square number nearest 200 is 196. 196 is the square of 14. Hence, to visualize 200 sq. mi., think of a square each side of which is slightly larger than 14 miles.

36. By use of this table, visualize each of the following: 401 square miles of land; 573 sheep; 223 apple trees; 440 pupils; 623 hills of corn.

37. From a rectangular piece of glass 19.4 in. long and 14.8 in. wide, the largest possible square piece is cut. What is the area of the remaining rectangle and also of the square?

38. If 28 tennis players are to engage in a progressive tournament of singles, how many byes will there be on the first round?

SUG. In a tournament, a player who draws a blank for the first round and hence is regarded as a winner in the first round without playing any competitor, is said to have a *bye.*

The number of players and byes together on the first round should equal that power of 2 which is next greater than 28, which in this case is 32. Hence, 4 byes will be necessary, since $32 - 28 = 4$.

39. How many byes would be necessary for a tennis tournament engaged in by 27 players? By 60? By 33? By 160?

40. In a certain city 118 baseball nines composed of schoolboys playing in public playgrounds engaged in a tournament. How many byes would there be on the first round?

41. How many examples in Exercise 19 (p. 26) can you now work at sight?

EXERCISE 121

ORAL

1. State rapidly the squares of all the numbers up to 20.

2. What is the square of 18? 14? 17? 21? 19? 15?

3. What is the square of $\frac{1}{2}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{2}{7}$? $\frac{14}{23}$? $\frac{17}{15}$?

4. What is the square of .2? .3? .8? .13? 1.6?

5. What is the square of 1.8? 2.1? 2.5? 30? 70? 600?

6. What is the cube of 9? 8? 3? 7? 6? 5? 11?

7. What is the cube of $\frac{2}{5}$? $\frac{5}{8}$? $\frac{4}{9}$? $\frac{8}{11}$? $2\frac{1}{2}$? $3\frac{1}{2}$?

8. What is the cube of .2? .4? 20? 30? 1.2?

9. Tell the value of 8^3, 12^3, 2^4, 3^4, 6^3, 3^5, 2^7, 4^3, 4^4, 5^4, 2^8, $(.07)^3$.

10. Tell the value of 1^2, 1^3, 1^5, $.1^2$, $.1^3$, $.1^5$, $(.01)^2$, $(.01)^3$, $(.01)^5$.

11. How many examples in Exercise 25 (p. 39) can you work at sight?

152. Squaring a Number by Parts. If the second figure of a given two-figured number is 5 or a smaller number, it is often an advantage to separate the tens from the units before squaring.

Thus, $43^2 = (40 + 3)^2$

Whence we have

$$\begin{array}{r}
40 + 3 \\
40 + 3 \\
\hline
40 \times 3 + 3^2 \\
40^2 + 40 \times 3 \phantom{{}+3^2} \\
\hline
40^2 + 2 \times 40 \times 3 + 3^2 = 1600 + 240 + 9 = 1849 \textit{ Ans.}
\end{array}$$

Or, in general, the square of a two-figured number equals (*the square of the tens*) + (*twice the tens* × *the units*) + (*the square of the units*).

Denoting the tens by t, and the units by u,

$$(t+u)^2 = t^2 + 2 \times t \times u + u^2$$

EXERCISE 122

Find the square of each of the following numbers by the method of Art. 152:

1. 21
2. 52
3. 61
4. 45
5. 77
6. 91
7. 82
8. 92
9. On squared paper mark off a square each side of which contains 23 spaces. Separate this square into smaller squares and rectangles so as to show that $23^2 = 20^2 + 2 \times (20 \times 3) + 3^2$.
10. Treat 32^2 in the same way. Also 19^2.

EVOLUTION

153. A **root** of a number is a number which, used as a factor a certain number of times, will produce the given number.

Thus, 4 is the third, or *cube*, root of 64, since $4 \times 4 \times 4 = 64$.

Let the pupil make a definition of *square root, fourth root, sixth root.*

Evolution is the process of determining any required root of a given number.

154. The **Symbol of Evolution** is $\sqrt{}$. This sign is a form of the letter *r* (or the first letter of the word *root*).

Square root is used so much oftener than other roots that the sign $\sqrt{}$ is used to indicate square root. Other roots are indicated by writing a small figure, called the *index*, corresponding to the number of the root, above the first part of the sign.

Thus, $\sqrt[3]{64}$ means the third or cube root of 64, and 3 is the index of the root.

155. Uses of Evolution. Evolution is useful in enabling us to express numbers in a brief form, and also in visualizing numbers.

Later we shall find that evolution has other important uses, as in enabling us to determine unknown lines or distances from certain known lines or distances. (See Art. 170.)

156. The **Factorial Method of Extracting a Root** of a number which is an exact power is to

Separate the number into prime factors;

Arrange these factors as a number of like groups, the number of groups being equal to the index of the required root;

The product of the factors in one of these groups is the required root.

Ex. Extract the square root of 324.

Separating into prime factors, $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Arranging in two like groups, $= (2 \times 3 \times 3) \times (2 \times 3 \times 3)$

$= (2 \times 3 \times 3)^2$

Hence, $\sqrt{324} = 2 \times 3 \times 3 = 18$ *Ans.*

EXERCISE 123

By the factorial method, obtain the square root of each of the following:

1. 1764
2. 44100
3. 5625
4. $12 \times 54 \times 98$
5. $12 \times 3 \times 81$
6. 3969
7. 655.36
8. 36864
9. 63×112
10. Find the value of $\sqrt{48 \times 16 \times 18 \times 24}$ by the factorial method.

11. Take the results of Ex. 33, Exercise 120, arrange them as a table of roots, and commit this table to memory. Thus, $\sqrt{289} = 17$, $\sqrt[3]{512} = 8$, etc.

12. Read each of the following and give the value of each:

(1) $\sqrt{36}$
(2) $\sqrt[3]{8}$
(3) $\sqrt[4]{16}$
(4) $\sqrt[3]{27}$
(5) $\sqrt{81}$
(6) $\sqrt[5]{32}$
(7) $\sqrt{169}$
(8) $\sqrt{256}$
(9) $\sqrt[3]{64}$
(10) $\sqrt[5]{243}$
(11) $\sqrt[4]{256}$
(12) $\sqrt[3]{729}$
(13) $2 + \sqrt{9}$
(14) $\sqrt{9} + \sqrt{16}$
(15) $\sqrt{9 + 16}$

13. Make up and work three examples similar to those in Ex. 12.

14. How many examples in Exercise 26 (p. 41) can you now work at sight?

Square Root

EXERCISE 124

1. What is the smallest number containing two figures? What is the largest number containing two figures?

2. How many figures are there in the square of 10? Of 99?

3. What is the smallest number of figures that can occur in the square of a two-figured number? What is the largest number of figures?

4. What is the smallest number containing three figures? The largest number? How many figures are there in the square of 100? Of 999?

5. What is the smallest number of figures that can occur in the square of any three-figured number? What is the largest number of figures?

6. Similarly, find the smallest and largest number of figures that can occur in the square of a four-figured number. Of a five-figured number.

7. Tell how many figures there will be in the square of

(*1*) 15 (*2*) 8 (*3*) 486 (*4*) 2015 (*5*) 208 (*6*) 90145

8. Tell how many figures there will be in the square root of (*1*) 64 (*2*) 121 (*3*) 3969 (*4*) 729 (*5*) 46656 (*6*) 44100

157. Number of Figures in a Square Root. From Exercise 124 it follows that if a number is divided into periods of two figures each, beginning at the decimal point,

The number of figures in the square root of the number = the number of periods of two figures each in the number.

Also, the square root of the largest square number represented in the left-hand period gives the first figure of the root.

158. The Method of Extracting Square Root is obtained by using the relation (see Art. 152) between the parts of a number and the parts of the square of a number.

Ex. 1. Extract the square root of 1156.

$$
\begin{array}{rr|l}
 & \multicolumn{2}{l}{\overparen{11}\overparen{56})\underline{30+4}} \\
\text{Tens squared} = 30^2 = & \multicolumn{2}{l}{\;\;900} \\
\text{Trial divisor} = 2 \times 30 = & 60 & 256 \\
 & \underline{4} & \\
\text{Complete divisor} = & 64 & 256
\end{array}
\qquad \text{or more briefly} \qquad
\begin{array}{r|l}
\multicolumn{2}{r}{\overparen{11}\overparen{56})\underline{34}} \\
\multicolumn{2}{r}{\underline{9}\;\;\;\;} \\
64 & 256 \\
 & \underline{256}
\end{array}
$$

Since 1156 contains two periods of two figures each, the square root of 1156 must contain two figures, a tens' figure and a units' figure.

Since the largest square in 1100 is 900, and the square root of 900 is 30, the number of tens is 3.

Subtracting the square of the tens (900) from 1156, we get 256 as the remainder; 256 must be *2* × *tens* × *units* + (*units*)^2. Since (*units*)^2 is much less than *2* × *tens* × *units*, much the largest part of 256 must be *2* × *tens* × *units*.

If 256 is divided by *2* × *tens*, or 60, it will give the units' figure or a slightly larger number. We obtain 4 as the approximate quotient, and, by trial, determine that it is the exact number of units, since 64 × 4 = 256.

Note in connection with Art. 152 that if the method there used is applied to the squaring of any three-figured number, as 346, we have

$$346^2 = 340^2 + 2 \times 340 + 6^2$$

Hence, having found the first and second figures of a square root, we may repeat our method to find the third figure.

Ex. 2. Extract the square root of 119,716.

$$\begin{array}{rl}
\text{We have} & \overset{\frown}{11}\overset{\frown}{97}\overset{\frown}{16})\underline{346}\ \textit{Root} \\
& \underline{9} \\
64 & |\,297 \\
& |\,\underline{256} \\
686 & |\,4116 \\
& |\,\underline{4116}
\end{array}$$

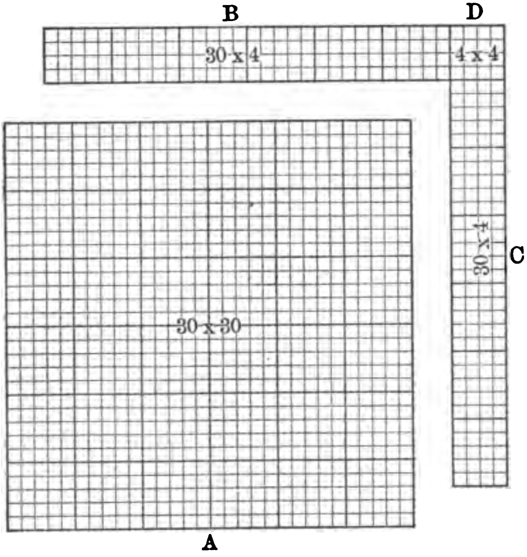
159. Diagram Showing the Process of Extracting Square Root.

By Art. 152, $34^2 = 30^2 + 2 \times 30 \times 4 + 4^2$.

On squared paper we mark off a square each side of which is 34 spaces. We separate this square into a smaller square, A, representing 30×30; two rectangles B and C, each representing 30×4; and another square, D, representing 4×4.

In extracting the square root of 1156, the square of the tens, 900 square units (= A), is first removed, leaving a surface of 256 square units (= B + C + D).

Much the largest part of this remaining surface is the two equal rectangles



(B + C). Hence, dividing the area 256 by the combined length of these rectangles, 2 × 30, or 60, gives the width approximately, or 4.

The entire length of the three figures (B + C + D) remaining after the large square (900) is removed, is 30 + 30 + 4, or 64.

64 × 4 = 256, the remaining area. Hence, 4 is the correct width, or the second figure of the root.

160. Rule for Extracting a Square Root. Hence, in general, to extract the square root of a number,

Separate the number into periods of two figures each, beginning at the decimal point;

Find the greatest square in the left-hand period, and set down its root as the first figure of the required root;

Square this figure, subtract the result from the left-hand period, and to the remainder bring down the next period;

Double the root already found for a trial divisor, divide it into the remainder (omitting the last figure of the remainder), and annex the quotient obtained, to the root and to the trial divisor;

Multiply the complete divisor by the figure of the root last found, and subtract the result from the remainder;

Proceed in like manner until all the periods of figures have been used.

161. Square Root of Fractions. In extracting the square root of a decimal number, we may proceed thus, for example:

$$\sqrt{.0225} = \sqrt{\tfrac{225}{10000}} = \tfrac{15}{100} = .15 \textit{ Root}$$

It is better, however, to put the work in a different form, by marking off the given number into periods of two figures each, beginning at the decimal point. Thus, we have,

$$\begin{array}{r|l}
 & .\overset{\frown}{02}\overset{\frown}{25}).\underline{15}\ \textit{Root} \\
 & \underline{1} \\
25 & 125 \\
 & \underline{125}
\end{array}$$

In extracting the square root of a decimal fraction, as many zeros as are desired may be annexed to the right of the fraction without altering the value of the fraction.

Ex. 1. Extract the square root of .369 to 4 decimal places.

$$\begin{array}{r|l}
\multicolumn{2}{l}{.\overset{\frown}{36}\overset{\frown}{90}\overset{\frown}{00}\overset{\frown}{00}\underline{).6074+}\ \textit{Root}} \\
\multicolumn{2}{l}{\quad 36} \\ \cline{2-2}
1207 & 9000 \\
 & \underline{8449} \\
12144 & 55100 \\
 & \underline{48576}
\end{array}$$

To extract the square root of a common fraction, extract the square root of the numerator and also of the denominator, and divide the first result by the second. Thus,

Ex. 2. $\sqrt{\frac{16}{81}} = \frac{4}{9}$ *Ans.*

If the denominator of the given fraction is not a square number, it is best to change the fraction into a decimal fraction and to extract the square root of the result.

Ex. 3. Extract the square root of $\frac{2}{3}$.

$$\tfrac{2}{3} = .66666666+$$

$$\begin{array}{r|l}
\multicolumn{2}{l}{.\overset{\frown}{66}\overset{\frown}{66}\overset{\frown}{66}\overset{\frown}{66}+\underline{).8164+}\ \textit{Root}} \\
\multicolumn{2}{l}{\quad 64} \\ \cline{2-2}
161 & 266 \\
 & \underline{161} \\
1626 & 10566 \\
 & \underline{\ 9756} \\
16324 & 81066 \\
 & \underline{65296}
\end{array}$$

162. Partly Factorial Method of Extracting Roots. The square roots of certain small numbers are used so frequently that it is convenient to calculate them and record them for further reference.

Thus, $\sqrt{2} = 1.4142+$ and $\sqrt{3} = 1.73205+$

Roots thus calculated and recorded are also useful in connection with the factorial method of extracting the square root of a number.

Thus,

$$\begin{aligned}\sqrt{200} &= \sqrt{100 \times 2} = 10\sqrt{2} \\ &= 10 \times 1.4142+ \\ &= 14.142+\end{aligned}$$

163. Cube Root and Higher Roots; Logarithms. The cube root of a number may be extracted by a method somewhat similar to that used for extracting square root. But cube and higher roots may be extracted with much less labor by the use of a series of auxiliary numbers called *logarithms*. When logarithms are used, the process of extracting a root is abbreviated to a matter of looking up a number in a table and performing a short division. The detailed study of logarithms, however, lies beyond the scope of this book.

164. The **Slide Rule** is a small instrument much used by engineers. By means of this instrument, a root of a number is extracted by the simple mechanical process of sliding one rod along another and reading a number on one of the rods.

By use of the slide rule, numbers may also be multiplied, divided, and raised to a required power.

The detailed study of the use of the slide rule also lies beyond the scope of this book.

EXERCISE 125

Find the square root of

1. 676
2. 841
3. 961
4. 1764
5. 3364
6. 4489
7. 6889
8. 18,496

Find the square root of

9. 173,889

10. 710,649

11. 879,844

12. 54,804,409

13. 64,272,289

14. 96,177,249

15. 2,181,637,264

16. 61.7796

17. .822649

18. 1.752976

19. .50665924

20. 11.67657241

21. 25.81554481

Find the square root to three decimal places:

22. 40

23. 8

24. 17.2

25. $13\frac{1}{2}$

26. .9

27. $\frac{7}{10}$

28. $5\frac{1}{4}$

29. $3\frac{1}{3}$

30. $30\frac{3}{100}$

31. $7\frac{5}{12}$

32. $26\frac{1}{11}$

33. $100\frac{1}{9}$

Find the value of

34. $\sqrt{2+\sqrt{3}}$

35. $\sqrt{\sqrt{13}-\sqrt{3}}$

36. $\sqrt{3\sqrt{3}+\sqrt{5}}$

37. Compute $\sqrt{300}$ by the method of Art. 158. Also by the method of Art. 162. Compare the amount of work in the two processes.

38. Compute in the shortest way $\sqrt{288}$; $\sqrt{675}$; $\sqrt{30{,}000}$; $\sqrt{50}$

39. Compute the square root of $\sqrt{5 \times 14 \times 21 \times 10}$ using the factorial method as far as possible.

40. How many miles are in the side of a square equal in area to New York State?

41. Express the area of Rhode Island in the square form and visualize it.

42. Treat the area of Oklahoma in the same way.

43. Treat, in the same way, the area of your own state. The areas of the states that bound your state.

44. From squared paper cut a square each side of which is 23 spaces. By use of this square, show the meaning of the process of extracting the square root of 529.

45. Is the square root of a proper fraction greater or less than the fraction itself? Illustrate by an example.

46. Visualize 20,000 sheep by picturing them as arranged in a square.

47. Visualize an audience of 10,000 people by regarding them as arranged in a square.

48. Make up and work a similar example.

49. How long will it take to walk around a square field whose area is 16 A., at the rate of 4 miles an hour?

50. How many yards of wire netting will be required to inclose a square poultry yard containing one acre?

51. Three square flower beds, with sides 4 ft. 6 in., 5 ft. 9 in., and 7 ft. 9 in. respectively, are to be replaced by a single square bed equal in area to the sum of the three. Find a side of the new square in feet and inches.

52. A rectangular lawn $196' \times 169'$ is to be replaced by a square lawn of equal area. What is the length of the side of the square lawn? Find the ratio of the lengths of fencing required by the two lawns.

53. In a recent year 40,250,000,000 board feet of lumber were cut in the United States. Visualize this as a rectangular solid with a square base and 100 yd. high. One quarter of a mile high.

54. Make up and work an example similar to Ex. 50.

EXERCISE 126

ORAL

State the square root of each of the following:

1. 256
2. 361
3. 400
4. 144
5. 289
6. 121
7. 324
8. 169
9. 529
10. 196
11. $\frac{64}{225}$
12. $\frac{49}{100}$
13. $\frac{81}{16}$
14. $1\frac{7}{9}$
15. $6\frac{1}{4}$
16. .09
17. 1.21
18. .0036
19. 2500
20. $.062\frac{1}{2}$

Give the value of:

21. $2 + \sqrt{9}$
22. $\sqrt{16} - \sqrt{9}$
23. $\sqrt{625} - \sqrt{196}$
24. $\sqrt{25} \times \sqrt{16}$
25. $\sqrt{64} \div \sqrt{4}$
26. $10 - \sqrt{16}$
27. $\sqrt{625} - \sqrt{1}$
28. $9 \div \sqrt[3]{27}$
29. $\sqrt[3]{125} - \sqrt[3]{64}$
30. $5 + 2\sqrt{9}$
31. $4\sqrt{16} - 3\sqrt{9}$
32. $\dfrac{2 + 4\sqrt{25}}{\sqrt{4}}$
33. $\dfrac{3\sqrt{64} - \sqrt{9}}{3}$

34. Practice oral work with small decimal fractions, as in Exercise 29 (p. 49).

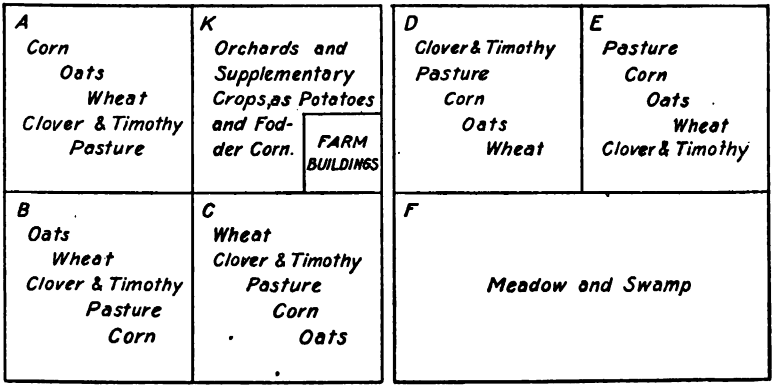
EXERCISE 127

INDUSTRY REVIEW: GENERAL AGRICULTURE

In the farm mapped on p. 301, each of the fields A, B, C, D, E contains 80 acres; F contains 160 acres.

1. In a certain year field A was planted with corn, field B with oats, C with wheat, D with clover and timothy, and part of K with potatoes, and E was used as pasture. Of these crops, the farmer sold 1624 bu. of wheat at 96¢ a bushel, 2140 bu. of corn at 72¢, 1920 bu. of oats at 42¢, 320 bu. of potatoes at 46¢, and 12 tons of hay at $10.75. What did he receive from these crops?

2. The farmer also received $627.40 from the sale of beef cattle, and his wife received $80.40 from the sale of butter and $120.50 from poultry. What was the entire income of the farm?



3. The expenses for the year were $600 for fertilizers, $924 for labor, $300 for machinery, and $62.50 for taxes. What was the net profit from the farm?

4. On the map of the farm, a five-year plan of rotating crops is indicated on fields A, B, C, D, E. Rotation of crops prevents the insect and other enemies of a crop from accumulating, gives opportunity to replace in the soil what any one crop has removed, and thus opens the way to economies in fertilizers. Can you find out any other advantages in rotating crops? When wheat is raised on field A by this plan, what would be raised on B, C, D, and E?

SUG. See the third item in the list in fields B, C, etc.

5. Suppose that field A were planted in corn five years in succession, and during those years produced 52, 45, 40, 35, 32 bushels per acre respectively. If corn was worth 65¢ a bushel, what would be the value of corn raised in the five years?

6. Suppose that, instead of corn, during the last four years of the five the following crops had been raised on field A: oats, 55 bu. per acre at 45¢; wheat, 24 bu. per acre at 95¢; potatoes,

140 bu. per acre at 60¢; clover and timothy hay, $2\frac{1}{2}$ tons per acre at $10.50 a ton. What would have been the total income from field A for the five years? By how much does this exceed the income in Ex. 5?

7. If the roots of the clover crop add 40 lb. of nitrogen per acre to the soil, what are the roots worth in the whole of field A, if the nitrogen is worth 15¢ a pound?

8. By failure to rotate crops in a period of 20 years, it is estimated that the value of the farms in a certain state was diminished by $60,000,000. This was 2% of their value at the beginning of the 20 years. What was their value at the beginning of the 20-year period?

9. Care in selecting the seed of the sugar beet in 100 years has increased the per cent of sugar in the beet from 8.5% to 17%. If sugar is worth 2¢ a pound to the farmer, what is the increase of sugar in 3000 tons of sugar beets worth?

10. If care in selection of seed corn, wheat, and oats should increase these crops by 45%, by how much would the income as given in Ex. 1 be increased?

11. On the farm map, F is a meadow and swamp worth $150 a year as pasture. To underdrain F would require 62,000 ft. of tile, costing $17 per 1000 feet. If the cost of laying the tile is 25¢ per rod, what would be the entire cost of the underdraining?

12. It was estimated that, after being underdrained, field F would produce 60 bu. of corn per acre. If the net profit on a bushel of corn was 35¢, the profit on all the corn would be what per cent of the cost of the underdraining?

In what way does underdraining aid in cultivating land?

13. Suppose that there are 200 wild birds on the farm of Ex. 1, that each eats 15 insects per day for 100 days in the year, and

that each insect, if it had lived, would have done ½¢ worth of damage. How much loss do the birds prevent per year?

14. To irrigate a 20-acre field, water is run through a pipe which delivers 10 cu. ft. per second. How long will it take to deliver water sufficient to cover the whole field 1 in. deep?

15. The first 1,300,000 acres in the United States reclaimed by irrigation cost $40,000,000. How much was this per acre? If the reclaimed land was worth $150 per acre, what was the total gain? Was there any gain beside the financial one?

Regarding nitrogen, phosphoric acid, and potash as worth 15¢, 4½¢, and 5½¢ a pound respectively, determine the difference between the actual value and the selling price of the following available fertilizers:

16. One which contained 1.6% nitrogen, 7% phosphoric acid, and 2% potash, and sold at $29 a ton.

17. 2.3% nitrogen, 8% phosphoric acid, 3.5% potash, and sold at $35 a ton.

18. 2.27% nitrogen, 1.40% phosphoric acid, 2.06% potash, and sold at $20 a ton.

19. .29% nitrogen, 1.82% potash, and sold at $55 per ton.

20. .1% nitrogen, .19% potash, and sold at $4 per ton.

21. Stable manure containing .5% nitrogen, .6% potash, and .3% phosphoric acid, at $2 per ton.

22. A farmer bought 500 lb. of nitrate of soda at 15¢ a pound, 1 ton of phosphoric acid at 4½¢, and ½ ton of potash at 5¢, and made a fertilizer by mixing them. How much did this cost him? This money would have bought how many tons of the fertilizer described in Ex. 16? How much would the latter have been worth?

23. A boy and a girl who together managed a garden plot 100 ft.

square, one year made a profit of \$64.50. The next year, they used 30 lb. of nitrate of soda, 30 lb. of bone meal, and 30 lb. of wood ashes, and found that their income was increased by \$34.75. If the nitrate of soda cost 15¢ a pound, the bone meal 5¢ a pound, and the wood ashes 4¢ a pound, what was their net gain?

24. In a certain year, the farms and their equipment in the United States were worth \$28,508,000,000 and the crops produced by them were worth \$7,500,000,000. The yield was what per cent of the sum invested?

25. By means of machinery, the hours of work required to raise a crop of wheat on an acre of ground has been reduced from 61 to 3 hours, of hay from 41 to 4 hours, of oats from 66 to 7 hours, of potatoes from 109 to 38 hours. Find the average number of hours saved per acre in raising these crops.

26. If the Department of Agriculture of the United States government costs \$15,000,000 a year and increases the agricultural products of the country \$300,000,000 a year, what per cent does the investment pay the people?

27. For growing potatoes, a farmer wishes to compound a fertilizer which shall contain 6% of phosphoric acid, 2% of ammonia, and 7% of potash. In order to make 600 lb. of the fertilizer, how many pounds shall he use of phosphoric acid 16% pure, of cottonseed meal containing 8% of ammonia, and of muriate of potash 50% pure? (An acid "16% pure" is a material containing 16% acid and 84% of other matter, as water.)

SUG. The plant foods mentioned are to occur in the fertilizer in parts which shall be as 6, 2, 7.

But the weight of the acid is increased by foreign matter $\frac{100}{16}$, or $6\frac{1}{4}$ fold; the ammonia $\frac{100}{8}$, or $12\frac{1}{2}$ fold; the potash $\frac{100}{50}$, or 2 fold.

Hence, the materials specified in Ex. 27 are to be used in parts which shall be as $(6 \times 6\frac{1}{4})$, $(2 \times 12\frac{1}{2})$, (7×2), or as $37\frac{1}{2}$, 25, 14.

Let the pupil complete the solution (use the method of partitive proportion, p. 274).

28. If you live on a farm, make an inventory of the value of the farm on Jan. 1, including value of land, animals, buildings, tools, machinery, furniture, and crops on hand. If possible, compare two inventories made in successive years.

CHAPTER XII

MENSURATION

EXERCISE 128

1. For what two reasons is it easier to measure the length of the schoolroom with a tape than with a foot rule?

2. The result of which of these measurements is likely to be more accurate?

3. What is area? Volume?

4. What method is generally used to get the area of a rectangle? The volume of a rectangular solid?

165. **Mensuration** is that branch of mathematics which treats of the measurement of lines, surfaces, and volumes.

Mensuration has already been studied in part in Chapters III and IV.

The principles involved cannot be demonstrated fully in this book; but these principles are so presented and illustrated as to make their truth clear to the pupil and to enable him to recall them readily. The complete demonstration of the rules and formulas used in this chapter belongs to another branch of mathematics, the study of Geometry.

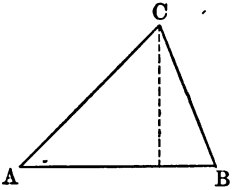
MENSURATION OF LINES

166. A **plane surface** is a surface such that if any two points in it are joined by a straight line, the line will be wholly in the surface.

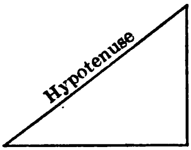
A **triangle** is a portion of a plane surface bounded by three straight lines, as the figure ABC.

The **base** of a triangle is the side upon which it is regarded as standing.

The **altitude** of a triangle is the perpendicular distance to the base from the **vertex**, or point where the other two sides meet.



Triangle

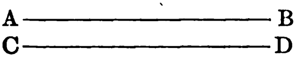


Right Triangle

167. A **right triangle** is a triangle, one of whose angles is a right angle.

The **hypotenuse** of a right triangle is the side opposite the right angle.

The **legs** of a right triangle are the sides of the triangle which form the right angle.



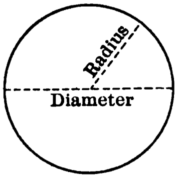
Parallel Lines

168. Parallel lines are straight lines in a plane surface which do not meet, however far they may be extended.

169. A **circle** is a portion of a plane surface bounded by a curved line, every point of which is equally distant from a point within called the **center**.

The **circumference** of a circle is the line which bounds the circle.

What is the diameter of a circle? Radius?



EXERCISE 129

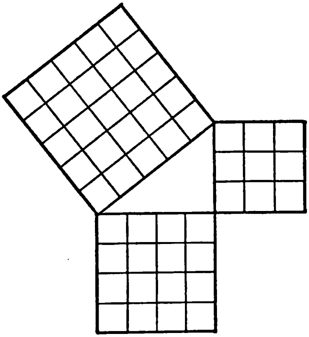
1. Using squared paper as a help, construct a diagram like the diagram on p. 308. Show that the area of the square on the hypotenuse equals the sum of the areas of the other two squares.

2. In like manner, make a right triangle whose short sides are 12 and 5, and show that $12^2 + 5^2 = (\text{hypotenuse})^2$.

3. Treat in the same way a right triangle whose short sides are 8 and 15. Also 6 and 8.

4. Measure the diameter of any circular object, as a silver dollar, and then measure its circumference. Divide the length of the circumference by the length of the diameter. What is the quotient? Treat the circular lid of a kettle in the same way.

170. Rules and Formulas for Mensuration of Lines.



1. *The square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides.*

Denoting the hypotenuse by c, and the other two sides by a and b,

$$c^2 = a^2 + b^2$$

and $$a^2 = c^2 - b^2$$

whence $$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

Hence, given any two sides of a right triangle, the third side may be computed without the labor of measuring it.

Ex. Find the hypotenuse of a right triangle of which the two other sides are 50 ft. and 60 ft.

$$\text{Hypotenuse} = \sqrt{50^2 + 60^2}$$
$$= 78.102+ \text{ ft.}$$

2. *Circumference of a circle = diameter* × 3.1416 (*approx.*).

$\frac{22}{7}$ is sometimes used instead of 3.1416, though it is not quite so accurate.

The symbol π is used for 3.1416, or $\frac{22}{7}$. This symbol is read "pī." It is the Greek form of p, the first letter in the word *perimeter*.

EXERCISE 130

In the following examples, a and b represent the legs of a right triangle, and c represents the hypotenuse.

1. Given $a = 8$, $b = 15$, find c.
2. Given $b = 35$, $c = 37$, find a.
3. Given $b = 28$, $a = 45$, find c.
4. Given $a = 112$, $c = 113$, find b.
5. Given $c = 73$, $b = 55$, find a.
6. Given $a = 24$, $b = 143$, find c.
7. Given $c = 1105$, $a = 561$, find b.

Find the remaining side correctly to 3 decimal places, when

8. $a = 5, b = 8$
9. $b = 2, c = 11$
10. $c = 43, a = 34$
11. $a = 92, b = 65$

Find the circumference of each circle, when

12. Radius = 7
13. Diameter = 46
14. Radius = $8\frac{1}{2}$
15. Diameter = $13\frac{1}{3}$
16. Radius = 74.6
17. Diam. = 175.4

Find the diameter of the circle, when

18. Circumference = 40
19. Circumference = 375
20. Circum. = 57.3
21. Circum. = 103.8

22. A ladder 25 ft. long stands against the side of a house, with its foot 7 ft. from the wall. How high is the top of the ladder?

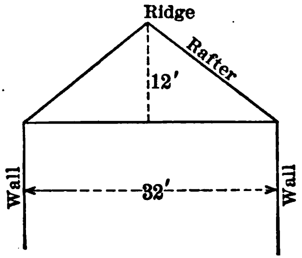
23. Find the hypotenuse of a right triangle whose sides are 3″ and $1\frac{1}{4}$″. Verify your result by the use of squared paper.

24. Treat in the same way a triangle whose sides are 2″ and $3\frac{3}{4}$″. 1.75″ and 6″.

25. Construct a right triangle by the use of strips of wood 3, 4, and 5 in. long. Also by strips 6, 8, and 10 in. long.

26. Construct a right triangle by stretching parts of a 50-foot tape so as to form a right triangle. Can you construct a right angle more accurately by use of a 100-foot tape?

27. A field 156 rods long and 133 rods wide is cut by a path running diagonally across it. Find the length of the path.



28. A house is to be 32 ft. wide and the ridge is to be 12 ft. higher than the top of the walls. How long must the rafters be (neglecting the projections of the rafters at the eaves)?

29. If the diameter of a pipe is $8\frac{1}{2}$ in., what is its circumference? What is the diameter of another pipe whose circumference is $8\frac{1}{2}$ in.?

30. A wagon wheel 3 ft. 4 in. in diameter will go how far in 121 revolutions?

31. Show how to find the length of a line on the ground (as the dimensions of a lot) by measuring the circumference of a wheel, marking a spot on the circumference with chalk, and pushing the wheel along the line whose length is to be measured.

32. A baseball diamond is 90 ft. square. How long is a straight line connecting the home plate and second base?

33. Tell how you could find out the diameter of a tree without cutting the tree down. Make up and work an example involving your method.

34. A wheel 6 ft. in diameter is revolving 4000 times per minute. A point on the rim of the wheel goes what fraction of a mile in 1 sec.?

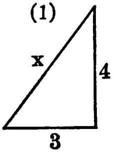
35. A boy walks 5 miles north, then 5 miles east; what direction and how far is he from the starting point? (Use $\sqrt{2} = 1.4142+$ to shorten the computation. See Art. 162, p. 296.)

36. A city park is 1600 ft. long and 1200 ft. wide. If you walk at the rate of 4 mi. an hour, how much time is saved by walking diagonally across it, instead of going along the sides from one corner to the opposite corner? If you need to cross the park twice a day, on 300 days of the year, how much time do you save in a year?

37. Make up and work a similar example concerning a circular pond $\frac{1}{4}$ mile in diameter and a bridge across its center.

38. At sight give the value of x in each of the following right triangles:

(1)



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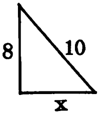

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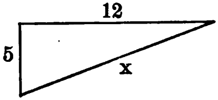

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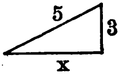

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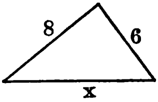

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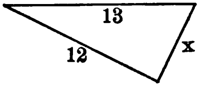

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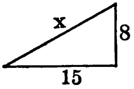

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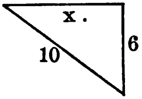


MENSURATION OF PLANE FIGURES

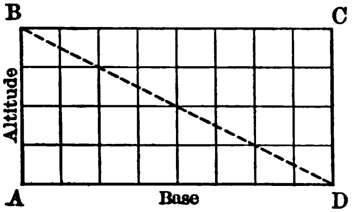
EXERCISE 131

1. What is a unit of area? Give three examples of a unit of area.

2. What is area? Give two examples of area.

3. On squared paper mark off a rectangle 8 spaces long and 4 spaces wide. In getting the area of this rectangle, is there any shorter way than counting the number of small squares in the rectangle?

4. Cut the rectangle ABCD out of the squared paper. Then fold and cut the rectangle along the line BD. Can you make the part ABD fit the part BCD exactly? Is the area of BCD therefore = $\frac{1}{2}$ AD × AB, that is $\frac{1}{2}$ of 8 × 4, or 16?



5. Count the small squares in BDC (piecing together the parts of squares). Compare the work of counting the small squares with that of saying $\frac{1}{2} \times 8 \times 4 = 16$.

6. Make up and work a similar example.

7. On squared paper, the lines of which are 1 in. apart, draw a circle whose radius is 5 in. Count the number of square inches in the circle (combining parts of a square inch approximately).

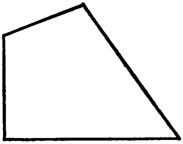
8. Now multiply 5^2, or 25, by 3.1416. Which method of getting the area of the circle is more rapid? Which is more accurate?

9. Make up and work a similar example.

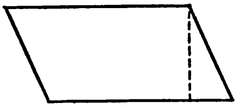
171. An **equilateral triangle** is a triangle which has all its sides equal.

A **quadrilateral** is a portion of a plane bounded by four straight lines.

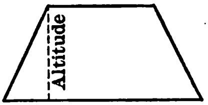
A **parallelogram** is a quadrilateral whose opposite sides are parallel.



Quadrilateral



Parallelogram



Trapezoid

A **trapezoid** is a quadrilateral which has two and only two of its sides parallel.

The **altitude** of a parallelogram or trapezoid is the perpendicular distance between parallel sides.

A **diagonal** of a rectangle (or of any four-sided figure) is a line joining two opposite corners of the figure.

172. A **polygon** is a portion of a plane surface bounded by straight lines. A polygon of *three* sides is called a **triangle**; of *four* sides, a **quadrilateral**; of *five* sides, a **pentagon**; of *six* sides, a **hexagon**, etc.

A **regular polygon** is a polygon in which the sides are all equal, and the angles are all equal.

The **perimeter** of a polygon is the sum of the lengths of its sides.

173. Rules and Formulas for Areas of Plane Figures.

Area of a rectangle = *(1 sq. unit)* × *(no. linear units in the base)* × *(no. linear units in the width)*, like units being used throughout.

It is convenient to abbreviate the above rule thus:

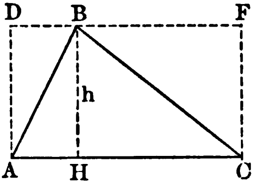
1. *Area of a* **rectangle** = *base* × *altitude.*

Abbreviating in like manner other rules for finding areas, we have:

2. *Area of a* **triangle** = $\frac{1}{2}$ *base* × *altitude.*
3. " " **parallelogram** = *base* × *altitude.*
4. " " **trapezoid** = $\frac{1}{2}$ *sum of parallel sides* × *altitude.*
5. " " **polygon** = *sum of the areas of the triangles into which the polygon may be divided.*
6. *Area of a* **circle** = 3.1416 × *radius squared* = πR^2, where R is the radius of the circle.
7. *Area of* **equilateral triangle** = $\frac{a^2\sqrt{3}}{4}$, where a denotes one of the sides.

The reason for the most important of the rules for computing areas, given on page 313, may be shown as follows:

Rule 2. Let ABC be any triangle, and BH be its altitude. Draw the rectangle ADFC having AC for its base and BH for its altitude.



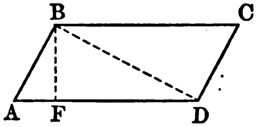
Then, by the method of Ex. 4 of Exercise 131,

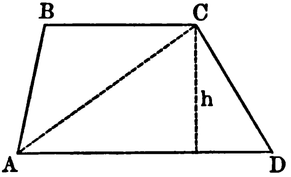
Triangle ABH = $\frac{1}{2}$ rectangle ADBH
" HBC = $\frac{1}{2}$ " HBFC

Hence, by adding, ABC = $\frac{1}{2}$ ADFC
The area of ADFC = AC × h
Hence, area of ABC = $\frac{1}{2}$ AC × h

Rule 3. It has been shown that if we take the parallelogram ABCD and draw the diagonal BD, triangle BCD = triangle ABD. Hence, ABCD = 2 ABD.

Hence, area ABCD = 2 ($\frac{1}{2}$ AD × BF)
= AD × BF



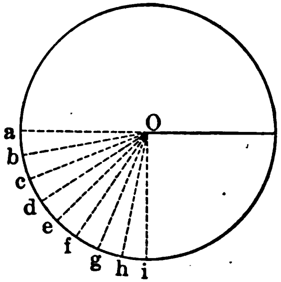


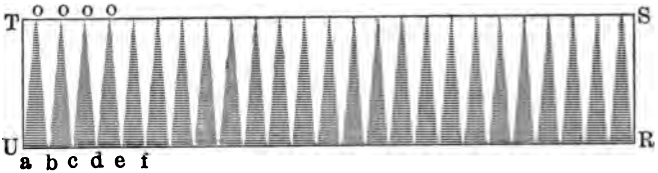
Rule 4. Also let ABCD be a trapezoid with the side BC parallel to the side AD. Draw the diagonal AC.

Then area ABCD = ACD + ABC
= $\frac{1}{2}$AD × h + $\frac{1}{2}$BC × h
= $\frac{1}{2}$ (AD + BC) × h

Rule 6. In order to understand the formula for obtaining the area of a circle, it will be useful to regard the circle as split up into parts, as in the figure opposite; and then conceive the parts *o a b*, *o b c*, *o c d*, etc., arranged as in the figure on page 315.

The smaller the parts into which the circle is divided, the more nearly will their bases, when taken thus, approximate to a straight line, and their areas taken together = $\frac{1}{2}$ rectangle RSTU, whose base is the circumference of the circle, and whose altitude is its radius.





Hence, area of circle $= \frac{1}{2}$ circumference $\times R$

$$= \frac{1}{2} \times 2R \times 3.1416 \times R = R^2 \times 3.1416$$

Observe carefully that the above areas are found by first measuring certain straight lines, and computing the area from the lengths obtained. This is much more expeditious than any direct counting of the units of area, which is indeed often impossible.

Ex. 1. Find the area of a parallelogram whose base is 5 yd. and whose altitude is 8 ft.

5 yd. = 15 ft.

Hence, Area $= \frac{1}{2} \times 15 \times 8$ sq. ft. = 60 sq. ft. *Ans.*

Ex. 2. What part of an acre is grazed over by a cow tied by a tether 100 feet long?

$$\text{Area} = 3.1416 \times R^2$$
$$= 3.1416 \times 100 \times 100 \text{ sq. ft.} = 31416 \text{ sq. ft.}$$
$$= \tfrac{31416}{43560} \text{ A.} = .721+ \text{ acre } \textit{Area.}$$

EXERCISE 132

Find the area of

1. A rectangle 8 yd. long and 11 ft. 8 in. wide.
2. The walls and ceiling of a room $17\frac{1}{2} \times 16\frac{2}{5} \times 8\frac{1}{3}$ feet.
3. A parallelogram whose base is 30 yd. and altitude 20 ft.
4. A straight rectangular street 7 mi. long and $2\frac{1}{2}$ rd. wide.
5. A page $8\frac{2}{3}$ in. long and $4\frac{1}{2}$ in. wide.
6. A triangle with base of 18 in. and altitude of 15 in.
7. A triangular field whose altitude is 40 yd. and base 45 rd.

8. A trapezoid whose bases are 60 and 75 ft. and whose altitude is 15 yd.

9. A circle whose radius is 6 in.

10. A circle whose diameter is 10 rd.

11. A circle whose circumference is 80 ft.

12. A triangle whose base is $10\frac{1}{2}$ ft. and altitude 5.5 yd.

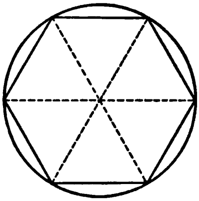
13. A trapezoid whose bases are 18.5 ft. and 14.4 ft. and whose altitude is 65 in.

14. An equilateral triangle whose sides are each 5 in.

15. A barn is 48 ft. wide and 90 ft. long. At the corner its height is 20 ft., but at the middle the height to the peak is 38 ft. Find the area of the end of the barn.

16. If the voice of an orator can be heard at an average distance of 400 ft., how many standing people can hear him, if each person occupies an area 2′ × 14″?

17. On squared paper draw a parallelogram like that on p. 314, making AD = 10 spaces, AF = 2 spaces, and BF = 4 spaces. Count the square spaces in this parallelogram and thus verify the rule in Art. 173 (*viz.:* area = AD × BF = 10 × 4 = 40).



18. Draw a circle and divide its circumference into six equal parts by points, the distance between each two successive points being equal to the radius of the circle. Join these points. What kind of polygon have you made? Show that this polygon may be divided into 6 equilateral triangles, or 12 equal right triangles.

19. If a wireless message can be heard a distance of 500 miles from a given station, taking the area covered as a circle, over how many square miles can the message be heard?

20. How many acres are within a mile's distance of a city hall?

21. In heating a house by a hot-air furnace, the area of the end of the cold-air box should equal the sum of the areas of the cross sections of the hot-air pipes. In a certain house, there are four 6-in. and two 5-in. hot-air pipes and the cold-air box is to be 15 in. wide. How deep should the cold-air box be?

22. Show that finding the area of a square may be reduced to the measurement of one straight line and one multiplication.

Show that finding the area of a circle may be reduced to the measurement of the length of one line and two multiplications.

23. A rectangle is 80 ft. by 30 ft. By what per cent will the area be increased if each of the dimensions of the rectangle is increased by 20%? By 50%? By 100%? By 200%? Show each of these results on a diagram.

24. Make up and work a similar example.

25. By how long a rope must a cow be tethered if she is to graze over one acre?

26. Express, by use of letters, the area of a field a rods long and b rods wide. Also of one whose dimensions are p ft. and $2\,q$ ft. Also c yd. and d ft.

27. Of a circle whose radius is $2\,p$ ft.; whose diameter is k ft.

28. Work again Exercise 33 (p. 67) or similar examples concerning compound numbers.

MENSURATION OF THE SURFACES OF SOLID FIGURES

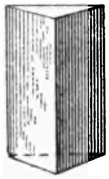
174. A **solid** is that which has length, breadth, and thickness.

A **prism** is a solid bounded by two equal and parallel polygons called bases, and by parallelograms. These parallelograms taken together form the *lateral surface.*

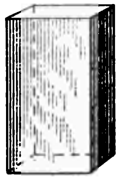
The *altitude* of a prism is the perpendicular distance between the bases.

Prisms are **triangular, quadrangular, pentagonal,** etc., according as their bases are triangles, quadrilaterals, pentagons, etc.

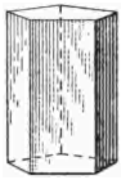
A **regular prism** is a prism which has regular polygons for its bases and whose lateral edges are perpendicular to the bases.



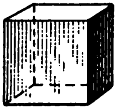
Triangular Prism



Quadrangular Prism



Pentagonal Prism



Cube

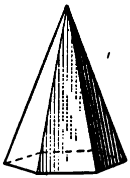
A **right prism** is a prism in which the other faces are perpendicular to the bases. An ordinary box is a right rectangular prism.

A **cube** is a prism bounded by squares.

175. A **pyramid** is a solid bounded by a polygon called the base, and by triangles meeting at a point called the vertex.

The triangles which meet at the vertex taken together form the *lateral surface.*

The *altitude* of a pyramid is the perpendicular distance from the vertex to the base.



Pyramid

A pyramid is **triangular, quadrangular, pentagonal,** etc., according as the base is a triangle, quadrilateral, pentagon, etc.

A **regular pyramid** is a pyramid which has a regular polygon for its base, and the triangles bounding the pyramid all equal.

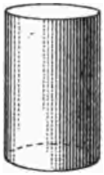
The *slant height* of a regular pyramid is the perpendicular distance from the vertex to one edge of the base.

176. A **cylinder** is a solid bounded by two parallel circles (called bases), and by a curved surface every point of which is equally distant from a line joining the centers of the two circles.

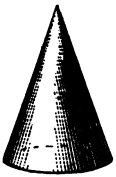
177. A **cone** is a solid bounded by a circle as a base, and by a curved surface which tapers uniformly from the circle to a point called the vertex.

The curved surface of a cone or cylinder is usually termed a *convex surface.*

The *altitude* of a cone is the perpendicular distance from the vertex to the base. The *slant height* is the distance from the vertex to any point in the circumference of the base.



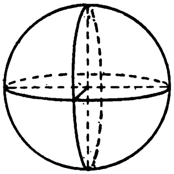
Cylinder

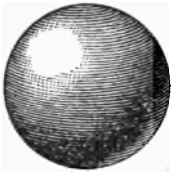


Cone

178. A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within, called the *center.*

The **radius** of a sphere is a line drawn from the center to any point of the surface. The **diameter** is a line passing through the center and terminated by the surface.





Sphere

179. Rules and Formulas for Areas of Surfaces of Solids. Abbreviating rules for areas, as in Art. 173, we have

1. *Lateral surface of a* **right prism** = *perimeter of base* × *altitude.*

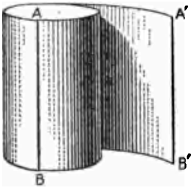
2. *Convex surface of a* **cylinder** = *circumference of base* × *altitude.*

3. *Lateral surface of a* **regular pyramid** = $\frac{1}{2}$ *perimeter of base* × *slant height.*

4. *Convex surface of a* **cone** $= \frac{1}{2}$ *circumference of base* × *slant height.*

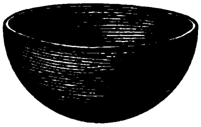
5. *Surface of a* **sphere** = 3.1416 × *diameter squared* $= 4\ \pi R^2$.

These rules (except 5) are derived from those given in Art. 173. Thus, the lateral surface of a right prism is composed of rectangles, all having the same altitude; that of a regular pyramid is composed of equal triangles. Also the convex surface of a cylinder, unrolled, forms a rectangle.



The convex surface of a cone may, for our purpose, be regarded as the lateral surface of a pyramid having an indefinite number of sides in its face.

To obtain a rule for computing the area of the surface of a sphere, we take a hemispherical shell and a circular plate of the same radius, both being made from the same sheet of metal, and weigh them. The weight of the shell is found to be double that of the circular plate.



The area of the circle is πR^2.

Since the hemisphere weighs twice as much as the circle, the area of the surface of the hemisphere must be twice the area of the circle, or $2\ \pi R^2$.

Since the surface of the sphere is twice that of the hemisphere, the area of the surface of the sphere is twice $2\ \pi R^2$, or $4\ \pi R^2$.

EXERCISE 133

Find the area of the lateral surface of

1. A right prism 10 in. high, on a square base 3 in. on a side.

2. A right prism 8 ft. high, on an octagonal base 9 in. on each side.

3. A regular pyramid on a hexagonal base 5 in. on a side, having a slant height of 10 in.

4. A cylinder 7 ft. long, the radius of whose base is 3 ft.

5. A pipe 18 in. in diameter and a mile long.

6. A silo in the shape of a hexagonal prism 24 ft. high, with each side of the base = 5 ft. 6 in.

7. A steam boiler 5 ft. in diameter and 22 ft. 4 in. in length.

8. Find the area of the surface of a sphere whose radius is 3 ft.

9. Find the area of the surface of a sphere whose diameter is 19 in.

10. At 12¢ a square foot, what is the cost of painting a pyramidal spire, whose base is a hexagon of 9 ft. on a side, and whose slant height is 90 ft.?

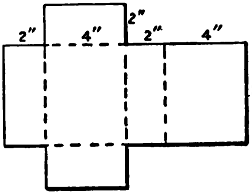
11. What will it cost to paint a cylindrical water tower at 20¢ a square yard, if the diameter of the tower is 10 ft. and its height is 80 ft.?

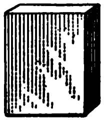
12. Compute the cost of gilding a dome in the shape of a hemisphere whose radius is 18 ft., at $1.75 a square yard.

13. Find the total convex surface of 40 steam pipes, each 18 ft. long and 3 in. in diameter.

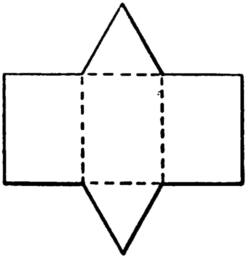
14. A conical tent is to be 8 ft. high, and the radius of its base is to be 6 ft. Neglecting the seams, what amount of canvas is needed for its construction?

15. From cardboard, cut a piece of the shape and dimensions shown in the adjoining diagram. (Squared paper will be an aid in doing this.) Cut the cardboard half through at the dotted lines; then fold it and paste its edges so as to form a rectangular prism. What will be the dimensions of this prism? Find its lateral area and also its total area.





16. Make up and work an example similar to Ex. 15.



17. Cut a piece of cardboard which may be folded so as to form a triangular prism 1½ inches long, with ends that are equilateral triangles with 1-inch sides. Find the lateral area and the total area of this prism.

18. Make up and work a similar example.

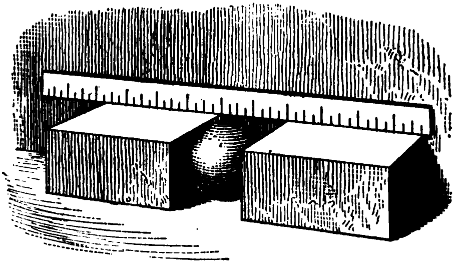
19. Cut out of cardboard a rectangle 8 inches long and 4 inches wide. Roll it up and paste it so as to form the lateral surface of a cylinder. Compute this lateral surface. Also the total surface.

20. An iron roller is a cylinder 8 ft. long and 27 in. in diameter. What area does it cover in one revolution?

21. Apart from the multiplications involved, the measurement of the surface of a sphere may be reduced to the measurement of the length of what straight line? What curved line?

22. Show how the measurement of the surfaces of other solids reduces to the measurement of straight lines. (Specify the number of lines and their names for each solid.)

23. Measure the diameter of several spheres by the method indicated in the diagram. Then compare the areas of the surfaces of these spheres.



24. Work again Exercise 69 (p. 146), or similar oral examples concerning percentage, dictated by the teacher or a pupil.

MENSURATION OF VOLUMES

EXERCISE 134

1. What is a unit of volume? Give three examples of a unit of volume.

2. What is volume? Give two examples of volume.

3. By use of inch blocks, build a solid 5 in. long, 3 in. wide, and 4 in. high. In getting the volume of this solid, is there any shorter way than counting the number of inch blocks?

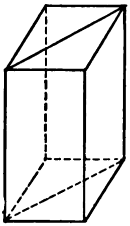
4. Which is easier: to saw a croquet ball into cubic inches as nearly as possible and to count the cubic inches (piecing together parts of cubic inches as nearly as you can), or to measure the diameter of the croquet ball and to multiply the cube of the diameter by .5236?

5. Which process will be more accurate? Which will leave the croquet ball fit for future use?

180. Rules for Volumes. Abbreviating rules for volumes in a way similar to that used for areas (see Arts. 173, 179), we have

1. *Volume of a* **prism** *= area of base × altitude.*
2. *Volume of a* **rectangular prism** *= length × breadth × thickness.*
3. *Volume of a* **cube** *= cube of its edge.*
4. *Volume of a* **cylinder** *= area of base × altitude.*
5. *Volume of a* **pyramid** *=* $\frac{1}{3}$ *area of base × altitude.*
6. *Volume of a* **cone** *=* $\frac{1}{3}$ *area of base × altitude.*
7. *Volume of a* **sphere** *=* $\frac{1}{3}$ *surface × radius* $= \frac{4}{3}\pi R^3$.

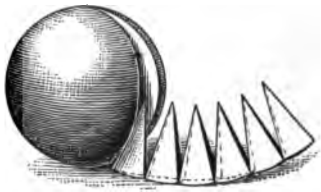
The student needs to study Solid Geometry in order to understand fully the reasons for these formulas.



On p. 113 it was found that the volume of a rectangular prism is equal to the number of linear units in the three edges multiplied together.

A rectangular prism may be conceived as divided into two equal triangular prisms with equal bases and the same altitude. Hence, the volume of each will equal half the volume of the rectangular prism, or the volume of a triangular prism = *base* × *altitude*.

Any other prism may be split up into triangular prisms and its volume obtained by the same rule.



A cylinder does not differ perceptibly from a prism with a very large number of sides.

For pyramid and cone see Exs. 17, 18, p. 325.

A sphere, for our present purposes, may be regarded as composed of very small pyramids, whose common vertex is the center of the sphere and whose common altitude is the radius of the sphere. The bases of the small pyramids taken together form the surface of the sphere and equal $4\pi R^2$.

Hence, volume of sphere $= \frac{1}{3}(4\pi R^2) \times R = \frac{4}{3}\pi R^3$

EXERCISE 135

Find the volume of

1. A prism the area of whose base is 10 sq. ft. and whose height is 12 ft.

2. A rectangular prism whose dimensions are 8 ft., 9 ft., and 10 ft.

3. A room whose dimensions are 15 ft. 3 in., 13 ft. 4 in., and 11 ft. 6 in.

4. A pyramid 9 ft. high and the area of whose base is 100 sq. ft.

5. A pyramid whose altitude is 28 ft. and whose base is 60 sq. yd. in area.

6. A cylinder whose radius is 9 in. and altitude is 10 ft.

7. A piece of wire $\frac{1}{2}$ in. thick and 75 yd. long.

8. A sphere of radius 5 in. One of diameter 11 in.

9. How many cubic feet of water will be held by a cylindrical water tank 8 ft. in diameter and 12 ft. high? How many gallons? (Use 1 cu. ft. = $7\frac{1}{2}$ gal.)

10. How many cubic yards of gas are needed to fill a spherical balloon 80 yd. in diameter?

11. Find the difference between the volume of a 4-inch cube and that of a 4-inch sphere.

12. If a bushel measure in the form of a cylinder is 18 in. in diameter, how deep is it? If it is 18 in. deep, what is its diameter?

13. Into a cylindrical water tank 13 in. in diameter and standing on end, an irregular piece of metal is sunk. If the surface of the water rises 3 in., what is the volume of the piece of metal?

SUG. The volume of the irregular piece of metal will be equal to the volume of a cylinder 13 in. in diameter and 3 in. high.

14. A heap of wheat in the shape of a cone is 8 ft. deep and the diameter of the base is 15 feet. How many bushels in the heap?

15. How many ice cream cones 2 in. high and $1\frac{1}{2}$ in. in diameter can be formed from 1 qt. of ice cream?

16. From $2\frac{1}{4}$ gal. of ice cream?

17. Out of pasteboard, make a triangular prism and pyramid of the same base and altitude.

18. Fill the hollow pyramid with sand and pour the sand into a hollow prism which has the same base and altitude as the hollow pyramid. How many times must you do this in order to fill the hollow prism with sand?

19. Make and use a hollow cone and a hollow cylinder as in Ex. 18.

20. A rectangular solid is 20″ × 30″ × 10″. If each dimension is increased by 20%, by what per cent is the volume increased? By what, if each dimension be increased by 50%? By 100%? By 200%?

21. An iron cylinder is 7 ft. long and 1 ft. in radius. Find its weight, taking the weight of 1 cu. ft. of iron as 450 lb.

22. What is the diameter of a circular cistern which holds 1 barrel for every 2 inches of its depth?

SUG. The volume of a barrel = $31\frac{1}{2} \times 231$ cu. in. Dividing this by 2, we obtain the area of the base of the cylinder.

23. Make up three examples similar to such of the examples in this Exercise as you think are most interesting or instructive.

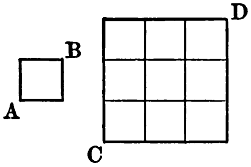
24. Work again Exercise 58 (p. 120) or similar examples.

LINES, AREAS, AND VOLUMES OF SIMILAR FIGURES

181. Similar surfaces are those which have the same shape. Thus, any two squares are similar plane figures. Two regular polygons having the same number of sides are similar.

Similar solids are solids which have the same shape. Thus, any two cubes, or two spheres, are similar.

EXERCISE 136



1. On squared paper mark off a square whose side is 1 space; also a square whose side is 3 spaces. Does the ratio of the areas of these squares = $1^2 : 3^2$?

2. On squared paper mark off a square whose side is 2 spaces. Mark another square whose side is 3 spaces. Find out the area of each square. Does the ratio of these areas $= 2^2 : 3^2$?

3. Make up and answer a similar example.

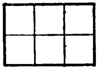
4. In the diagrams of Ex. 1, draw a diagonal from A to B and also from C to D. Of the triangles thus formed, cut one from each diagram. Are these triangles similar? Find the area of these triangles. Does the ratio of the areas of the triangles $= 1^2 : 3^2$?

5. On squared paper draw two similar rectangles and treat them in the method of Ex. 2.

6. Compute the area of a circle whose radius is 3 inches. Also of one whose radius is 4 inches. Does the ratio of these areas $= 3^2 : 4^2$?

7. Draw two rectangles which are not similar. Also two triangles.

8. Pick out each pair of similar figures from the following and, if possible, give the ratio of their areas: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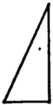
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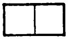
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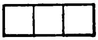
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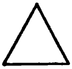
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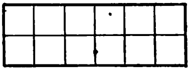
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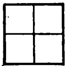
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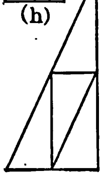
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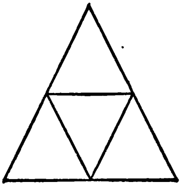
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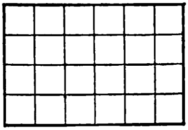
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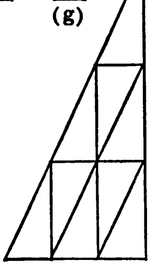

(l)

9. By use of blocks, build a cube whose edge is 2 inches; also one whose edge is 3 inches. What is the volume of each cube? Does the ratio of these volumes $= 2^3 : 3^3$?

10. Make up and answer a similar example.

182. Properties of Similar Figures. In any two similar figures

1. *Any two* **corresponding lines** *have the same ratio as any other two corresponding lines;*

2. *The* **areas** *of any two similar figures are to each other as the* **squares** *of any two corresponding lines;*

3. *The* **volumes** *of any two similar solids are to each other as the* **cubes** *of any two corresponding lines.*

Ex. If a pipe 1 in. in diameter discharges 50 gal. in a minute, how much will a pipe 2 in. in diameter discharge?

The quantity discharged by a pipe is in proportion to the area of the section of the pipe, and hence, in proportion to the square of its diameter. Hence,

$$1^2 : 2^2 = 50 : x$$

Hence, x or no. gal. required $= \frac{50 \times 4}{1} = 200$ *Ans.*

EXERCISE 137

1. The bases of two similar triangles are 15 in. and 18 in. respectively. If the area of the first triangle is 135 sq. in., what is the area of the second triangle?

2. A polygon whose base is 12 ft. contains 62 sq. ft. What is the area of a similar polygon whose base is 42 ft.?

3. The area of a circle whose radius is 5 in., is 78.54 sq. in. Find the area of a circle whose radius is 7 in.

4. A cylinder whose altitude is 8 ft. has a convex surface of 44 sq. ft. What is the convex surface of a similar cylinder whose altitude is 20 ft.?

5. The volume of a solid is 400 cu. ft. and one edge is 12 ft. Find the volume of a similar solid, if the corresponding edge in it is 21 ft.

6. If the sides of two squares are as 2 : 3, what is the ratio of their areas? If the edges of two cubes are as 3 : 5, what is the ratio of their volumes?

7. Two spheres have radii equal to 7 and 9 inches respectively. What is the ratio of their circumferences? Of the areas of their surfaces? Of their volumes?

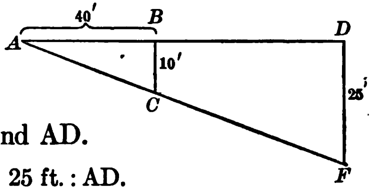
8. If there are 300 yd. in a 4-inch ball of yarn, how many yards will there be in a 6-inch ball? In a 2-inch ball?

9. If it costs $250 to paint a certain house, how much will it cost to paint a similar house, all of whose dimensions are double those of the first?

10. If the planet Jupiter has 11 times the diameter of the earth, how do their surfaces compare? How do their volumes compare?

11. If a grindstone 18 in. in diameter costs $4, what should be the cost of another, having the same thickness but 24 in. in diameter?

12. If the average weight of a man 5 ft. 6 in. tall is 150 lb., what should be the weight of a man 6 ft. tall?



13. The triangles ABC and ADF are similar. If BC = 10 ft., DF = 25 ft., and AB = 40 ft.; find AD.

SUG. Use the proportion 10 : 40 = 25 ft. : AD.

14. A brook has a fall of 1 ft. in 75 yd. If a dam 10 ft. high

is constructed across the brook, how long a pond will this dam make?

SUG. Use two similar triangles, the legs of one of which are 75 yd. and 1 ft.

15. On another stream, a dam 8 ft. high makes a pond ½ mile long. How great a fall is there per 100 feet in this stream?

16. The projectiles being of the same shape, how does the projectile of a 3-inch gun compare in weight with that of a 6-inch gun? Of a 12-inch gun? Of a 10-inch? 14-inch?

17. If a 2-inch pipe will fill a certain cistern in 1 hr. 12 min., how long will it take a 3-inch pipe to fill the cistern?

18. The Mt. Washington railway rises 3596 ft. in 3 miles. What is the average rise in 100 ft.?

19. Make up and work an example concerning the steepest part of the Mt. Washington railway (Jacob's Ladder), where the rise is at the rate of 13½ in. for 36 in. of track.

20. Will a 1-inch pipe discharge one half as much water as a 2-inch pipe? Why?

21. The load which can be suspended from a wire is proportional to the area of the cross section of the wire. If a steel wire of .36 in. diameter can support 7 tons, how much will a steel wire of ½ in. diameter support?

22. Make up and work three examples similar to such of the examples in this Exercise as you think are most interesting or instructive.

23. Practice the oral computation of interest as in Exercise 82 (p. 187).

EXERCISE 138

REVIEW

1. Find the area in acres of a triangular field each of whose sides is 10 chains.

2. Of a triangle whose base is 2 yd. 6 in., and altitude is 3 ft. 8 in.

3. Find the radius of a circle whose area is one square yard.

4. Find the weight of a granite pyramid 4 ft. square and 6 ft. high, if granite is 2.6 times as heavy as water.

5. Make up and work a similar example concerning the weight of a cylindrical column of marble. (Use specific quantity of marble as 2.7.)

6. If the parallel sides of a trapezoid are 12 in. and 20 in. and the area is 128 sq. in., find the altitude of the trapezoid.

7. A circular fountain 18 ft. in diameter is to be surrounded by a cement walk 5 ft. wide. What will the walk cost at $1.25 per square yard?

8. If a hogshead is 12 ft. in circumference around the middle, determine whether it will go through a doorway that is 3 ft. 9 in. wide.

9. If a room is $20' \times 16' \times 8'$, find the distance from one corner of the floor to the opposite corner of the ceiling.

10. The gable end of a house is 36 ft. wide and the ridge of the roof is 15 ft. higher than the top of the walls. (See diagram, p. 310.) If the rafters project 1 ft. at the eaves, how long are the rafters? Draw to scale a diagram of the gable end of the house.

11. If cranberries are shipped in a crate 20 in. × $10\frac{1}{2}$ in. × $6\frac{3}{4}$ in. and apples in a box 18 in. × $11\frac{1}{2}$ in. × 11 in., find the difference between the volumes of the crate and the box.

12. Make up and work an example concerning the cost of slating a church spire which is the shape of an hexagonal pyramid, given a side of the base and the slant height.

13. Find the convex surface of a boiler 36 in. in diameter and 12 ft. long. Also find the total convex surface of 60 tubes each 1 in. in diameter and 12 ft. long. Find the difference between the two results.

14. A vein of coal has a fall of 4.5 in. per yard. What will be the total fall in a distance of 440 yd.?

15. If the rise of a side of a pile of dry sand is greater than 71.4 in. for a horizontal distance of 91 in., the sand begins to slip. Find the greatest possible height of a heap of sand, the base of which is 213.2 in. in diameter.

16. A rectangular block of iron weighs 14,200 lb. What is the weight of another block of iron whose dimensions are one half those of the first block?

17. A body floating in water displaces its own weight of water. Hence, a log which weighs 410 lb. will displace how many cubic feet of water?

18. If a log displaces 6 cu. ft. of water, how much does the log weigh?

19. If a beam $16' \times 8' \times 4'$ floats one fourth out of water, how much does it weigh?

20. If the projectile of a steel cannon is taken as a steel cylinder 12 in. in diameter and 20 in. long, with a conical end added which is 8 in. long, how many pounds will the projectile weigh, if steel weighs 492 lb. per cubic foot?

21. In a certain city the number of cases in the juvenile courts one year was 2761. Four years later the number was 1543. Find the per cent of decrease. Can you suggest any reason for this decrease?

22. Of what advantage is it for different parts of the country (or different countries) to exchange goods? Give an illustration. How do the processes of arithmetic aid in this exchange? Give an illustration.

23. In a city of 325,500 inhabitants, the birth rate is 27.3 and the death rate 14.5. Find the increase in the number of people in the city in one year.

24. The top of Pike's Peak is 14,147 ft. and Denver is 5175 ft. above the sea level. If an increase of 300 ft. in altitude means a decrease of 1° in the mean annual temperature, how much colder is Pike's Peak than Denver?

25. If 40 cu. ft. of fodder in a silo make a ton and a cow eats 30 lb. of silage per day, how long will 5 cows be supplied by the fodder in a cylindrical silo 14 ft. in diameter and 35 ft. high? (Why is silage used in feeding cattle?)

26. How is the borrowing and lending of money aided by the giving of a written note by the borrower to the lender? Why is this note sometimes a time note and sometimes a demand note?

27. The sounding wire used by Peary, the Arctic explorer, in his last voyage was made of steel and was .028 in. in diameter. How much would 1000 fathoms of it weigh? (1 cu. ft. of steel weighs 490 lb.)

28. Two ships start from a certain buoy at the same time. One sails due north at the average rate of 8.4 miles per hour and the other due east at the rate of 7.5 miles per hour. How far apart will they be at the end of 5 hours?

29. Taking the moon to be 2000 miles and the earth 8000 miles in diameter, find in the shortest way the ratio of their surfaces.

Also of their volumes. If these bodies were once of the same temperature, which would be likely to cool off the sooner?

30. In buying a bond as an investment, why should the date at which the bond matures be considered?

EXERCISE 139

ORAL

Using $\frac{22}{7}$ instead of 3.1416, give:

1. The circumference and area of a circle whose radius is 14 in.

2. Of a circle whose diameter is 14 in.

3. The lateral surface of a cylinder whose radius is 7 ft. and altitude 10 ft.

4. The volume of this cylinder.

5. The lateral surface and volume of a cone with these dimensions.

6. The surface of a sphere whose radius is 14 in. Whose diameter is 14 in.

7. The volume of each of these.

8. The area of a parallelogram whose base is 8 ft. and altitude 6 ft. Of a triangle having these dimensions. Of a rectangle.

9. The area of a trapezoid whose bases are 10 in. and 14 in. and altitude 1 ft.

10. Find the base of a parallelogram whose altitude is 10 ft. and area 50 sq. ft. Of a triangle having these dimensions.

11. Using the square root sign, state the area of an equilateral triangle whose side is 4 inches. 3 ft. 1 ft. 2 in.

12. Of a regular hexagon whose side is 2 ft. 3 ft. 4 in.

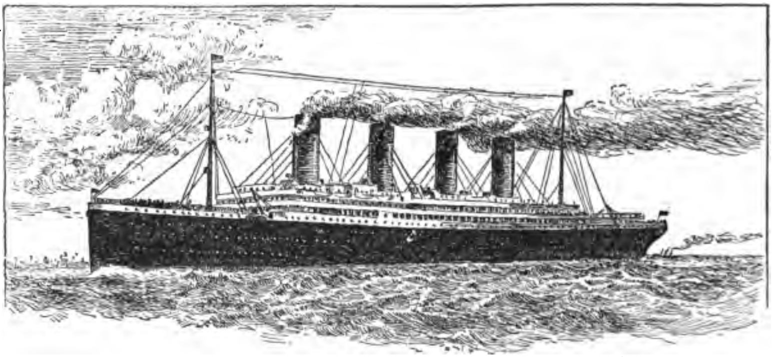
13. Compare the volume of a cube whose edge is 8 in. with the volume of a cube whose edge is 2 in.

14. Compare the volumes of two spheres whose radii are 2 in. and 8 in. Compare their surfaces.

15. How many examples in Exercise 15 (p. 22) can you now work at sight?

EXERCISE 140

INDUSTRY REVIEW: TRAVEL AND COMMERCE



1. A steamer crossed the Atlantic Ocean by a route 3100 miles long in 4 da. 11 hr. 30 min. What was her average rate per hour? What was her rate in knots per hour, if 1 knot = 1.15 miles?

2. If a steamer travels 673 miles in one day, what is her rate in miles per hour? In knots per hour?

3. If the distance between New York and San Francisco by rail is 3250 miles and the schedule time of a train between the two places is 103 hours, by what per cent does the rate of the train per hour exceed the rate of the steamer in Ex. 1?

4. In 1856 the record for the voyage across the Atlantic Ocean by the route referred to in Ex. 1 was 9 da. 1 hr. 45 min. How much less than half of this is the time given in Ex. 1?

5. The steamer referred to in Ex. 1 is 790 ft. long, 88 ft. wide, and 60 ft. deep. If we reckon $\frac{5}{8}$ of the products of her dimensions as the space available for freight, and if 40 cu. ft. equals 1 ton, how many tons can she carry?

6. State how you would visualize a steamer of the size mentioned in Ex. 5.

7. A passenger pays $70 as fare each way across the ocean, spends $10 each way in extras on the ship, and spends 97 days in Europe at an average cost of $9.50 per day. What are his expenses for the trip? Name some of the ways in which he may expect to get a return for this outlay.

8. In the year 1800 the sum of the exports and imports of the United States was $160,244,000, and the population was 5,308,409. What was the per capita foreign commerce?

9. In the year 1900 the sum of the imports and exports of the United States was $2,244,000,000 and the population was 76,303,387. Find the foreign commerce per capita in this year.

10. Find the foreign commerce per capita in the year 1910, when the population was 92,174,000, and the foreign commerce totaled $3,301,932,150. Why is foreign commerce carried on, and why should it increase from year to year?

11. In one year 1,200,000 immigrants arrived in this country. This is equal to how many towns of 5000 inhabitants each? Of 25,000 each? To how many states of the population of Nevada?

12. The following table gives facts concerning tonnage in a recent year:

For Vessels Clearing Port	*Tonnage*
New York City	10,472,000
Rest of the United States	12,361,000
London	8,192,000
For Vessels Entering Port	*Tonnage*
New York City	11,383,000
Rest of the United States	12,567,000
London	11,223,000

Find the total tonnage of vessels entering and leaving the harbor of New York. Find the same for London. Obtain the difference between the two results.

Make up and work two other examples covering the facts in this table.

13. The internal commerce of the United States in one year was $25,000,000,000, while the foreign commerce was $3,200,000,000. Find the ratio of these two numbers.

CHAPTER XIII

METRIC SYSTEM

183. The **Metric System** of weights and measures is a system based on a certain unit of length called the *meter*, all other units of the system being derived from the meter. In the metric system, the only multiplier or divisor used is 10 or a power of 10.

Such multipliers and divisors form what is called the *decimal scale*.

The **meter** is the length of a certain platinum bar kept in the archives of the International Bureau of Weights and Measures at Sèvres in France. The meter is approximately 39.37 inches long.

It was intended to make the meter equal $\frac{1}{10,000,000}$ part of the distance from the earth's equator to its pole, but the relation is not exact, owing to a slight error in the original computation.

184. Derived Units. Other units are derived from the meter taken as the fundamental unit, in two different ways:

(1) By the use of 10 as a multiplier or divisor.

The size of the units derived thus is indicated by the prefixes, **deka-**, **hekto-**, **kilo-**, **myria-**, meaning 10, 100, 1000, 10000, respectively; and by **deci-**, **centi-**, **milli-**, meaning $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, respectively.

The decimal *divisions* of the unit are expressed by *Latin* prefixes; the decimal *multiples* by *Greek* prefixes.

(2) By making a new unit of a certain portion of space or matter, determined by the metric unit of length.

Thus, the volume of 1 cubic decimeter = the *liter* (pronounced "leeter");
the volume of 1 cubic meter = the *stere* (pronounced "stair");
the weight of 1 cubic centimeter of water = the *gram*.

These new units, taken as fundamental units, may in turn be multiplied or divided in the decimal scale. Hence, we obtain the various tables for metric weights and measures.

In these tables, only the units printed in **black letters** are much used in practice. The other units serve for computation, or other theoretic purposes merely.

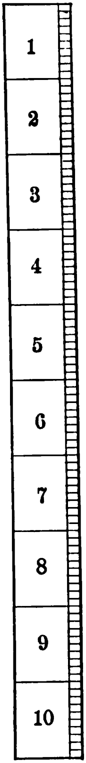
185. The **Chief Advantages in the Metric System** are that

(1) One unit (as of weight) may be reduced to another unit of the same kind merely by moving the decimal point.

(2) The fundamental units of length, capacity, weight, etc., are chosen so as to have a simple relation.

Note that the metric system has the disadvantage that the number 10 is not divisible by 3, as is the number 12, which is used in subdividing a foot into inches.

So great are the advantages of the metric system, that it has come into use in all civilized countries, except Great Britain, the United States, and Russia. It is used in the Department of Medical Service of the United States, and in Cuba, Porto Rico, and the Philippine Islands.



1 Decimeter

In the measurement of the weight of objects and also of their temperatures, velocities, etc., it is found that only approximately accurate results can be obtained. Hence, the quotients obtained by dividing such results by 2, 3, 4, 5, etc., are also all approximate. Hence, in such measurements 12 has no advantage over 10 as the basis of numeration. For this and other reasons, in work of this kind the metric system is used almost exclusively by scientific men in all lands.

METRIC TABLES

186. Measures of Length. The fundamental unit of length is the *meter*.

TABLE OF LENGTH

10 **millimeters** (mm.) = 1 **centimeter** (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 **meter** (m.)
10 meters = 1 dekameter (Dm.)
10 dekameters = 1 hektometer (Hm.)
10 hektometers = 1 **kilometer** (Km.)
10 kilometers = 1 myriameter (Mm.)

EXERCISE 141

On squared centimeter paper

1. Mark a line 1 decimeter in length.

2. Divide this line into millimeters.

3. Mark off a line 57 mm. long. Also one 1.27 dm. long. 137 cm.

4. Out of tough paper cut a tape or ribbon 1 meter long.

5. With this tape, measure the length and width of the room in meters.

6. Find how many centimeters long is a foot rule.

7. Measure your height in meters and centimeters.

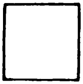
State at sight the number of

8. Meters in a kilometer. In a Dm. 5 Hm.

9. Centimeters in a meter. In a Hm. 8 dm.

10. Millimeters in a cm. In 3 m. 5 dm.

187. Measures of Area. The unit of area is the *square meter.*



Square Centimeter

TABLE OF SQUARE MEASURE

100 **sq. millimeters** (sq. mm.) = 1 **sq. centimeter** (sq. cm.)
100 sq. centimeters = 1 sq. decimeter (sq. dm.)
100 sq. decimeters = 1 **sq. meter** (sq. m.)
100 sq. meters = 1 sq. dekameter (sq. Dm.)
100 sq. dekameters = 1 sq. hektometer (sq. Hm.)
100 sq. hektometers = 1 **sq. kilometer** (sq. Km.)
100 sq. kilometers = 1 sq. myriameter (sq. Mm.)

188. Measures of Land Surface. The ratio, 100, between two successive units of square measure is too great for many practical purposes, as in measuring land. Hence, a new unit, the *are,* is selected, which is increased or decreased by using 10 as a multiplier or divisor.

The unit of land measure is the **are** (pronounced "air"), which equals 100 square meters.

When the multiplier used throughout a table is 10, 10 is said to be the *scale* of the table; similarly, when 100 is the multiplier, the scale is 100, etc.

TABLE OF LAND MEASURE

10 centares (ca.) = 1 **deciare** (da.)
10 deciares = 1 are (a.)
10 ares = 1 dekare (Da.)
10 dekares = 1 **hektare** (Ha.)

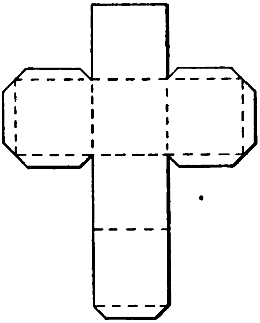
A centare equals 1 sq. m.

189. Measures of Volume. The *cubic meter* is the unit of volume.

TABLE OF CUBIC MEASURE

1000 cubic millimeters = 1 **cubic centimeter** (cu. cm.)
1000 cubic centimeters = 1 cubic decimeter (cu. dm.)
1000 cubic decimeters = 1 **cubic meter** (cu. m.)

Let the pupil make a cubic centimeter by cutting out a piece of pasteboard of the shape indicated in the figure, cutting the pasteboard half through at the dotted lines, then folding together in the shape of a cube and pasting.



190. Measures of Wood, etc. The ratio, 1000, between two successive units in the above table is too great for many practical purposes, as in measuring wood. Hence, a new unit, the *stere,* is taken, which is multiplied and divided according to the scale of 10.

The unit of wood measure is the **stere,** which equals one cubic meter.

Hence, a millistere equals 1 cu. dm.

TABLE OF WOOD MEASURE

10 millisteres (ms.) = 1 **centistere** (cs.)
10 centisteres = 1 decistere (ds.)
10 decisteres = 1 **stere** (s.)

EXERCISE 142

ORAL

How many

1. Square meters in a sq. Hm.? In a sq. Km.? An are?
2. Square centimeters in a sq. m.? In a sq. dm.? 7 sq. m.?
3. Centares in an are? In a Da.? 15 a.?

4. Ares in 25 Ha.? In 500 ca.? 7 ca.?

5. Hektares in 3 a.? In 303 a.? 36 ca.?

6. Square meters in 2 sq. Dm.? In 22 sq. dm.? 22 sq. cm.?

7. Square meters in 5 a.? In 75 a.? 83 Ha.?

8. Cubic centimeters in a cu. m.? In 16 cu. dm.? 38 cu. mm.?

9. Cubic meters in 76 cu. dm.? In 768 cu. cm.? 500 cu. mm.?

191. Measures of Capacity. The *liter* is the unit of capacity. It equals a cubic decimeter; that is, the volume of a cube whose edge is $\frac{1}{10}$ of a meter.

TABLE OF CONTENTS

10 milliliters (ml.) = 1 **centiliter** (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 **liter** (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 **hektoliter** (Hl.)
10 hektoliters = 1 kiloliter (Kl.)
10 kiloliters = 1 myrialiter (Ml.)

Let the pupil construct a liter out of pasteboard.

192. Measures of Weight. The *gram* is the unit of weight. It is the weight of a cubic centimeter of distilled water at its greatest density; that is, when at a temperature of 39.2° Fahrenheit.

TABLE OF WEIGHT

Write out the metric table for weight, by substituting *gram* for *liter* in the table of Art. 191.

EXERCISE 143

Oral

State the number of

1. Liters in a Kl. In a Hl. 5 Dl. 75 dl.
2. Dekaliters in 7 Kl. In 40 Hl. 7 l. 58 dl.
3. Liters in a cubic meter. In 206 cu. cm.
4. Hektoliters in 25 Kl. In 6 l. 44 Dl. 325 cl.
5. Liters in 30 cu. dm. In 6 cu. cm. 45 cu. m.
6. Grams in a Kg. In 5 Hg. 87 dg.
7. Centigrams in 4 g. In 31 Dg. 7 mg. 70 g.
8. Kilograms in 50 g. In $\frac{1}{2}$ g. 7 Dg. 4 dg.
9. Grams in the weight of 1 cu. m. of water. In 7 cu. dm. of water.
10. Kilograms in 30 cu. m. of water. In 16 cu. Km. of water.

Notation, Numeration, Reduction

193. The methods of **Notation and Numeration** used in the metric system have been indicated in the preceding tables. With respect to **notation**, note further that

(1) Of the abbreviations used, those for units larger than the fundamental units begin with capital letters; the others, with small letters.

(2) The place occupied by each unit with reference to the decimal point (when the scale is 10) should be fixed in mind. Thus, in 58723.486, with reference to the meter, the place value of each figure is as follows:

Mm.	Km.	Hm.	Dm.	m.	dm.	cm.	mm.
5	8	7	2	3	4	8	6

(3) In writing a metric number for which the scale is 100, the number of units of each denomination must occupy two places (by the aid of zeros if necessary); when the scale is 1000, three places.

Thus, 35 sq. Km. 8 sq. Hm. 17 sq. m. is written 35080017 sq. m.

With respect to **numeration**, or *reading* metric numbers, note that a metric number may be read in either of two ways:

(1) *By specifying each unit.*

Thus, 37.56 m. may be read as "3 dekameters 7 meters 5 decimeters 6 centimeters."

(2) *By reading the entire number as an abstract number, and then affixing the name of the fundamental unit.*

According to this method, the number in (1) would be read as "thirty-seven and fifty-six hundredths meters."

In Europe, a metric number is often written with the symbol for the metric unit in place of the decimal point.

Thus, 382.64 m. is often written in the form 382m64 and read "382 meters 64 centimeters." Similarly, 15.62 francs is written 15f62 and is read "15 francs 62 centimes."

The period is often omitted after the abbreviations cm., m., km., etc.

194. Reduction. To reduce a metric number to a *lower* denomination in the same table,

Move the decimal point to the right, one place for each reduction of the unit when the scale is 10; two places when the scale is 100; three places when the scale is 1000.

Thus, 6.728 m. = 672.8 cm.

In like manner, to change a metric number to a *higher* denomination, *move the decimal point to the left.*

Thus, 8596.2 mm. = 8.5962 m.

Instead of making a single direct reduction, it is often an advantage to make two steps of the reduction, by

(1) Changing the number of *given* units into *fundamental* units, and
(2) Changing the number of *fundamental* units into *required* units.

Ex. Convert 938765.23 dm. into kilometers.

We first change decimeters into meters, and then change meters into kilometers.

Hence, 938765.23 dm. = 93876.523 m. = 93.876523 Km.

EXERCISE 144

1. How many places to the right (or to the left) must the decimal point be moved to change a number of meters to dekameters? To kilometers? To centimeters?

2. How many places to the right (or left) must a decimal point be moved to change square meters to square centimeters? To square millimeters? To square kilometers? To change sq. Km. to sq. Hm.? Sq. dm. to sq. mm.? Sq. mm. to sq. dm.? Illustrate by examples.

Reduce:

3. 7.324 Km. to dm. To Dm. To cm.
4. 712.45 sq. m. to sq. cm. To sq. Hm. To Ha.
5. 503.217 dl. to ml. To Dl. To Kl.
6. .025 cu. m. to cu. cm. To l. To ds.
7. 3267.1 cg. to Dg. To Kg. To mg.
8. 106.73 dl. to ml. To Dl. To Kl.
9. 46.71 Dl. to cu. m. To cu. cm. To dl.
10. 375.5 a. to Ha. To ca. To sq. Dm.
11. 345.75 m. to cm. To Hm. To Km. To mm.
12. 863,200 sq. cm. to sq. m. To Ha. To sq. Dm.

13. How many Kg. are there in the weight of 75 l. of water? Of 1 cu. m. of water? 170 cu. mm. of water? 35 cu. cm. of water? 11 Kl. of water?

14. How many l. of water are needed to weigh 75 Kg.? To weigh 5000 Kg.? 30 g.? 58.7 cg.? 138.75 Dg.?

15. Change 42.3 Hl. to dl. To cu. cm. To cu. m. Find its weight (if water) in g. In Kg.

16. Reduce 3 rd. 5 yd. 3 ft. 8 in. to inches. Also 10 m. 5 dm. 4 cm. to centimeters. Compare the amount of work in the two processes.

17. Reduce 120 sq. yd. to square rods. Also 10,000 sq. dm. to square meters. Compare the work in the two processes.

18. Make up and work an example similar to Ex. 16. Also to Ex. 17.

Processes with Metric Numbers

195. Addition. Metric numbers may be added in the same way as any other decimal numbers. It is necessary, in all cases, to reduce to the same denomination the metric numbers which are to be added.

Ex. Add 37 cm., 36.26489 Hm., 3 Km.

We reduce to meters all the numbers to be added.

37 cm. = .37 m.
36.26489 Hm. = 3626.489 m.
3 Km. = 3000. m.
6626.859 m. *Sum*

196. Subtraction.

Ex. Subtract 387.92 cg. from 5.827 Dg.

Reducing both numbers to grams and subtracting,

5.827 Dg. = 58.27 g.
387.92 cg. = 3.8792 g.
54.3908 g. *Difference*

197. Multiplication.

Ex. Find the area of a rectangular field which is .5 Km. long and 27.8 m. wide.

.5 Km. = 500 m.
Area = 27.8 × 500 sq. m. = 13900.0 sq. m.
= 1.39 Ha. *Area*

198. Division.

Ex. 1. Divide 21.856 Dm. by 3.2 cm.

21.856 Dm. = 218.56 m.
3.2 cm. = .032 m.
218.56 m. ÷ .032 m. = 6830 *Quotient*

Ex. 2. How high must a box be, in order that it may hold 30 liters, if it is 50 cm. long and 2 dm. wide?

Since 1 l. = 1 cu. dm.,
30 l. = 30 cu. dm. = volume of the box.
Since 50 cm. = 5 dm.,
the area of base of the box = 5 × 2 sq. dm. = 10 sq. dm.
= 30 ÷ 10 = 3
Height of box = 3 dm. *Ans.*

EXERCISE 145

Add:

1. 3.6 m. + 45 cm. + .06 Km. + 3.2 Dm.
2. 9 cu. dm. + 300 cu. mm. + 50.6 cu. m.
3. 43 cl. + 7.1 Hl. + 305 ml. + 2.5 Kl. + 27.8 l.
4. 75.38 Ha. + 438.1 a. + 9587 ca.
5. 12 sq. m. + 200 sq. cm. + 37 sq. mm.
6. 14 l. + 1.403 Kl. + 378.12 cl. + 99 Hl. + 14 dl.
7. 38 sq. Dm. + 19.3 Ha. + 1435 sq. m. + 281.5 a.

Subtract:

8. 7.3 cm. from .08 Km.
9. .46 Hl. from 8769.1 cl.
10. 5.7 sq. dm. from .09 Ha.
11. 3.57 g. from 2.538 Kg.

12. What is the difference between 7 Hm. 5 Dm. 3 m. 4 cm. and 8 Km. 3 Hm. 4 dm. 2 mm.?

13. From 57.3128 Kl. take 4875.625 cu. cm.

Multiply:

14. 3 Dm. 5 m. 4 cm. by 5.

15. 7 Kg. 3 Hg. 2 Dg. 7 dg. 8 cg. by 35.

Find the area of a surface

16. 3.02 m. × 25 cm.

17. 94.5 Km. × 6.8 m.

18. 3.6 mm. × 7.5 Hm.

19. 8.8 dm. × 2.4 Dm.

Divide:

20. 1800 dl. by 90 Dl.

21. 540 Ha. by 6 a.

22. 3.8 cu. m. by 1.9 cu. mm.

23. 77 sq. mm. by 5 sq. Dm.

24. Subtract 3 bu. 3 pk. 5 qt. from 5 bu. 1 pk. 2 qt. Then subtract 3.56 l. from 5.42 l. Compare the amount of work in the two processes.

25. Make up and work a similar example concerning the addition of compound numbers.

26. Multiply 3 gal. 2 qt. 1 pt. by 5. Then multiply 3.14 liters by 5. Compare the amount of work in the two processes.

27. Make up and work a similar example concerning the division of compound numbers.

28. A field containing 56 Ha. is 35 Dm. long. Find its width. If it had been 26 Dm. 5 cm. long, find its width.

29. How many cubic meters in a box 30.6 dm. × 204 cm. × .5 Dm?

30. How many liters in a box 3 m. × 8 dm. × 5 cm.?

31. A vat is 4 m. × 36 dm. × 250 cm. Find its capacity in kiloliters. Find in kilograms the weight of water it will contain.

32. A box 15 dm. long and 80 cm. wide contains a cubic meter. How deep is it?

33. A room is 5.2 m. long, 4.5 m. wide, 3.2 m. high. Find the cost of plastering it at 42 cents per square meter. Find the volume of the room in cubic meters.

34. A bin 7.4 m. × 3.6 m. × 2.5 m. will contain how many hektoliters of grain? How many steres of wood?

35. How many liters in a cube whose edge is 20 cm.?

36. A bar of metal is 3 m. × 2.8 m. × 1.5 m. and weighs 4.5 times as much as an equal volume of water. Find the weight of this bar in kilograms.

37. If a piece of ore weighs 45 Kg., and of this 675 g. are silver, what per cent of the ore is silver?

38. Divide .005 cu. dm. by .02, and express the quotient as a decimal of a cubic meter.

39. A bin which holds 70 Hl. stands on a base 26 dm. × 2 m. How high is the bin?

40. Find the cost of 79.24 meters of silk at 8.75 francs per meter.

41. Find the cost of 72.5 Hl. of grain at 16.85 fr. per hektoliter.

42. If 7.4 Ha. of land cost 2223.7 fr., what does 1 Ha. cost?

43. If a cubic millimeter of normal human blood contains 5,000,000 red corpuscles, and a physician finds that in a patient ill with anæmia only 3,500,000 are present, what per cent is the blood of the patient below normal in this respect?

44. Work again such examples in Exercise 78 (p. 174) as the teacher may select.

METRIC EQUIVALENTS

199. Direct Equivalents. The following table shows the relation between important metric units and the units of weight and measure in common use in the United States and Great Britain. The equivalents printed in black letters are of especial importance.

1 meter	**= 39.37 in. = 1.1 yd.**	(39$\frac{3}{8}$ in. approx.)
1 kilometer	**= .6214 mile**	($\frac{5}{8}$ mi. approx.)
1 sq. meter	= 1.196 sq. yd.	(1$\frac{1}{5}$ sq. yd. approx.)
1 hektare	**= 2.471 acres**	(2$\frac{1}{2}$ A. approx.)
1 cu. meter	= 1.308 cu. yd.	(1$\frac{1}{3}$ cu. yd. approx.)
1 stere	= .2759 cord	($\frac{3}{11}$ cd. approx.)
1 liter	**= 1.057 liquid quart**	(1$\frac{1}{16}$ l. qt. approx.)
1 liter	**= .9081 dry quart**	($\frac{9}{10}$ d. qt. approx.)
1 liter	= 61.022 cu. in.	(61 cu. in. approx.)
1 hektoliter	= 2.8376 bushels	(2$\frac{5}{6}$ bu. approx.)
1 hektoliter	= 26.417 gallons	(26$\frac{2}{5}$ gal. approx.)
1 gram	= 15.432 grains	(15$\frac{2}{5}$ gr. approx.)
1 kilogram	**= 2.2046 pounds av.**	(2$\frac{1}{5}$ lb. av. approx.)
1 metric ton	= 1.1023 ton	(1$\frac{1}{10}$ T. approx.)

200. The **Inverse Equivalents,** showing the value of units in use in the United States and Great Britain in terms of the metric units, are often useful.

1 yd.	= .9144 m.	($\frac{9}{10}$ m.)
1 mi.	= 1.609 Km.	(1$\frac{3}{5}$ Km.)
1 sq. yd.	= .8361 sq. m.	($\frac{5}{6}$ sq. m.)
1 acre	= .4047 Ha.	($\frac{2}{5}$ Ha.)
1 cu. yd.	= .7645 cu. m.	($\frac{3}{4}$ cu. m.)
1 cord	= 3.624 st.	(3$\frac{2}{3}$ st.)
1 liquid qt.	= .9463 l.	($\frac{16}{17}$ l.)
1 dry qt.	= 1.101 l.	(1$\frac{1}{9}$ l.)
1 bu.	= .3524 Hl.	($\frac{6}{17}$ Hl.)
1 grain	= .0648 g.	($\frac{5}{77}$ g.)
1 lb. av.	= .4536 Kg.	($\frac{5}{11}$ Kg.)
1 ton	= .9072 T.	($\frac{10}{11}$ T.)

Ex. 1. Reduce 30 m. to feet.

$$30 \text{ m.} = 30 \times 39.37 \text{ in.}$$

$$= \frac{30 \times 39.37}{12} \text{ ft.} = \frac{393.7}{4} \text{ ft.} = 98.425 \text{ ft.}$$

Ex. 2. Find the area in hektares, and also in acres, of a field 50 Dm. long and 175 m. wide.

50 Dm. = 500 m.
175 × 500 sq. m. = 87500 sq. m.
= 8.75 Ha. (since 10000 sq. m. = 1 Ha.)
= 8.75 × 2.471 A.
= 21.62125 A. *Area*

EXERCISE 146

Change:

1. 5 mi. to kilometers.
2. 56 A. to hektares.
3. 45 gal. to liters.
4. 13 cu. yd. to cubic meters.
5. 60 lb. to kilograms.
6. 30 m. to feet.
7. 72 rd. to hektometers.
8. 5 cu. m. to cubic feet.
9. 40 sq. m. to square feet.
10. 16 mm. to inches.
11. 4 bu. 3 pk. 3 qt. to hektoliters.
12. 3 gal. 2 qt. 1 pt. to liters.
13. 3 T. 15 cwt. 40 lb. to kilograms.
14. 18.32 Hl. to bushels and lower denominations.
15. 7528 Kg. to tons and lower denominations.

Find the weight of

16. 6 cu. m. of water in tons.
17. 1000 gal. of water in kilograms.
18. 1 cu. in. of water in grams.

19. How many cubic yards in a room 3 Dm. long, 25 m. wide, and 120 dm. high?

20. How many tons of water will a cistern 26 m. × 20 m. × 155 dm. contain?

21. If I buy 360 bu. of wheat at $.95 a bushel, and sell it at $2.95 a hektoliter, how much do I gain?

22. A gram is approximately what fraction of an ounce? (Let the answer have unity for a numerator.)

23. Find the dimensions of a baseball diamond in meters.

24. Also of a tennis court.

25. Compare the amount of work in finding the number of gallons in a tank 4 yd. × 3 yd. × 2 ft., with that of finding the number of liters in a tank 4 m. × 3 m. × 60 dm.

26. How many acres in a field 75 m. long and 64 m. wide?

27. Atmospheric pressure is about 1 Kg. per square centimeter. How many pounds is that to the square foot?

28. The inside measurements of a cubical vessel are each 23 inches. How many liters of water will the vessel contain? What will this water weigh in pounds?

29. How many quarts of milk will a box 3 dm. × 24 cm. × 15 cm. contain? How many quarts of corn?

30. I bought a farm of 220 acres at $60 an acre. For what must I sell it per hektare to gain 20%?

31. In a recent year radium cost $3500 per gram. How much was this per grain? Per pound?

32. If a man weighs 150 lb., how many kilograms does he weigh?

33. How many pounds do you weigh? How many kilograms?

34. How many feet and inches is your height? How many meters? What is the length of your forefinger in centimeters?

35. What is the height of the Washington Monument in meters?

36. The Eiffel Tower is 300 m. high. What is its height in feet?

37. A record of 1 m. 54 sec. for the 800-meter race is equivalent to what time for the half-mile run?

38. Find the area of your state in square kilometers.

39. If a meter = 39.37079 inches, show that the difference between 5 miles and 8 kilometers is 51 yd. approximately.

40. Find the duty on 397 m. of linen fabrics, 120 cm. wide (rate 6¢ per square yard, and 30% ad val.), invoiced at 615.35 fr., from which a deduction of 19.85 fr. is to be made (1 fr. = $.193).

41. Make up and work an example similar to Ex. 1. To Ex. 21. To Ex. 25. To two other examples in this Exercise which you think are interesting or instructive.

42. Work again Exercise 20 or similar oral examples concerning fractions.

201. The **Specific Gravity** of a body is the ratio between the weight of the body and the weight of an equal bulk of water.

Since a kilogram is the weight of a liter, or cubic decimeter, of water, the metric system affords peculiar advantages in dealing with problems relating to specific gravity.

Ex. A bar of iron 5 dm. long, 4 cm. wide, and 10 mm. thick, has a specific gravity of 7.8. Find its weight in kilograms and pounds av.

4 cm. = .4 dm.; also 10 mm. = .1 dm.
Volume of the bar = 5 × .4 × .1 cu. dm.
= .2 cu. dm.

Weight of 2 cu. dm. of water = .2 Kg.
" " the bar of iron = .2 Kg. × 7.8 = 1.56 Kg.
= 1.56 × 2.2046 lb. = 3.439 lb. } *Ans.*

EXERCISE 147

1. The specific gravity of lead is 11.3; how many pounds of lead are in a bar 2 m. × 4 dm. × 2 dm.?

2. A bar of iron (sp. gr. 7.8) is 12 ft. × 3 in. × 2 in. Find its weight in kilograms.

3. A bar of steel 35 m. long and 2.6 sq. dm. on the end weighs how many pounds? (Sp. gr. 7.8.)

4. If a bar of metal containing 86 cu. dm. weighs 1267.64 lb., what is its specific gravity?

5. If a cubic meter of cork weighs 660 lb., find the specific gravity of cork.

6. A tank 8 m. × 6 m. × 45 dm. contains 178.2 tons of oil. What is the specific gravity of the oil?

7. A cubic yard of a certain substance weighs as much as a cubic meter of water. What is its specific gravity?

8. What is the specific gravity of air, when the air in a room 7.2 m. × 6.5 m. × 35 dm. weighs 212.94 Kg.?

9. A jar when empty weighs 2.5 Kg., and when full of mercury (sp. gr. 13.5) weighs 121502.5 Kg. Find the capacity of the vessel.

10. An empty dish weighs 3.3 lb., and when full of milk weighs 22.1078 lb. How many liters will the dish contain? (Specific gravity of milk is 1.03.)

11. If a bar of metal (sp. gr. 12.3) is 3 dm. × 6.4 cm. × 55 mm., how many coins, each weighing 4.5 grams, can be made from the bar?

12. A kilogram of gold (sp. gr. 19.36) is beaten into sheets .01 mm. thick. How many square meters will it cover?

13. If sea water is 2.8% salt, and has a specific gravity of 1.025, how many kilograms of salt can be obtained from 1000 cu. m. of sea water?

14. A rectangular block of a certain alloy is 3.4 m. × 16 dm. × 25 cm. If the specific gravity of the alloy is 15, and the block contains 6.12 Kg. of brass, what per cent of it is brass?

15. A rectangular vessel, 10 cm. wide and 3 cm. deep, contains 3 Kg. of sea water (sp. gr. 1.025). How long is the vessel?

16. A mixture is formed by diluting 1 l. of nitric acid with 2 l. of water. If the specific gravity of the acid is 1.5, what is the weight of the mixture? What per cent of this weight is nitric acid?

17. Work again Exercise 82 (p. 187).

EXERCISE 148

ORAL

In answering the questions of this exercise, use the following approximate equivalents:

1 meter = $39\frac{3}{8}$ in. = $1\frac{1}{10}$ yd.
1 liter = $\frac{9}{10}$ dry qt. = $1\frac{1}{16}$ liquid qt.
1 gram = $15\frac{2}{5}$ gr. = $\frac{1}{30}$ oz.
1 Km. = $\frac{5}{8}$ mi. 1 Ha. = $2\frac{1}{2}$ A.
1 Kg. = $2\frac{1}{5}$ lb. 1 Hl. = $2\frac{5}{6}$ bu.

Tell how many

1. Pounds in 100 Kg. 50 Kg. 30 Kg. 5 Kg. $2\frac{1}{2}$ Kg. 4 Kg. 3 Kg. $\frac{3}{4}$ Kg. 1.5 Kg.

2. Yards in 100 m. 50 m. 30 m. $2\frac{1}{2}$ m. 1.5 m. 5 m. $\frac{2}{3}$ m.

3. Inches in $\frac{1}{2}$ m. $\frac{1}{10}$ m. 3 dm. 2 cm.

4. Miles in 10 Km. 20 Km. 100 Km. 120 Km. 2 Km. $\frac{1}{2}$ Km. $\frac{3}{4}$ Km. 2.5 Km.

5. Dry quarts in 10 l. 20 l. 30 l. 5 l. 4 l. 8 l. Liquid quarts in the same.

6. Acres in 10 Ha. 20 Ka. 4 Ha. 2 Ka. $\frac{1}{2}$ Ha. 1.5 Ha. $1\frac{3}{4}$ Ha. $\frac{3}{8}$ Ha. .125 Ha.

7. Ounces in 100 grams. 60 g. 5 g. 3 g. $2\frac{1}{2}$ g. 1.5 g.

8. Bushels in 12 Hl. 6 Hl. 18 Hl. 10 Hl. 1.2 Hl.

9. Yards in 40 m. Dry quarts in 40 l. Pounds in 40 Kg. Bushels in 30 Hl. Ounces in 30 grams. Miles in 24 Km.

10. Draw on the blackboard lines which you think are 1 m., 1 cm., 1 dm. long, and test the length of each line by actual measurement.

11. About how many liters are there in a peck? In a bushel? In a gallon?

EXERCISE 149

GENERAL REVIEW

1. Write a number containing 12 places but not more than 4 different digits.

2. Write in words each of the following:

(*1*) 200,202 (*2*) XCIX (*3*) 327 (*4*) 205,000,217 (*5*) 300,000,000

3. Multiply 43.6 by .325. Also divide .003133 by .00104.

4. Perform each of the following operations:

(*1*) $2\frac{1}{2} \times \frac{2}{3}$ (*3*) $40 \times 5\frac{5}{7}$ (*5*) $2.12 \div .008$
(*2*) $17 \div \frac{3}{4}$ (*4*) $2\frac{1}{2} \div 3\frac{3}{4}$ (*6*) $.006 \times .01\frac{2}{3}$

5. Reduce 1,672,000 seconds to minutes, hours, and days.

6. Find the cost, at $27 per M, of 150 boards each 16 ft. long, 10 in. wide, and $\frac{1}{2}$ in. thick.

7. Reduce $\frac{360}{432}$ to its lowest terms. Also express .0825 as a common fraction.

8. A dealer bought 400 tons of ice for $950. He sold the ice at the rate of 30¢ per hundred pounds. Find out how much money he gained. Also what per cent he gained.

9. Find the interest on $250 for 2 years 5 months and 18 days at 6%. Also at $4\frac{1}{2}$%.

10. On March 17, 1913, Walter Harris bought of the James C. Williamson Co. an automobile for $4500. For it he paid $3000 cash, and for the remainder of the cost gave a promissory note for 3 months, with interest at 5%. Write the note and find the amount due on it at maturity.

11. How much tax must a man pay, if he owns a house assessed at \$3500, a factory assessed at \$12,750, and a store assessed at \$6550, the tax rate being \$1.75 per thousand?

12. Make a receipted bill of the following items bought in Morganville, Oklahoma, by Walter Johnson, from Smith and Adams, on Jan. 15, 1913: 25 lb. sugar @ $5\frac{1}{2}$¢; 20 lb. lard @ $8\frac{1}{2}$¢; 6 bu. potatoes @ 78¢; 3 gal. 2 qt. molasses @ 60¢.

13. Three boys formed a partnership to buy and sell vegetables. Walter invested \$40, Howard \$55, and Ernest \$60. In one month they made \$32.50. What was each partner's share of the profits?

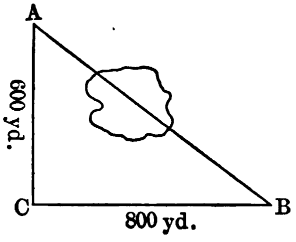
14. Extract the square root of 403,425.

15. How many tons of green fodder will be held by a cylindrical silo, 12 ft. in diameter and 35 ft. high, if 60 cu. ft. of fodder make one ton?

16. A note for \$460.25 dated April 7, 1910, with interest at 5%, was paid on Jan. 20, 1913. Find the sum of the principal and the interest on the note.

17. A farmer sold 250 bu. of potatoes at 75¢ a bushel, 14 doz. eggs at 32¢ a dozen, and 45 lb. of butter at 32¢ a pound. He invested $\frac{1}{5}$ of the proceeds in coal and deposited the remainder in the bank. How much money did he deposit?

18. A man bought a house for \$4000, and during the first year spent \$65 for repairs, \$70 for taxes, and \$8.75 for insurance. He rented the house during the year for \$36 a month. What per cent of his investment did he get back during the first year?



19. On the diagram, A is a house, and B a spring of water, the two being separated by a swamp. The line AC is perpendicular to BC. How long a pipe, laid through the swamp, will be needed to connect the house with the spring?

20. Find the cost of carpeting the floor of a room 27 ft. by 18 ft. with a carpet $\frac{3}{4}$ yd. wide at $1.15 a yard.

21. If the capital of a certain stock company is $80,000 and the profits for a given year are $8400, how much profit will there be on each $100 of the stock?

22. A merchant bought notebooks at $1.80 per dozen and sold them at 20¢ each. What per cent did he gain?

23. A man insured for one year a house valued at $4500 for $\frac{4}{5}$ of its value, at $\frac{3}{4}\%$. What was his loss, if his house burned down during the year? What was the company's loss?

24. How much more would it cost to build a fence around a square lot each side of which is 50 rd., than around a circular lot whose diameter is 50 rd., if the fence costs 80¢ per rod?

25. On Jan. 1, a merchant had $213.24 in the bank. At different times during the month he deposited in the bank $75.15, $82.23, $120, $67.42, $23.72, $155.05, $101.42, $93.52, $227.43, $65.23, and $48.76. Also at different times he took out $120, $23.15, $82.93, $74.13, $125.15, $65.23, $78.47, $59.62, $18.43, and $13.72. How much did he have in the bank on Feb. 1?

26. A bin is 5.5 m. long, 12.5 dm. wide, and 2 m. deep. How many hektoliters of grain will it hold?

27. At $14 a ton, how much would a farmer receive for four loads of hay which weighed respectively 1990, 1750, 2017, 1643 lb.?

28. Find the value of $37 \times 42 + 86 - (738 - 528 \div 4) + 19 \times 17 - 1300$.

29. Subtract the sum of $9\frac{3}{5}$, $8\frac{7}{8}$, and 4.9 from the sum of $7\frac{5}{12}$, 8.3, and $10\frac{1}{15}$.

30. A merchant marks goods 20% above cost, and sells them $12\frac{1}{2}$% below the price as marked. His gain is what per cent of the cost?

31. What is the difference in time between two cities whose longitudes are 90° 15′ 16″ W. and 30° 20′ 14″ E. respectively?

32. The expenses of a town for a year are $7324 and the balance in the treasury is $696. There are 468 polls to be assessed at $1.25 each, and the taxable property amounts to $1,965,000. Find the rate of the town tax.

Besides the town tax, there is a county tax of $1\frac{1}{4}$ mills and a state tax of $\frac{3}{4}$ mill on every dollar of taxable property. Mr. A. pays one poll tax and has property worth $28,970. Find his total tax.

33. After I sold $\frac{5}{8}$ of my crop to one man, and $\frac{3}{4}$ of the remainder to another, there were 186 barrels left. How many barrels were there in the crop?

34. A man sold a carriage for $234, and by so doing lost 10%. What should he have sold it for in order to gain 10%?

35. In the Louisiana Purchase, the United States acquired 875,000 square miles for $15,000,000. What was the cost per acre?

36. Light travels 186,300 miles per second. How many times would it go around the world in one second, taking the circumference of the earth as 25,000 miles? (Compute to two decimal places.)

37. If there are 78,000,000 acres of swamp land in the United States, which can be drained at a cost of $10 per acre and which will thereby be increased $100 an acre in value, how much would be gained by draining this land?

38. During a certain rain, a roof 32′ × 20′ collected 800 gal. of water. How much would be collected by a similar roof 64′ × 18′?

39. Make up and work an example showing the short way of multiplying by $12\frac{1}{2}$. By $37\frac{1}{2}$. By $66\frac{2}{3}$. By $62\frac{1}{2}$.

40. Make up and work an example showing the short way of dividing by $33\frac{1}{3}$. By $87\frac{1}{2}$. By $12\frac{1}{2}$. By 25.

41. 1 cubic foot of water weighs 62.426 lb. Soft iron is 7.21 as heavy as water, while steel is 7.83 times as heavy as water. Find, in the shortest way, the difference between the weight of 82.7 cu. ft. of steel and the same volume of soft iron.

42. As we go toward the center of the earth, the temperature increases 1° for every 62 feet, beginning with a mean temperature on the surface of the earth of 47°. At this rate of change, what is the temperature at the bottom of a boring 6573 ft. deep?

43. If railroad fare is 2.4¢ a mile, $10 will buy a ticket for how many miles?

44. A man in a balloon has attained a height of 33,000 ft. above the earth's surface. By what decimal of a mile does this exceed the altitude of Mt. Everest?

45. By the use of a simple example, explain the reasons for the rule used in multiplying one fraction by another. For dividing one fraction by another.

46. Find the interest on $5750 for 75 days at 5%, using 1 yr. = 360 da. Also find the exact interest. Find the difference between the two results.

47. If .9% of the atmosphere is argon, how many cubic feet of air must be taken to get 1 cu. ft. of argon?

48. Make up and work two examples concerning the ratio of the area of your state to that of other states or of the United States.

49. How many square inches of leather are needed to cover a baseball $2\frac{7}{8}$ in. in diameter?

50. What are the two ways of dividing one fraction by another? What is the advantage in the method in which the divisor fraction is inverted?

51. Find the weight of a barrel of flour in kilograms.

52. How many tons of hay, worth $12.75 a ton, must be given in exchange for 16 tons of coal, worth $6.50 a ton?

53. A portion of a breakwater is to be 120 ft. long, 54 ft. deep, 15 ft. thick, and is to be built of the largest possible cubical blocks. Find the size of one of these blocks.

SUG. The edge of the required block must be an exact divisor of the three given numbers.

54. By buying coal at $6 a ton on May 1, instead of Nov. 1, I save 60 cents on a ton in the price. How much do I save on 25 tons, allowing interest at 5% on the money invested?

55. If 1540 lb. of coke are obtained from 2240 lb. of coal, how many long tons of coal will be needed to produce 5665 lb. of coke?

56. The span of the Brooklyn Bridge is 1595.5 ft. long. What decimal part of a mile is this?

57. Make up and work an example showing the advantage in reducing a number of fractions to their L. C. D.

58. In the year 1910 the amount of the exchanges at the New York Clearing House was $163,704,000,000, while the balances paid in money were $3,932,621,000. The latter number is what per cent of the former?

59. Discuss the relative advantages in the use of decimal and common fractions, and illustrate by examples.

60. A balloon once rose a distance of 6 miles in $1\frac{1}{2}$ hours. During this rise, the temperature changed from 98° to 65°. Find the change in temperature per minute and per 100 ft.

61. Two steamers sail a given distance in 2 da. 1 hr. and in $3\frac{1}{2}$ da. respectively. What is the ratio of their speeds?

62. A merchant bought goods listed at $479.80, from which the following discounts were allowed: 20%, 5%, $1\frac{1}{2}$%. What was the net amount of the bill?

63. A man owns a field 660 ft. long and 150 ft. deep (wide). How many acres are in the field? Into how many lots, 33 ft. wide and 150 ft. deep, can it be divided? Draw a diagram showing the field divided into lots.

64. At one time, a single ranch in Texas contained 1,000,000 acres. How many square miles was this? How would you visualize 1,000,000 acres?

65. How great would the population of the United States be if the whole country was as densely populated as Pennsylvania in the year 1910? As Rhode Island?

66. The railroad up Pike's Peak has a rise of 7552 ft. in 8 miles, while that up Mt. Washington has a rise of 3596 ft. in 3 miles. Draw a diagram showing the rise per mile on each of the roads.

67. A certain railroad is 2512 miles long and its capital consists of $25,000,000 common stock, $10,000,000 preferred stock, and $15,000,000 bonds. What is its capitalization per mile?

68. Give three examples to show how the measurement of an area may be shortened into the measurement of straight lines.

69. One of the big trees in California is 112 ft. in circumference. Find its diameter.

70. Two trains whose lengths are 185 ft. and 211 ft. are traveling in opposite directions at 25 and 35 miles per hour respectively. How long will it take them to pass each other?

71. A broker borrowed $8000 in Boston at 4% and bought a

western mortgage which paid 6½%. How much did he make in 5 years? (Compute in the shortest way.)

72. A man left an estate of $72,120. He willed ⅓ of it to his wife, ⅛ to each of three children, and one half of the remainder to a hospital. By a written analysis, show how much the hospital received.

73. Discuss the advantages and disadvantages in the metric system, giving illustrations.

74. The pyramid of Cheops is 746 ft. square at the base and 480 ft. high. What is the stone in it worth at $1 per perch?

75. An Atlantic steamer makes her passage in 5 days, if she maintains a speed of 22½ knots per hour. What must be her average speed per hour, if her time is to be reduced to 4½ days? (Solve by use of an inverse proportion.)

76. The St. Gothard Tunnel is 14.9 Km. long. How many miles long is this? (What is the St. Gothard Tunnel?)

77. What discount, in addition to a discount of 40%, will reduce to $216 a bill whose original amount was $400?

78. On Nov. 1, an electric-light meter read 7862 kilowatts. On Dec. 1, the reading was 8012 kilowatts. What was the light bill for November, at 8¢ per kilowatt?

79. The area of North America is 6,446,000 square miles. What per cent of North America is the area of the United States proper?

80. A New York merchant sold 25,000 centals of wheat, at $1.62 per cental, to a merchant in Bordeaux, France, and drew on him for the proceeds at 5.19½. Find the face of the draft.

Sug. For the meaning of 5.19½ in this connection, see Art. 131 (p. 252).

81. The naked eye can see an object $\frac{1}{250}$ in. in diameter. Aided by the microscope, it can see an object $\frac{1}{100,000}$ in. in diameter. Find the ratio of these fractions.

82. State the different ways of sending money from one part of the country (or world) to another, and also state the advantages and disadvantages of each method.

83. Between Dec. 21 and June 21, the sun moves 47° north in the sky. If the diameter of the sun is 30′, by what fraction of its diameter does the sun change its position each day, on the average?

84. If a bullet travels 3 miles, with an average velocity of 2000 feet per second, how many seconds will the bullet arrive ahead of the report of the gun? (See p. 388.)

85. Sulphur melts at a temperature of 231°, lead at 617°, and iron at 2800°. Find the ratio of each pair of these numbers.

86. A given fertilizer contains 1.6% of nitrogen, 7% of available phosphoric acid, and 2% of available potash. What is the value of the fertilizer per ton, if nitrogen is worth 15¢ a pound, phosphoric acid 5¢, and potash 5¢?

87. If the atmosphere is 75.95% nitrogen, 23.1% oxygen, .9% argon, and .05% other elements, find the ratio of each pair of these constituents.

88. If dynamite is 290 parts clay, 73 parts nitroglycerine, and 2 parts coal dust, how many pounds of each constituent are there in a ton of dynamite?

89. If in laying a certain pavement 100 flagstones 6 ft. long are needed, how many stones 5 ft. long would be required?

90. A bone in a fossil horse is 4 in. long, and the corresponding bone in a modern horse 4′ 9″ high is 6 in. long. How high was the fossil horse, if the two animals were of the same shape?

91. If a salesman is selling $40,000 worth of goods a year, which is more profitable to him: a commission of 5% on all sales, or a salary of $15 a week and a commission of 2%?

92. If the velocity of light is 186,000 miles per second, and it takes light 8.6 years to travel to the earth from Sirius, the brightest star in the heavens, how many miles is Sirius from the earth?

93. If a horse 14 hands high weighs 800 lb., what should be the weight of a horse 17 hands high?

94. A farmer has a rectangular wheat field 80 rods long and 60 rods wide. If he cuts the wheat on a strip 8 rods wide around the outside of the field, how many acres does he cut?

95. Four notes for $750 each, given on March 1, are due on August 1, October 8, November 20, and December 5, respectively. On what single date might the entire sum be paid, without loss and without payment of interest?

96. At one store steam-heating apparatus for a house may be had for $650 with 20% and 5% off, and at another store for $750 with 30% and 10% off. At which store will the cost be less?

97. In order to determine the length of wire in a coil, the following operations were performed: (1) The coil was weighed; (2) a piece of wire was cut off and its length measured; (3) the piece cut off was weighed. If the weight of the coil was 976.35 grams, the weight of the piece was 1.03 grams, and the length of the piece was 1.005 cm., find the length of the coil.

98. The weight of a given body in air is 819.324 grams, and in water is 692.456 grams. Find the specific gravity of the body.

99. The height of the mercury in a barometer decreases as the elevation of the barometer above the sea level increases. Change the following table to a table giving the height of the barometer above the sea level in inches, and the elevation in feet:

Elevation in meters above sea level	0	100	200	300	400	500
Height of barometer in millimeters	760	751	742	733	724	716

100. A steam engine used in pumping water from a coal mine is found to consume an amount of coal proportional to the product of the amount of water pumped up and the depth from which the water is raised. 112 lb. of coal are burned in pumping 9600 gal. of water from a depth of 600 ft. How much coal will be burned, when 12,000 gal. are raised from a depth of 750 ft.?

101. What is the weight of 20 ft. of iron pipe whose outside and inside diameters are 9 in. and 7.8 in. respectively, if a cubic foot of iron weighs 450 lb.?

102. On a map drawn to the scale of 1 inch to a mile, a certain estate covers an area of 2.44 square inches. How many acres does the estate contain?

103. The population of a certain city in a given year was 63,376. This was 4% greater than that in the preceding year, which, in turn, was 5% greater than the population in the year immediately preceding it. Find the population in each of these years.

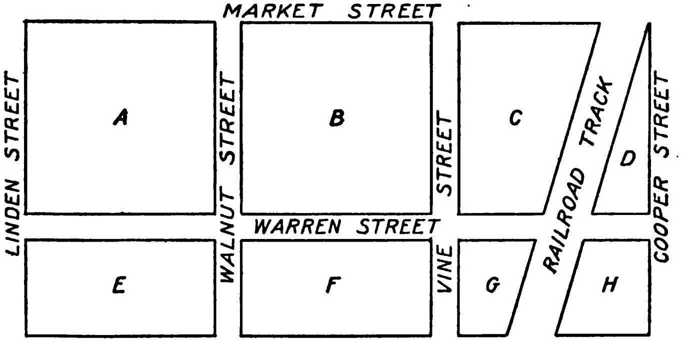
104. Reduce $\dfrac{\frac{10}{3}+\frac{4}{3}-1-\frac{24}{7}\times\left(.8+\frac{1}{9}\right)}{\frac{3}{2}\times\left(\frac{7}{4}-1\right)-\frac{4}{3}\times\left(\frac{1}{7}-.1\right)}$ to its lowest terms.

105. A given household has been using 14 tons of coal a year. If the price of coal is raised 10 per cent, find how many tons less the household must use, so as not to increase its expenditure for coal.

106. Review such of the oral exercises in this book as the teacher may select.

EXERCISE 150

Industry Review: Real Estate and Building



1. Before it was cut up by cross streets or the railroad track, a real estate dealer bought the piece of ground bounded by Linden, Market, and Cooper streets and the lower line of the diagram. If the scale of the drawing is $\frac{1}{4}$ in. to 100 ft., how many acres and what decimal of an acre were in the tract bought?

2. If the dealer paid $15,000 for the land, how much did he pay per acre? Per square foot?

3. At $6\frac{1}{2}$¢ per square foot, how much did the railroad company pay for the land which is marked as Railroad Track?

4. The dealer paid $450 to have the streets surveyed and lots laid out, and $725 for having the streets graded. How many lots would he have to sell, at an average price of $400 per lot, to pay for the cost to date?

5. If each lot was 40 ft. in front and 100 ft. deep, and if lots were first laid out to face on Market and Warren streets, how many lots would be made out of B? How much land would be left in the

middle of B? By using it, how many of the lots facing on Market and Warren streets could be increased in depth by 100 ft.?

6. What is C worth at 15¢ a square foot? Also D?

7. Copy the drawing on page 368, using a scale twice as large as that used in the book.

8. On your drawing, mark out C into lots as best you can, in accordance with the method of Ex. 5.

9. If F is divided into lots as in Ex. 5 and each lot is worth $400 on the average, would it pay the real estate dealer to leave F as a public park, provided that, by so doing, 20 other lots would be increased $125 each in value, on the average?

10. If, in addition, the town should pay him $3000 to leave F as a public park, would it then be profitable for him to do so?

11. A builder bought the lots in B facing on Warren Street. He paid $700 for each of the corner lots, and $600 for each of the others. He built on these lots 10 double houses at a cost of $5250 each. What did the lots and houses cost him?

12. He put a concrete sidewalk 6 ft. wide on the front and side streets at a cost of $1.80 per square yard, and put a curb along the streets at a cost of 27¢ a foot. What did these improvements cost him?

13. If each of the corner houses rents for 20% more than any of the others, what ought each of the houses to rent for per month in order to bring in 10% on the investment?

14. Make up and work three other examples concerning the diagram on page 368.

15. Show the meaning of sills, posts, joists, jambs, siding, studding, girders, plates, by drawing a diagram of the framework of a wooden house sufficiently accurate for the purpose.

16. Obtain a set of plans for a two-story house. Make up and

work examples concerning the painting, plastering, papering, and furnishing of the house.

17. In a given year, the loss due to damage to buildings by fire was \$2.72 per capita in the United States, while in Europe the loss was 33¢ per capita. If the population of the United States in that year was 95,000,000, how much would have been saved, if the loss due to fire had been the same as in Europe?

18. If the Metropolitan Life Building is 700 ft. high, how many stories would this make, counting 14 ft. to a story?

19. If the floor space in it available for rental is 1,085,663 sq. ft., how many acres is this? What is the rental value of the building at \$3.25 per square foot, on the average?

20. If one elevator is required for 25,000 sq. ft. of rental floor space, how many elevators will this building require?

21. If the corridors and unrentable floor space in a building are, in general, 20% of the rentable floor space, on this basis what would be the total floor space of the Metropolitan Life Building?

22. Find the length of the side of a square building which is 10 stories high and is to have the same amount of floor space as the Metropolitan Life Building. (See Ex. 19.)

23. Get the numerical facts concerning some large building or construction work in a neighboring city, as the Pennsylvania Railroad Terminal in New York, the Philadelphia City Hall, the Auditorium at Chicago, or the Union Ferry Depot at San Francisco. Make up and work numerical examples concerning it.

24. Suppose some member of the class purchases a tract of land, lays it out in building lots, and sells a lot to each of the other members of the class. In connection with this process, let the class use actual business methods and carry on numerical work, as in making surveys, deeds, notes, mortgages.

In like manner, let each member of the class build a house on his lot, furnish it, pay taxes, etc.

By including the cost of grading, light, heat, etc., this exercise may be extended so as to include almost any numerical activity or relation of life.

APPENDIX

THE EQUATION

202. An **equation** is an expression of equality between two numbers. In the equations now to be considered, the equal numbers are expressed by the aid of one or more letters.

Thus, $2x + 3 = 12$ is an equation, in which the letter x stands for an unknown number.

The **members** of an equation are the parts on each side of the equality mark.

Thus, in the above equation, $2x + 3$ is the first member, and 12 is the second member.

203. The **Solution of an Equation** is finding the value of the letter or letters in the equation. An equation is solved by using one or more of the following principles:

(1) *If equal numbers are added to (or subtracted from) equal numbers, the results will be equal numbers.*

Thus,

$$\begin{aligned} 7 &= 7 \quad \textit{(equals)} \\ 2 &= 2 \quad \textit{(equals to be added)} \\ 9 &= 9 \quad \textit{(equal results)} \end{aligned}$$

(2) *If equal numbers are multiplied (or divided) by equal numbers, the results are equal numbers.*

Thus,

$$\begin{aligned} 12 &= 12 \quad \textit{(equals)} \\ 12 \div 3 &= 12 \div 3 \quad \textit{(equals divided by equals)} \\ 4 &= 4 \quad \textit{(equal results)} \end{aligned}$$

Ex. 1. Find the value of x in $3x = 12$.

Dividing the members of the equation by 3, we obtain

$$x = 4 \; \textit{Ans.}$$

Ex. 2. Solve $x + 6 = 14$.

Subtracting 6 from each member, we obtain

$$\begin{array}{rcl} x + 6 & = & 14 \\ -6 & & -6 \\ \hline x & = & 8 \ \textit{Ans.} \end{array}$$

Ex. 3. Solve $\frac{2x}{3} - 4 = 5$

Multiplying each member by 3, we obtain

$$2x - 12 = 15$$

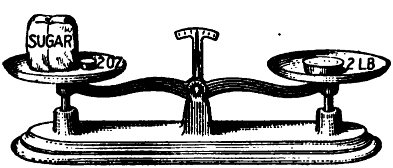
Adding 12 to each member, $2x = 27$

Dividing each member by 2, $x = 13\frac{1}{2}$ *Ans.*

EXERCISE 151

Solve the following equations, stating each principle of Art. 203 when it is used:

1. $x + 4$ in. $= 9$ in.
2. 3 in. $+ x = 8$ in.
3. $x - 3$ ft. $= 10$ ft.
4. $x + 4$ in. $= 2$ ft.
5. $x - 5$ in. $= 1$ ft.
6. $x + 4 = 9$
7. $3 + x = 8$
8. $x - 3 = 10$
9. $x + 4 = 24$
10. $x - 5 = 12$
11. $3x = \$12$
12. $5x = 15$ in.
13. $\frac{1}{4}x = 7$¢
14. $\frac{1}{6}x = \$3.50$
15. $8x = 16$
16. Make up and work an example similar to Ex. 1. To Ex. 3.
17. Also one like Ex. 7. Ex. 10.
18. Like Ex. 11. Like Ex. 13.



19. $x + 2$ oz. $= 2$ lb. From this equation, find x, which represents the weight of the sugar.

20. If a package of coffee and a $\frac{1}{4}$ lb. weight together weigh 3 lb., find the weight of the coffee.

Construct a diagram similar to that of Ex. 19.

21. Make up and work a similar example entirely your own.

Solve the following, stating the principles used:

22. $5x + 4\text{ in.} = 24\text{ in.}$

23. $5x - 4\text{ ft.} = 16\text{ ft.}$

24. $3x + 2 = 20$

25. $1 + 4x = 41$

26. $5x - 7 = 43$

27. $\frac{3x}{4} = \$12$

28. $\frac{4x}{5} = 20$

29. $\frac{5x}{6} = 25¢$

30. $\frac{11x}{3} = 22$

31. $\frac{2x}{3} + 6 = 10$

32. $\frac{2x}{3} - 6 = 2$

33. $\frac{x}{14} = \frac{3}{7}$

34. $\frac{4}{5} = \frac{2x}{3}$

35. $\frac{1}{2}x + \frac{1}{3}x = 20$

36. $\frac{1}{2}x - \frac{1}{3}x = 20$

37. $40 + \frac{1}{2}x = 60$

38. $6x = 20 + x$

39. $6x - 20 = 2x$

40. Make up and work an example similar to Ex. 25. To Ex. 28.

41. To Ex. 29. To Ex. 33.

204. To **Check the Answer of an Equation**, *substitute in each member of the equation the value of x obtained, and observe whether the two members reduce to the same result.*

If, in solving Ex. 38 of Exercise 151, we obtain $x = 4$, we test our result thus:

The first member $= 6x = 6 \times 4 = 24$.

The second member $= 20 + x = 20 + 4 = 24$.

As the two results are the same, 4 is the correct value for x. Check your answers to Exs. 1, 3, 7, 13, 22, and 33 of Exercise 151.

205. Solution of Problems by the Aid of Equations. The method of representing an unknown number by x (or some other letter) and using an equation is an important aid in solving many problems.

Ex. Walter and Harold made \$27 gardening one summer. Walter, who was older and stronger, received a double share of the profits. How much did each receive?

$$\begin{array}{rl}
\text{1 share} & = \text{Harold's part of the profits} \\
\text{2 shares} & = \text{Walter's part of the profits} \\
\hline
\text{1 share} + \text{2 shares} & = \$27 \\
\text{3 shares} & = \$27 \\
\text{1 share} & = \$9\text{, Harold's part} \\
\text{2 shares} & = \$18\text{, Walter's part.}
\end{array}$$

We shorten the work thus:

$$\begin{array}{rl}
x & = \text{Harold's part of the profits} \\
2x & = \text{Walter's part of the profits} \\
\hline
x + 2x & = \$27 \\
3x & = \$27 \\
x & = \$9\text{, Harold's part} \\
2x & = \$18\text{, Walter's part.}
\end{array}$$

About how much shorter is the second process than the first?

EXERCISE 152

1. A man and a boy worked a garden together one summer and made \$256. If the man received three times as large a share of the profits as the boy, how much did each receive?

2. Two girls take in \$21.60 at a refreshment stand. One girl furnishes the materials sold and receives four times as large a share of the profits as the other girl. How much does each girl receive?

3. Make up and work two similar examples.

4. Three boys caught 84 fish together. The boy who furnished the boat received twice as large a share as each of the other two boys. How many fish did each boy receive?

5. Divide 360 in two parts so that one part is twice as large as the other. Three times as large. Four times as large. Five times as large.

6. Shorten the following by the use of x for the unknown number, and find the number:

A number + $\frac{1}{3}$ of the number = 240.

7. Ada and Marion together have 120 post cards and Ada has $\frac{1}{2}$ as many as Marion. How many has each?

8. A farm is worked on shares. As the owner supplies the tools and fertilizers, the tenant receives only $\frac{2}{3}$ as large a share of the profits as the owner. If the profits for one year are $2450, what does each receive?

9. Two men manage a store, and as one of the men owns the building, the other receives only $\frac{3}{5}$ as large a share of the profits as the owner. If the profits for one year are $3600, what does each man receive?

10. A man left an estate of $84,000, of which his daughter was to receive half as much as his wife, and his niece half as much as his daughter. How much did each receive?

11. Separate 240 into two parts such that one shall exceed the other by 32.

SUG. Let x = the smaller part; then $x + 32$ = larger part; and $x + x + 32 = 240$, etc.

12. Two boys together have 180 marbles and one boy has 36 marbles more than the other. How many marbles has each boy?

13. At a certain election 12,784 votes were cast, and one candidate received 1732 more votes than the other candidate. How many votes did each receive?

14. Separate 28 into two parts such that one shall exceed the other by $2\frac{3}{4}$.

15. Make up and work three examples similar to such of the above examples as you think are most interesting and instructive.

EQUATIONS OF TWO UNKNOWN QUANTITIES

206. Elimination is the process of changing two equations containing two unknown numbers into one equation containing one unknown number.

Ex. Solve the following pair of equations for x and y:

$$\begin{aligned} 7x + 2y &= 20 \\ 5x - 2y &= 4 \end{aligned}$$

By Art. 203, we may add the corresponding members of the two equations. We thus obtain

$$12x = 24$$

Dividing each of these equals by 12 (Art. 203),

$$x = 2.$$

If $x = 2$, $7x = 14$.

Hence, we obtain from the first equation

$$14 + 2y = 20.$$

Whence, $y = 3$.

We may prove, or verify, our results by substituting them in the given equations. We obtain

$$14 + 6 = 20, \text{ or } 20 = 20$$

and $10 - 6 = 4$, or $4 = 4$.

Since the substitution makes the members of the two equations identically equal, the equations are said to be *satisfied* by the results obtained; and the values obtained for x and y are proved to be correct.

207. Kinds of Elimination. The process in the example of Art. 206 is called elimination by **addition.**

To solve $\begin{cases} 3x + 5y = 21 \\ x + 5y = 17 \end{cases}$

it would be necessary to use elimination by **subtraction.**

Sometimes it is more convenient to eliminate x than y.

EXERCISE 153

Solve the following for x and y:

1. $3x + 2y = 18$
   $x + 2y = 10$

2. $5x + 4y = 48$
   $3x - 4y = 16$

3. $5x + 4y = 32$
   $5x + 2y = 26$

4. $7y - 2x = 22$
   $3y + 2x = 18$

5. $6y + 5x = 34$
   $4y + 5x = 26$

6. $3x - 2y = 1$
   $2x + 2y = 4$

7. $15x + 6y = 108$
   $4x + 6y = 86$

8. $.7x - .02y = 2$
   $.7x + .02y = 2.2$

9. Make up and work an example similar to Ex. 1. To Ex. 2.

10. Also to Ex. 3. To Ex. 8.

208. Preliminary Multiplication in Elimination is often necessary.

Ex. Find the value of x and y in

$$12x + 5y = 75 \quad \ldots\ldots\ldots\ldots \quad (1)$$
$$9x - 4y = 33 \quad \ldots\ldots\ldots\ldots \quad (2)$$

Multiply equation (1) by 4, and (2) by 5

$$48x + 20y = 300 \ldots\ldots\ldots\ldots \quad (3)$$
$$45x - 20y = 165 \ldots\ldots\ldots\ldots \quad (4)$$

Add equations (3) and (4)

$$93x = 465$$

Divide by 93, $x = 5$ *Root*

Substitute for x its value 5 in equation (1)

$$60 + 5y = 75$$

Subtracting 60 from each member, $5y = 15$

Hence, $y = 3$ *Root*

Let the pupil verify these results.

EXERCISE 154

Solve the following for x and y:

1. $\begin{aligned} 3x - 2y &= 1 \\ x + y &= 2 \end{aligned}$

2. $\begin{aligned} 5x - 3y &= 1 \\ 3x + 5y &= 21 \end{aligned}$

3. $\begin{aligned} 3x - y &= 2 \\ 2x + 5y &= 7 \end{aligned}$

4. $\begin{aligned} 4x + 5y &= 10 \\ 7x + 3y &= 6 \end{aligned}$

5. $\begin{aligned} 3x + 2y &= 60 \\ 2x + 3y &= 60 \end{aligned}$

6. $\begin{aligned} 6y - 7x &= 20 \\ 25y + 12x &= 495 \end{aligned}$

7. $\begin{aligned} 2x + 3y &= 23 \\ 5x - 2y &= 29 \end{aligned}$

8. $\begin{aligned} 5y - 3x &= 2 \\ y + 2x &= 3 \end{aligned}$

9. $\begin{aligned} 2x + 1.5y &= 10 \\ .3x - .05y &= .4 \end{aligned}$

10. $\begin{aligned} \tfrac{1}{2}x + \tfrac{1}{3}y &= 5 \\ \tfrac{1}{3}x + \tfrac{1}{2}y &= 5 \end{aligned}$

209. The **Utility of Equations in x and y in Solving Problems** may be shown by an example.

Ex. 1. Find two numbers whose sum is 21 and whose difference is 3.

Let $x =$ the larger number
and $y =$ the smaller number

Then by the conditions of the problem

$$\begin{aligned} x + y &= 21 \\ x - y &= 3 \\ \hline 2x &= 24 \end{aligned}$$

Adding, $2x = 24$

Dividing by 2, $x = 12$
Hence, $y = 9$

$$\left.\begin{aligned} x &= 12 \\ y &= 9 \end{aligned}\right\} \textit{Ans.}$$

Let the pupil verify these results.

Ex. 2. How many pounds of 20¢ coffee must be combined with how many pounds of 32¢ coffee to make a mixture of 24 lb. worth 28¢ a pound?

Let x = the number of pounds of 20¢ coffee
y = the number of pounds of 32¢ coffee

$$\begin{array}{lrcll}
\text{Hence,} & x + y & = & 24 & \ldots\ldots\ldots\ldots\ (1) \\
 & 20x + 32y & = & 28 \times 24 & \ldots\ldots\ldots\ (2) \\
\text{Multiply (1) by 20,} & \underline{20x + 20y} & \underline{=} & \underline{20 \times 24} & \ldots\ldots\ldots\ (3) \\
\text{Subtract (3) from (2)} & 12y & = & 8 \times 24 & \\
\text{Hence,} & y & = & 8 \times 2, \text{ or } 16 & \multirow{2}{*}{$\left.\right\}\ \textit{Ans.}$} \\
\text{and} & x & = & 8 &
\end{array}$$

EXERCISE 155

Find two numbers

1. Whose sum is 28 and whose difference is 14.

2. Whose sum is \$32.24 and whose difference is \$8.14.

3. Make up and work an example similar to Ex. 2.

4. Find two numbers whose sum is 25, and such that the first number added to twice the second gives 34.

5. Find two numbers such that twice the first added to three times the second gives 13; and also such that four times the second subtracted from seven times the first gives 2.

6. Make up and work an example similar to Ex. 5.

7. 2 lb. of flour and 5 lb. of sugar cost 31¢, and 5 lb. of flour and 3 lb. of sugar cost 30¢. Find the value of a pound of each.

8. A man hired 4 men and 3 boys for a day for \$18; and for another day, at the same rate, he hired 3 men and 4 boys for \$17. How much did he pay each man and each boy for a day?

9. Make up and work a similar example.

10. How many pounds of 18¢ coffee must be combined with how many pounds of 30¢ coffee to make a mixture of 100 lb. worth 20¢ a pound?

11. How many pounds of 40¢ tea must be combined with how many pounds of 75¢ tea to make a mixture of 50 lb. worth 50¢ a pound?

12. Make up and work a similar example.

13. Separate 20 into two parts such that twice one part equals three times the other part.

14. Separate $\frac{7}{6}$ into two parts such that $\frac{5}{2}$ of one part equals the other part.

210. Algebra and Geometry. Other and more difficult problems than the above can be solved by an extended use of letters, symbols, and equations. The study of such topics and methods leads to the branch of mathematics called *Algebra.*

Similarly, the extended study of the use of lines, angles, surfaces, and solids leads to the branch of mathematics called *Geometry.*

IMPORTANT NUMERICAL FACTS

(Material for Examples)

AREAS AND POPULATION (YEAR 1910)

UNITED STATES PROPER	AREA IN SQ. MI.	POPULATION	UNITED STATES PROPER	AREA IN SQ. MI.	POPULATION
Alabama	52,250	2,138,093	**New York**	**49,170**	**9,113,279**
Arizona	113,020	204,354	North Carolina	52,250	2,206,287
Arkansas	53,850	1,574,449	North Dakota	70,795	577,056
California	158,360	2,377,549	Ohio	41,060	4,767,121
Colorado	103,925	799,024	Oklahoma	70,057	1,657,155
Connecticut	4,990	1,114,756	Oregon	96,030	672,765
Delaware	2,050	202,322	Pennsylvania	45,215	7,665,111
Dist. of Col.	70	331,069	**Rhode Island**	**1,250**	**542,610**
Florida	58,680	751,139	South Carolina	30,570	1,515,400
Georgia	59,475	2,609,121	South Dakota	77,650	583,888
Idaho	84,800	325,594	Tennessee	42,050	2,184,789
Illinois	56,650	5,638,591	**Texas**	**265,780**	**3,896,542**
Indiana	36,350	2,700,876	Utah	84,970	373,351
Iowa	56,025	2,224,771	Vermont	9,565	355,956
Kansas	82,080	1,690,949	Virginia	42,450	2,061,612
Kentucky	40,400	2,289,905	Washington	69,180	1,141,990
Louisiana	48,720	1,656,388	West Virginia	24,780	1,221,119
Maine	33,040	742,371	Wisconsin	56,040	2,333,860
Maryland	12,210	1,295,346	Wyoming	97,890	145,965
Massachusetts	8,315	3,366,416			
Michigan	58,915	2,810,173	OUTLYING POSSESSIONS		
Minnesota	83,365	2,075,708			
Mississippi	46,810	1,797,114	Alaska	590,884	64,356
Missouri	69,415	3,293,335	Guam (Ladrones)	210	....
Montana	146,080	376,053	Hawaii	6,449	191,909
Nebraska	77,510	1,192,214	Panama Canal Strip	474	....
Nevada	110,700	81,875	Philippine Islands	115,026	....
New Hampshire	9,305	430,572	Porto Rico	3,435	1,118,012
New Jersey	**7,815**	**2,537,167**	Samoan Islands	77	....
New Mexico	122,580	327,301	....	....	....

In the following list *av.* (for *average*) annexed to a fact means that the given fact varies with different conditions and that a mean or average figure is given. For instance, the velocity of sound increases slightly as the temperature of the air rises.

Also *approx.* (for *approximate*) annexed to a fact means that for certain reasons the given fact cannot be exactly determined and that we must be content with an approximation to the correct value.

Approx. also sometimes means an approximation to an average.

AREA

	SQUARE MILES
United States	3,025,600
North America	6,446,000
Land surface of earth	51,238,800
Entire surface of earth	196,976,970
Great Britain and Ireland	121,391
France	207,054
Europe	3,555,000

POPULATION

United States	92,174,515
New York City	4,766,883
London	6,580,616
The earth	1,520,000,000 (approx.)

RESOURCES (CROPS, ETC., YEAR 1910)

Coal lands in U. S.	400,000 sq. mi. (approx.)
Coal in U. S.	2,500,000,000,000 tons (approx.)
Iron ore in U. S.	15,000,000,000 tons (approx.)
Water-power of Niagara	7,000,000 H. P. (approx.)
Natural water-power in U. S.	75,000,000 H. P. (approx.)
Possible water-power in U. S. (developed by storage dams, etc.)	200,000,000 H. P. (approx.)
Reclaimable swamp lands in U. S.	80,000,000 acres (approx.)
Lands in U. S. reclaimable by irrigation	100,000,000 acres (approx.)
National forest reserves of U. S.	168,000,000 acres
Corn crop of U. S.	3,000,000,000 bushels
Wheat crop of U. S.	700,000,000 bushels
Cotton crop of U. S.	13,000,000 bales
Railways in U. S.	244,084 mi.
Railways of the world	510,175 mi.

Astronomical Facts

Planet	Diameter in Miles	Distance from Sun in Miles
Mercury	3,030	36,000,000
Venus	7,700	67,200,000
Earth	7,918	92,800,000
Mars	4,230	141,500,000
Jupiter	86,500	483,300,000
Saturn	73,000	886,000,000
Uranus	31,900	1,781,000,000
Neptune	34,800	2,791,000,000

Sun's diameter	866,400 miles
Moon's diameter	2,162 miles
Moon's distance	238,850 miles
Distance of nearest fixed star	21 millions of millions of miles, or 3.6 light years
Polar diameter of earth	7901.476 miles
Equatorial diameter of earth	7926.592 miles
Circumference of earth	25,000 miles (approx.)
1 tropical year	365.2422164 days

Dates (a. d. unless otherwise stated)

	Year
Rome founded	B. C. 753
Battle of Marathon	B. C. 490
Fall of Jerusalem	70
Fall of Rome	476
Battle of Hastings	(Oct. 14) 1066
Printing with movable type	1438
Fall of Constantinople	1453
Discovery of America	(Oct. 12) 1492
Jamestown founded	1607
Landing at Plymouth	(Dec. 21) 1620
Declaration of Independence	(July 4) 1776
Washington Inaugurated	(April 30) 1789
Battle of Waterloo	(June 18) 1815
Telegraph invented	(May 27) 1844
First transatlantic cable message	(Aug. 16) 1858
Telephone invented	1876
Battle of Manila Bay	(May 1) 1898

DISTANCES

	MILES
From New York to Boston	234
From New York to Albany	142
From New York to Buffalo	440
From New York to Chicago	912
From New York to Denver	1930
From New York to San Francisco	3250
From New York to Philadelphia	90
From New York to Washington	228
From New York to New Orleans	1372
From New York to Jacksonville	1077
From New York to London	3375
From San Francisco to Manila	4850
From New York to San Francisco via Panama	5240
From London to Bombay via Suez	6332

HEIGHTS OF MOUNTAINS

	FEET		FEET
Mt. Washington	6,290	Mt. Mitchell	6,711
Pike's Peak	14,147	Mt. Whitney	14,501
Mt. McKinley	20,464	Mt. Blanc	15,744
Mt. Everest	29,002	Acongua	23,802

HEIGHTS (OR LENGTHS) OF STRUCTURES

Statue of Liberty (N. Y. Harbor)	151 ft.
Same with pedestal	306 ft.
Bunker Hill Monument	221 ft.
Washington Monument	555 ft.
Metropolitan Life Building	700 ft.
Woolworth Building	750 ft.
Eiffel Tower	984 ft.
Lusitania	790 ft.
Olympic	882 ft.
Span of Brooklyn Bridge	$1595\frac{1}{2}$ ft.
Deepest shaft in world	5000 ft.
Deepest boring in world	6573 ft.
Simplon Tunnel	$12\frac{1}{4}$ mi.
Panama Canal	49 mi.
Suez Canal	100 mi.

LENGTHS OF RIVERS

	Miles		Miles
Hudson	280	Mississippi	3160
Ohio	950	Rhine	850
Colorado	1360	Amazon	3300
Missouri	3100	Nile	3400

RAINFALL (MEAN ANNUAL)

	Inches		Inches
Phœnix (Ariz.)	7.9	New York	44.8
Denver	14	New Orleans	57.4
Chicago	34	Cherrapongee (Asia)	610

RECORDS (YEAR 1910)

100 yard dash	$9\frac{3}{5}$ seconds
Quarter mile run	47 seconds
1 mile run	4 min. $15\frac{3}{5}$ sec.
25 mile run	2 hr. 29 min. $23\frac{3}{5}$ sec.
Running high jump	6 ft. $5\frac{5}{8}$ in.
Running broad jump	24 ft. $7\frac{1}{4}$ in.
Pole vault	12 ft. $10\frac{1}{4}$ in.
100 yard swim	$55\frac{2}{5}$ seconds
1 mile swim	23 min. $16\frac{4}{5}$ sec.
100 yard skate	$9\frac{4}{5}$ seconds
1 mile skate	2 min. 36 sec.
1 mile on bicycle	1 min. 7 sec.
1 mile in automobile	$27\frac{1}{3}$ sec.
1 mile by running horse	1 min. $35\frac{2}{5}$ sec.
1 mile by trotting horse in race	2 min. $3\frac{1}{4}$ sec.
Throw of baseball	426 ft. 6 in.
Transatlantic voyage (from N. Y.)	4 da. 10 hr. 51 min.
Typewriting from printed copy	123 words in one minute
Typewriting from new material	6135 words in one hour
Shorthand	187 words in one minute
Corn crop on 1 acre	$255\frac{3}{4}$ bushels
Corn crop on 1 acre raised by a member of a boys' corn club	$228\frac{3}{4}$ bushels
Milk yield, 1 cow, 1 year	27,432 pounds
Butter yield, 1 cow, 1 year	1,164.6 pounds
Eggs laid by 1 hen, 1 year	268

TEMPERATURES (FAHRENHEIT)

Normal temperature of the human body, 98.7°
Ether boils at 96°
Alcohol boils at 173°
Water boils at 212°
Sulphur fuses at 238°
Tin fuses at 442°
Lead fuses at 617°
Iron fuses at 2800° (approx.)
Temperature of arc light, 5400° (approx.)
Increase of temperature below earth's surface, 1° per 62 ft. (av.)
Decrease above earth's surface, 1° per 183 ft. (av.)

VELOCITIES

Wind	18 miles per hr. (av.)
Sensation along a nerve	120 ft. per sec. (av.)
Sound in the air	1,090 ft. per sec. (av.)
Rifle bullet	2,500 ft. per sec. (av.)
Message in submarine cable	2,480 miles per sec. (av.)
Light	186,000 miles per sec. (approx.)

WEIGHTS AND SPECIFIC GRAVITIES

Weights

Boy 12 years old	75 lb. (av.)
Man 30 years old	150 lb. (av.)
Horse	1000 lb. (av.)
Elephant	$2\frac{1}{2}$ tons (av.)
Whale	60 tons (approx.)
1 cu. ft. air	$1\frac{1}{4}$ oz. (approx.)
1 cu. ft. water	62.5 lb.

Specific Gravities

Air	$\frac{1}{800}$	Stone (average)	2.5
Cork	.24	Aluminum	2.6
Maplewood	.75	Glass	2.6–3.3
Alcohol	.79	Iron (cast)	7.4
Ice	.92	Iron (wrought)	7.8
Sea water	1.03	Lead	11.3
Water	1.00	Gold	19.3
Clay	1.2	Platinum	21.5

Miscellaneous

Number of teeth in human body	32
Heart beats per minute — Frog	10
Heart beats per minute — Man	72
Heart beats per minute — Bird	120
Smallest length visible to unaided eye	$\frac{1}{250}$ inch
Smallest length visible by aid of microscope	$\frac{1}{125,000}$ inch
Accuracy of work in a machine shop	$\frac{1}{1000}$ inch
Accuracy of most refined measurements	$\frac{1}{50,000,000}$ inch
Height of earth's atmosphere	50 miles (approx.)
Dimensions of double tennis court	78′ × 36′
Dimensions of single tennis court	78′ × 27′
Dimensions of football field	300′ × 160′
Standard width of railroad track	$4' \, 8\frac{1}{2}''$

Blank pages are left at the end of the book on which the teacher may have the pupil write other numerical facts which are important in the locality in which the school is situated; as distance from the nearest metropolis to important places; lengths of the nearest large rivers; heights of mountains or high buildings; and important numerical facts relating to local industries. Thus, in agricultural regions, the pupil may record and use facts like the following: 3 gal. milk make 1 lb. butter (av.); the shrinkage on corn in a crib is 3% per month.

Numerical data for an abundance of interesting examples will also be found in the almanacs published by the great metropolitan dailies, as by the *New York World*, or *New York Tribune*.

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Considered as a whole

The books are too bulky.

They are books for teachers, not for pupils.

They are not well put to-gether. They are scrappy. A pupil never knows where he is or what he has accomplished.

The make-up is wretched. The pages are never units, nor are they pleasing to the eye.

The books are made up with no idea of the order of difficulty. Book I is altogether too hard at the beginning and many of the questions in Book III are too trivial to have any consideration.

There is a plausible feature in some of the applied problems, but it will be found that the are not arranged to mee the interests and needs of the children of the several grades for which the problems are written.

They are books that may have a little temporary sale, but they have so many weak points that they cannot hold their place.

Compare Bk I, Chapte I and II (for Grades I and II with the Wentworth-Smith Work and Play with Numbe and the subsequent parts with the corresponding portions of the Wentworth-Smith series, and the weakness of Durell will